The Behaviour of Reactive Power Marginal Prices in an Electricity Spot Market

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"The advantage of the general theory is that it provides a concrete and rigorous starting point, from which successive approximations can be made."

Caramanis et al *[1982, p. 3236]*

May the contents of this thesis be used with wisdom, understanding and compassion when regarding those whose lives may be affected.

Errata

(20 June 1999)

Corrections:

Page 65, third bullet point: "identifing" should be "identifying".

Page 78, second bullet point: "all V^* terms have been removed;" should read "all $\sum_{n \in \mathrm{PvG}}$ terms have been removed;".

Page 143: the equations:

$$
\begin{aligned}
Q_{D3} &= Q_{D3}^{opt} + 4\ \mathrm{MW} & \quad Q_{D8} &= Q_{D8}^{opt} - 4\ \mathrm{MW} \\
Q_{D15} &= Q_{D15}^{opt} + 4\ \mathrm{MW} & \quad Q_{D17} &= Q_{D17}^{opt} - 4\ \mathrm{MW} \\
Q_{D24} &= Q_{D24}^{opt} - 4\ \mathrm{MW} & \quad Q_{D26} &= Q_{D26}^{opt} + 4\ \mathrm{MW}
\end{aligned}
$$

should read:

$$
\begin{aligned}
Q_{D3} &= Q_{D3}^{opt} + 4\ \mathrm{MVAr} & \quad Q_{D8} &= Q_{D8}^{opt} - 4\ \mathrm{MVAr} \\
Q_{D15} &= Q_{D15}^{opt} + 4\ \mathrm{MVAr} & \quad Q_{D17} &= Q_{D17}^{opt} - 4\ \mathrm{MVAr} \\
Q_{D24} &= Q_{D24}^{opt} - 4\ \mathrm{MVAr} & \quad Q_{D26} &= Q_{D26}^{opt} + 4\ \mathrm{MVAr}
\end{aligned}
$$

Figure 6.1, Page 64: The filename '(IMPEDANCE.DAT)' should read '(LINKDATA.DAT)'.

Page 184: The sentence "Often this constrained relative minimum is also the absolute minimum, especially when $f(x, y)$ is a function of generation costs." has not been experimentally substantiated and must be removed.

Addition To Future Work:

Section 8.2.3.3 concluded that the equations of "the pq pricing model and the PvG pricing model are not equivalent". This conclusion was extrapolated to the corresponding equations of the pq–type and PvG–type OPFs. Experiments were used to establish the conclusion. However, the experiments actually only establish that pq–type and PvG–type OPFs can produce numerous valid optimal dispatches. The experiments do not prove the non–equivalence of the two OPF equation sets.

Future work must therefore determine whether or not the two OPF types are equivalent. If the two OPF types are equivalent the pq–type spot market and the PvG–type spot market will also be equivalent (ref. Section 12.3). This implies the two markets can produce identical optimal dispatches and identical spot prices. Note that

OPF equivalence does not imply redundancy of the PvG/pq classification system. The original purpose of this system will remain, to describe the behaviour of OPF power system variables. It will also remain that this classification system indicates whether the focus of the spot market is on independently controlling voltage magnitude or reactive power generation.

Future work should initially investigate the natures of the objective functions and equation sets of the two OPF types. Furthermore, the nature of the underlying constrained optimisation function used by Matpower and QOPF must be investigated. One starting point is to determine whether the different dispatches reflect different constrained relative minima or different absolute minima. Different relative minima would indicate the constrained optimisation algorithm is influenced by initial conditions. This result can mask (but does not prove or disprove) OPF equivalence. Different absolute minima would demonstrate that the two OPF types are not equivalent.

The following investigations have already been made:

- LaGrange multiplier theory finds constrained relative minima [Anton 1988, Section 16.10];
- The Mathworks constrained optimisation function used by Matpower and QOPF may sometimes only give local solutions[1].
- An experiment: QOPF was used to solve the IEEE 30–bus power system. Q_{G2}^{max} was non–binding. Increasing Q_{G2}^{max} from 50 MVAr to 500 MVAr and resolving results in different values of θ_i, P_{Gi} and Q_{Gi} at the third and fourth decimal places.
- The pq pricing model and PvG pricing model are equivalent whe the cost of reactive power generation is zero[2].

[1] 'Optimisation Toolbox User's Guide' (1994), *p. 3–12 'Limitations'*, Matlab Ver. 4.2c; Toolbox Version 1.0d, The Mathworks Inc., 24 Prime Park Way, Natick, MA 01760-1500.

[2] WARD, A. G., 'NODAL2 Training Manual', *Prepared for Transpower New Zealand Ltd, PO Box 1021, Wellington, New Zealand,* May 1999, Chapter 6.

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Most importantly, I give glory and thanks to God for His love and grace. He has brought me through times of severe arthritic attacks, hip replacements and fatigue by supplying me with His strength and a joy inexpressible. Without these, I could not have persevered in order to complete this thesis. He has proved to me beyond any shadow of doubt that all who call upon Jesus Christ His Son will be saved.

CONTENTS

ABSTRACT

This thesis investigates the behaviour of marginal prices for reactive power in a competitive electricity spot market. In the proposed spot market, non–zero unit costs are assigned to the generation of reactive power as a method of paying for reactive power ancillary services. These costs enable reactive power to be optimally dispatched in the same manner as real power. This is unlike previous research, which has only described the behaviour of reactive power marginal prices in spot markets where the unit generation costs of reactive power equal zero.

The theory of Dispatch Based Pricing, proposed by Ring [1995] is used to calculate and describe the behaviour of reactive power marginal prices for this spot market. Dispatch Based Pricing is an *ex post* variant of spot pricing, with the rare ability to accept non–zero unit generation costs for reactive power. It was originally derived from an optimal power flow (OPF) formulation. A new classification system for power system nodes in any OPF formulation is defined to enable the behaviour of reactive power marginal prices to be clearly described. Hence, Dispatch Based Pricing is redefined with respect to this classification system. An OPF is developed to validate this redefined Dispatch Based Pricing model and the marginal prices generated thereby. This OPF accepts non–zero unit generation costs for both real and reactive power, and uses them to optimally dispatch real and reactive power generation.

The mechanisms determining the behaviour of reactive power marginal prices are investigated for optimal and sub–optimal dispatches of an unconstrained power system. Price behaviour is also investigated for optimal dispatches of voltage–constrained and reactive–power–generation–constrained power systems. The implications of this reactive power marginal price behaviour are discussed. It is shown that Dispatch Based Pricing can be used to calculate marginal prices when a load–following generator is used to supply reactive power.

The conclusions regarding the behaviour of reactive power marginal prices are used to propose a spot market with non–zero reactive power unit generation costs, for the South Island section of New Zealand's National Grid. The use of Dispatch Based Pricing to calculate reactive power marginal prices for this spot market is detailed. The effects of this spot market on the operation of this South Island power system are then discussed.

Chapter 1

INTRODUCTION

1.1 REACTIVE POWER SPOT MARKETS

The generation of reactive power has been traditionally viewed as costless. However, this view is changing as deregulation of the power sector continues. Deregulation of a power sector allows for the introduction of competitive electricity markets. But, it must also address the question of responsibility for the ancillary services of system security, power quality, and voltage support [Baughman *et al* 1997a]. These ancillary services and others, depend on reliable sources of reactive power generation. In New Zealand, extraordinary levels of voltage support were required in recent hot weather conditions [Inder 1997]. Such incidents highlight the need for a pricing regime that will compensate those who generate the necessary reactive power for these ancillary services.

The primary aim of an electricity market is to foster competition, so as to encourage efficient use of resources and improve ancillary services to the consumer [Hogan *et al* 1996]. To achieve this aim, any pricing regime used within this market must be able to provide economic signals that reflect both the short–run and the long–run requirements of the power system. This thesis is concerned with 'short–run marginal cost spot pricing', which is designed to encourage efficient day–to–day usage of resources.

A spot pricing regime produces spot prices that announce to the market the marginal costs of power system resources. These spot prices are designed to encourage market participants to consume or generate only what they can afford, or alternatively, what they are willing to sell. In a deregulated electricity market therefore, a spot pricing regime affects the daily dispatch of all resources in the power system.

The mathematical model for any spot pricing regime produces marginal prices (i.e. spot prices) that act as signals to encourage the efficient and economic dispatch of real power. These real power marginal prices are effective signals because the costs of real power generation from all of the generators are described by real power generation cost functions within the model. These cost functions (in various forms) are inherent in all spot pricing models.

Most spot pricing models also calculate reactive power marginal prices. However, almost all models do not accommodate reactive power generation cost functions. This is because reactive power supplied by generators has been traditionally assumed to have a zero unit generation cost. Without reactive power generation cost functions, reactive power marginal prices are insignificant, rendering them ineffective as economic signals. For this reason, reactive power marginal prices have generally been regarded as by–products of the spot pricing model and not utilised as a means of funding ancillary services.

Little is therefore known about the behaviour of reactive power marginal prices because they are not used. This absence of knowledge has not been helped by the lack of published experimental work on this topic. Moreover, any work that has been published is often related to very specific experiments with little generality for the application of reactive power marginal prices to actual spot markets.

1.2 THESIS OBJECTIVES

The central objective of this thesis is to increase the understanding of how reactive power marginal prices behave. In particular, the emphasis is on the behaviour of marginal prices calculated for a dispatch from a hypothetical spot market in which both real and reactive power have non–zero unit generation costs at the generators. That is, the price behaviour is investigated in a spot market where real and reactive power generation cost functions are used to economically dispatch real and reactive power. To achieve this objective, the behaviour of reactive power marginal prices is investigated for optimal and sub–optimal dispatches, unconstrained and constrained dispatches. Once investigated, the implications of this price behaviour are discussed with reference to the South Island section of New Zealand's National Grid.

The behaviour of real and reactive power marginal prices is dependent, to a certain extent, on the spot pricing model used to generate the marginal prices. Dispatch Based Pricing, a variant of spot pricing, is used in this thesis to investigate price behaviour. This is because Dispatch Based Pricing is currently being considered for use in wholesale electricity markets similar to the New Zealand spot market. Hence, the behaviour of reactive power marginal prices in the context of the Dispatch Based Pricing framework is the focus of this thesis. However, the conclusions of this thesis are applicable to reactive power marginal price behaviour in general.

The Dispatch Based Pricing framework was proposed by Ring [1995] (see also Read and Ring [1995d]). It has two features that make it particularly suited to the research described in this thesis. First, it is one of the few spot pricing regimes capable of accommodating reactive power generation cost functions when calculating marginal prices for real and reactive power. Second, any spot pricing model developed using Dispatch Based Pricing is highly transparent. This makes it possible to identify which marginal costs of the power system are dictating the behaviour of real and reactive power marginal prices.

Read and Ring developed Dispatch Based Pricing using linear programming optimisation techniques. In the context of Power Systems Engineering, these techniques equate to optimal power flow (OPF) techniques. However, the Dispatch Based Pricing equations do not conform to Power Systems Engineering practises, even though they are used to calculate real and reactive power marginal prices. Specifically, the OPF–type Dispatch Based Pricing equations have been formulated using power–flow node classifications (i.e. PV, PQ, S). This results in a conflict of definitions, making the equations very difficult to use. Therefore, an initial objective in this thesis is to redefine Dispatch Based Pricing in terms of OPF node nomenclature, developed specifically for this purpose. This OPF node classification system is developed because of the apparent absence of any sort of OPF node nomenclature in literature.

Transpower New Zealand Ltd has implemented the Dispatch Based Pricing equations in a software package called NODAL2. This software therefore, is used to calculate the reactive power marginal prices presented in this thesis. Dispatch Based Pricing (and NODAL2) is based on OPF techniques. Therefore, OPF software is used to generate benchmark prices required to establish the reliability of real and reactive power marginal prices from NODAL2. This OPF has to be capable of accepting both real and reactive power generation cost functions, to calculate marginal prices that match the marginal prices from NODAL2.

The search for an OPF capable of accepting reactive power generation cost functions proved fruitless. This was due to the common assumption that reactive power generation must be costless. Hence, a benchmark OPF was developed for this work, where development consisted of modifying an OPF so that it will accept reactive power generation cost functions. Development was undertaken with the permission and assistance of the OPF authors from PSERC of Cornell University[1].

By validating the NODAL2 marginal prices using this modified OPF, the redefined Dispatch Based Pricing equations are also validated. The validation demonstrates that the equations are consistent with Power Systems Engineering practises. Until these processes of redefinition and validation have been performed, it is not possible to adequately fulfil the central objective of this thesis.

1.3 THESIS OUTLINE

The main body of this thesis can be divided into four sections: Chapter 2; Chapters 3 to 5; Chapters 6 to 8; Chapters 9 to 11.

[1]PSERC is an acronym for the Power System Engineering Research Center.

Chapter 2 documents the development of spot pricing from its origins through to spot pricing models that include reactive power generation cost functions. An emphasis is placed on spot pricing models that produce marginal prices for reactive power. This historical development is applied to spot pricing in its most general sense. That is, not being limited to the philosophy or operating conditions of any one country. The purpose of this is two–fold: to identify the current understanding of how reactive power marginal prices are generally perceived to behave and to place Dispatch Based Pricing in the wider context of spot pricing. A history of electricity pricing in New Zealand and its current spot pricing market are also presented.

The general methodology of Dispatch Based Pricing along with other general power system theory are presented in Chapter 3. The new OPF node classification system is also presented in Chapter 3.

Dispatch Based Pricing is derived from an OPF formulation. Hence, Chapters 4 and 5 are used to redefine Ring's Dispatch Based Pricing model with respect to the OPF node nomenclature. In Chapter 4 a redefined version of Ring's OPF formulation is presented. This OPF formulation is then linearised to form the linear program required for deriving the Dispatch Based Pricing model. Also in this chapter, the concept of linear programming optimisation is summarised with respect to Dispatch Based Pricing.

Chapter 5 is used to derive a Dispatch Based Pricing model (referred to as the pq/PvG pricing model) from the OPF linear program of Chapter 4. This derivation is based on linear programming duality theory. Hence, duality theory is described here. In addition, Dispatch Based Pricing is described in the context of using unit generation costs to economically (i.e. optimally) dispatch reactive power. Even Read and Ring assumed zero unit costs and thus focussed their discussions primarily on using Dispatch Based Pricing only for real power spot pricing.

From Chapter 6 onwards, all work with respect to Dispatch Based Pricing is original. In Chapters 6 to 8, the new Dispatch Based Pricing model and its software implementation (called NODAL2) are validated. In Chapter 6, the benchmark OPF and its software implementation (called QOPF) are presented. The work of Chapter 7 onwards was only possible after the development of the QOPF software.

The Dispatch Based Pricing model is identified in this work as the combination of two other Dispatch Based Pricing models. These are presented in Chapters 7 and 8 respectively with examples of their application. The first pricing model is successfully validated using QOPF in Chapter 7. Chapter 8 describes why the second pricing model cannot be fully validated.

The investigation into the behaviour of reactive power marginal prices is detailed in Chapters 9, 10 and 11. In Chapter 9, Dispatch Based Pricing is used to identify the power system marginal costs influencing the behaviour of reactive power marginal prices. Pr isidered for optimal dispatches of unconstrained, voltage constraine eneration constrained, fixed generator v able generation voltage, power systems.

Chapter 10 presents reactive power price behaviour for sub–optimal dispatches. This behaviour implies a load–following generator scenario. It is demonstrated that a Dispatch Based Pricing model can be used to calculate the true cost to the power system of supplying the next unit of real or reactive power from a load–following generator.

The results of Chapters 9 and 10 are used in Chapter 11 to propose a spot market for the South Island section of New Zealand's National Grid. It is discussed how the pq pricing model and PvG pricing model can be used to calculate real and reactive power marginal prices for this spot market. Some implications regarding the effect of reactive power spot pricing on the operation of this power system are also discussed.

1.4 TERMINOLOGY

The following terms are used in the place of Power Systems Engineering terms. They are used to maintain continuity with the work of Ring [1995], and of Read and Ring [1995d]. 'Busbar' defines the connection point of a number of power system components. This has been replaced with the term 'node'. 'Node' includes the artificial nodes created in Dispatch Based Pricing by using pi–models to model devices such as three–winding transformers. The two exceptions are the 'Swing Bus' and in Chapter 11 where 'busbar' is used to identify actual busbars in the New Zealand National Grid. It must be noted however, that 'node' and 'busbar' are used inter–changeably within literature.

'Demand' is used instead of 'load' to indicate the consumer's demand for more power at a particular node. Again, the exception is in Chapter 11 when discussing physical loads.

Prices are usually described for power usage over a defined period of time, such as an hour. Ring [1995] however, assumes an arbitrary, unitless period. Hence, \$/MW and \$/MVAr are used in place of \$/MWhr and \$/MVArhr in this thesis, except when quoted from other works.

The following terms differ from those in Ring's work. Physical generating turbines are referred to as 'generators' and the companies owning these machines are referred to as 'generating companies'.

In spot pricing, 'marginal price' and 'spot price' are synonymous. However, 'marginal price' is used herein, as it describes the marginal nature of the resource associated with that price. That is, the marginal price is the price of obtaining one more unit of that resource. The exception is Chapter 2 where 'spot price' is used for consistency with the cited literature.

A distinction is made between 'marginal price' and 'marginal cost', where the marginal price is equal to the marginal cost plus some profit. For example, the marginal price of reactive power at a node is equal to the costs of marginal losses, plus the marginal costs of any binding power system constraints, plus any unit generation costs of reactive power, plus any profit added to the unit generation cost by the generating company.

Chapter 2

SPOT PRICING OF REACTIVE POWER

2.1 AN HISTORICAL OVERVIEW

A well–designed pricing system is essential to the effective operation of a power system. Many influences impact on the design process of a pricing system that will meet the needs of both energy supplier and customer. In the past thirty years two influences have had a continued impact: the need for efficient power system operation, and the steady increase in reliance on and demand for power.

Efficient operation of the power system has been a primary focus since the 1930s, with the introduction of Economic Dispatch. This was one of the earliest forms of pricing and the precursor to the present Optimal Power Flow [Huneault and Galiana 1991]. By dispatching power, so as to minimise operational costs, the power system can be operated to make more efficient use of resources and equipment. In the United States of America, efficient power system operation was traditionally achieved through centralised planning by a single, vertically integrated, organisation. Although not vertically integrated in New Zealand, the New Zealand Electricity Department provided centralised planning and dispatch, being the entity owning and operating the generation and transmission system.

In the past two decades, the approach used to obtain efficient power system operation has changed dramatically. Scherer [1977] used electricity prices in the United States as an example to help explain this change. In the mid-1920's electricity prices were very high. Primarily, this was due to the state of infancy of the power system industry. Over the next four and a half decades the price in electricity dropped dramatically, even with a steady rise in the consumer price index, until 1969. This price decrease was seen as the combined result of economics of scale, and the dramatic advancement and refinement of technology over this period.

During the 1960s and 1970s however, environmental and technological issues began to impact on the price of electricity. In particular, an increasing awareness for the need to reduce pollution from power stations, coupled with crises such as the oil shortages of the 1970s began to push the cost of electricity up. During this period, power system technology was seen by Scherer to making less and less efficiency gains for cost of development. As a consequence, the financial benefits obtained through gains in efficiency were no longer able to counteract this price increase. Therefore, new methods of obtaining efficiency gains in the power system had to be found.

Caramanis *et al* [1982] of the Massachusets Institue of Technology, suggested that a market-based pricing system held the key to efficient power system operation. This novel pricing system (named the MIT Model due to its origins and discussed later in this chapter) was based on the philosophy that only the physical power system was a natural monopoly, and that efficiency gains could be made by setting up an energy market place, through deregulation. In this energy market generating companies and customers would buy, sell and trade energy, just as in any other economic market, in order to foster competition within the generation and consumption sectors. The aim of such a market was:

> "... to maximise social welfare, that is to maximise consumers' plus producers' surplus, subject to the operational constraints *(of the power system).*" [Baughman and Siddiqi 1991]

Various forms of this proposed energy market have since been implemented in countries such as Chile, the UK, Norway, Argentina, New Zealand and Australia [Hogan *et al* 1996]. Consequently, there has been a proliferation of publications on pricing systems. Some new pricing systems reject the concept of an energy market and deregulation. Some refine, extend, or modify the original MIT pricing model. All differ in order to meet the needs of the individual power system. However, all are designed to economically optimise the operation of the power system. A discussion on some of these pricing systems follows.

2.2 THE DESIGN OF A PRICING SYSTEM

Computer models of physical power systems need to be accurate if the system operator is to determine the current state of the physical network. In contrast, pricing systems need not calculate prices for every quantity within the power system. In fact, a pricing system that produces a price for every part of a power system will be in danger of becoming too complex and impractical to implement in an energy market. The main purpose of a pricing system therefore, is to generate energy prices that give signals encouraging welfare maximisation. If a market is to encourage efficient operation, these prices must represent physical quantities (i.e. resources) that all market participants have the ability to influence by modifying their electrical behaviour. It is the price signals that force participants to behave according to the objectives of the energy market.

In the two decades following the proposal of the MIT pricing model, many authors have proposed different resources (and their availability) that should be considered for a pricing system that encourages efficient power system operation. Some examples are: real power losses and constraints on power flow, voltage and real power generation [Caramanis *et al* 1982]; pre/post contingency power system conditions [Dandachi *et al* 1996]; fixed assets such as capacitor banks [Chattopadhyay *et al* 1995]; control of frequency and tieline flows, power harmonic control and removal, and spinning reserve [Baughman *et al* 1997a].

There is growing concern for system security and quality of supply in deregulated markets. Thus, an emphasis has also been placed on pricing reactive power resources such as reactive power reserves; supply of capacitive and inductive reactive power; dynamic and static reactive power devices; reactive power capacity and generation costs of generators; and joint allocation for reactive power [Hao and Papalexopoulos 1997]. However, it is the needs of the individual power system that influences which of these resources will be priced.

Green [1997] proposed six principles that should be considered when designing a pricing system. These principles aid the process of identifying the resources that are relevant to a specific power system. Green said that if a pricing system was to be effective in signalling relative costs of power from different energy suppliers, and effective in determining the amount of power transferred in each transaction, prices should:

1. promote the efficient day–to–day operation of the bulk power market;
2. signal locational advantages for investment in the transmission system;
3. signal the need for investment in the transmission system;
4. compensate the owners of existing transmission assets;
5. be simple and transparent;
6. be politically acceptable and able to be implemented.

In essence, pricing systems should be practical from the perspective of the people who will use it. For example it is not practical for domestic power users to participate in a pricing scheme based on spot prices if the costs of metering and other equipment required out–weighs the cost benefits those users will ever gain from buying power at the spot price. In particular, a pricing scheme along with its associated software and equipment must be accepted by all parties involved. It must be able to withstand any challenges made against any part of the pricing system (Principle 6).

2.3 REAL TIME SPOT PRICING

In 1982, Caramanis *et al* [1982] attempted to address these pricing scheme requirements with a pricing system called 'Optimal Spot Pricing' (the MIT Model). This model was based on two earlier variants of spot pricing by Vickrey [1971] and Schweppe *et al* [1980]. It used a full ac optimal power flow to produced spot prices for both real and reactive power[1]. Caramanis *et al* used the word 'optimal' to describe the function of spot pricing theory in their proposed deregulated energy market. In particular, spot prices were designed to send signals to all participants in this energy market (i.e. generating companies and customers), encouraging them to make efficient use of electricity resources. As a result, spot prices utilise competitive market forces so as to maximise the value of a global social welfare function. The welfare function used by Caramanis *et al* is essentially:

$$\begin{aligned} \text{Welfare} = \quad & \text{Value of Electricity Usage} \\ - \quad & \text{Variable Power System Operating Costs and Fuel Costs} \\ - \quad & \text{Cost of Rationing} \\ - \quad & \text{Cost of Equipment} \end{aligned}$$

The components of this welfare function represent the cost of resources that all market participants see as beneficial to use efficiently. In maximising the value of this welfare function an optimal spot price for electricity is obtained, which is agreed on by both generating companies and customers. This optimal price occurs because customers will not use power if the spot price is seen as too expensive and generating companies may not generate power if they decide the spot price is too low. Thus, the process of supply and demand to obtain optimal prices causes power within the power system to be dispatched economically, at minimum cost. If the dispatch is not economic (or sub–optimal) someone will loose money, either the generating companies or the customers.

The optimal spot price for both real and reactive power (suggested by Caramanis *et al*) has the following components in order to provide signals encouraging market participants to behave efficiently and competitively:

$$\begin{aligned} \text{Optimum Spot Price} = \quad & \text{Marginal Fuel Cost} \\ + \quad & \text{Cost of System Marginal Losses} \\ + \quad & \text{Cost of System Constraints} \end{aligned} \tag{2.1}$$

Each component represents the marginal cost of a power system resource. Each marginal cost reflects the increment in total cost to the power system when meeting the

[1]The marginal prices for reactive power demand are calculated using the costs of the real power losses caused by changes in demand. Reactive power cost functions were not accommodated in this OPF.

demand for an additional unit of the corresponding resource (or its availability when considering constraints) [Riggs and West 1986]. Therefore, marginal fuel costs represent the change in total generation cost when the power system meets the customer–demand for one more unit of real or reactive power. In generating that extra unit of power, the real and reactive power losses in the system will also change. The cost of the extra power generated to meet this change in losses is represented by the marginal loss component of the spot price. Customers in different areas will therefore incur different prices, since spot prices are a function of system losses.

The cost of system constraints is included in the spot price to reflect the burden a customer places on the system. If that burden forces the operation of the power system to be constrained in some way, the marginal costs of these constraints will be non-zero. For example, overloaded transmission lines and binding voltage limits are common constraints. These marginal costs represent the price customers are willing to pay to relax each constraint by one unit. Hence, marginal costs of constraints are used to encourage customers to reduce their electricity consumption in order to relieve these constraints on the system.

In reflecting the actual operation of the power system in the price of electricity, the MIT Model was "shown to encompass and achieve more fully the objective of rate structures and load management techniques proposed". Caramanis *et al* believed that spot prices would encourage more efficient use of fuels, provide incentives to use power at less expensive times and locations and make cogeneration and energy sources such as solar power viable options. Since the MIT proposal, experience has shown that spot pricing must be accompanied by two other mechanisms if efficient market operations are to occur: financial hedging for protection against spot price volatility and a recovery system of fixed network costs which provides efficient long run investment signals [Hogan *et al* 1996].

For spot pricing models to achieve an optimal dispatch, prices must be published to market participants in real-time. This means that prices must be announced moment by moment to market participants, and the participants must respond instantaneously to these prices. This cannot be achieved with current technology. However a suggested solution to this problem was to calculate prices either prior to the dispatch (*ex ante*) or after the dispatch (*ex post*) [Read and Ring 1996].

Ex ante prices are determined by taking the customer bids and generating company offers of market participants and going through a clearing process to determine optimum spot prices which are acceptable to both parties. These spot prices are usually published for certain time periods such that, market participants know the cost of power prior to consumption. A period of time could be a year, 24 hours, 1/2 hour, or 5 minutes. However, using time periods can result in problems if inter–temporal constraints are not considered [Read and Ring 1995b]. The term 'inter–temporal' identifies constraints that can span several time periods. Two examples of inter–temporal constraints are the ramping requirements of thermal generators, and the water storage of hydro generators. Therefore, Caramanis *et al* included equations to constrain power system quantities from one period of time to the next[2]. The aim of such constraints is to influence energy consumption decisions across several time periods, especially during periods of heavy loading on the electricity network.

Prices calculated *ex post* allow the power system to be co–ordinated by a central dispatcher, in a more or less traditional manner. The power system is dispatched using supply and demand curves based on offers from generating companies and bids from wholesale customers. *Ex post* prices are calculated after the dispatch has occurred, for each time period. The time period for calculating spot prices in the Australian market is 5 minutes. Hence, these prices reflect the cost of the actual dispatch [Read and Ring 1996].

Since the initial proposal for Optimal Spot Pricing of real power, many different formulations have been suggested for calculating *ex ante* and *ex post* marginal spot prices. Variations in formulations are the result of extra cost components. These components represent the cost of resources such as reactive power generation, and are added to the basic spot price equation (2.1).

2.4 SPOT PRICING OF REACTIVE POWER

2.4.1 A Spot Market

A 'Spot Market' is usually associated with the trading of real power using marginal pricing techniques. In this thesis, the concept of a spot market is extended to include the trading of reactive power. In this extended spot market, market participants are able to buy and sell reactive power in an identical manner to the usual buying and selling of real power.

The extended spot market can be divided into a sub–market for real power and a sub–market for reactive power. This allows market participants to trade solely in real power, or solely in reactive power, or both. However, the spot prices in the sub–markets must be calculated together as they are co–dependent, reflecting the co–dependent nature of real and reactive power.

2.4.2 Optimal Power Flow Formulations as Spot Pricing Tools

A spot price market effectively optimises power system resources subject to system constraints, so as to maximise a global social welfare (or objective) function. This describes the function of optimal power flow (OPF) technology.

[2]These constraint equations use time as a frame of reference. Constraints representing losses, thermal limits, voltage limits use space (that is, the location of the customer in the power system) as a frame of reference.

Optimal Power Flow formulations have been used as dispatch planning tools for decades with Economic Dispatch, the precursor to the OPF, being introduced in the 1930s [Huneault and Galiana 1991]. The Economic Dispatch model represented the power system as a single equality constraint. Since then, OPFs have been developed to describe the ac power system in full.

The aim of an OPF is to optimise the resources of a power system so as to minimise some objective function, with respect to a set of resource constraints. Common objective functions are the total cost of system losses, and the total real power generation costs. The latter is generally the summation of the offers that set the price at which each generation company is willing to sell their real power in the spot market.

Constraints are used to describe the power system. The equality constraints are the (ac) power balance equations. The inequality constraints represent the finite nature (or amount available) of resources such as real power generation, reactive power generation and voltage magnitude. The objective function and the constraints comprise an 'OPF formulation'. Therefore, a full ac OPF incorporates both real and reactive power sub–markets (and correctly models their co–dependence) through the ac power balance equations. Furthermore, OPF technology models the primary aim of an ideal spot market, to optimise the dispatch of real and reactive power generation with respect to an objective function and a set of constraints.

When solving any economic dispatch problem, the OPF assigns a cost to each resource constraint[3]. These costs reflect the change in the objective function for an incremental change in the relevant power system resource. Therefore, each cost is the marginal price or spot price of that resource. An OPF therefore, calculates the real and reactive power spot prices for each sub–market.

The introduction of spot pricing has caused a change in emphasis in the development of OPFs. Before the work of Caramanis *et al* [1982], most of the research was on producing faster, more accurate, and more reliable numerical algorithms for solving the optimisation problem described by the OPF formulation. However, with the realisation that OPFs can be used as spot pricing tools, there has been a certain change in the direction of OPF research. There is now a significant emphasis on developing OPF formulations to meet the pricing needs of individual power systems. In particular, a number of papers have been published which focus on developing OPF formulations to obtain reliable reactive power spot prices.

2.4.3 Dc OPF Approximation

Caramanis *et al* [1982] proposed a full ac OPF spot pricing model. Until 1993 however, most of the literary emphasis was on calculating real power spot prices for a real power sub–market, using a dc OPF. The dc OPF is based on standard dc power–flow

[3]These costs are the LaGrange multipliers described in Appendix B.

equations. These equations ignore the flow of reactive power by assuming that reactive power, costs almost nothing to generate and assuming that there 'is a weak relationship between real and reactive power' [Hogan 1992] (see Section 3.6). Consequently, reactive power was not considered as absolutely necessary in the development of spot pricing models, such as the model developed by Schweppe *et al* [1988]. These simplifications were important at that time because OPF technology and computers were slow.

Hogan [1992] used a dc OPF approach for illustrative purposes when addressing serious problems with regard to wheeling transactions in the United States[4]. However, Hogan [1996] showed that real power spot prices were heavily dependent on reactive power flows and reactive power spot prices.

In particular, Hogan showed that a dc OPF only produces prices comparable to prices from an ac OPF when a power system is either unconstrained, or congested as a result of thermal constraints. When a power system is congested by binding voltage constraints, Hogan demonstrated that reactive power prices can be 800% greater than their corresponding real power prices, thus negating the above assumptions of the dc OPF. Therefore, Hogan concluded that the dc OPF cannot be used to calculate real power spot prices if voltage constraints and reactive power are issues. He stated that an ac OPF must be used because it calculates reactive power prices, upon which real power prices are dependent.

Dc OPF formulations can be either *ex ante* or *ex post*. Hogan's ac and dc formulations calculated *ex post* spot prices.

2.4.4 Ex ante Reactive Power Spot Pricing Models and Results

Generally full ac OPFs are now used to calculate reliable spot prices for real power. Until recently however, reactive power has been assumed to have a zero generation cost. Any non–zero reactive power spot prices were only a function of the marginal costs of real power losses and binding constraints, and thus treated as a by–product. These reactive power spot prices were recognised as necessary for calculating real power prices when optimising the real power dispatch, but were not further utilised.

With the deregulation of the power sector, there has been much discussion over who should pay for ancillary services that depend on reactive power, and how these services should be financed. Since many spot pricing markets (designed solely to economically dispatch real power) are already operating, the use of reactive power spot prices to charge for these services is being seriously considered. This would require the addition of a reactive power sub–market to the existing spot market.

In this section, a number of OPF formulations are presented, which were designed to investigate the feasibility of using reactive power spot prices to pay for reactive power.

[4]Wheeling is defined as the transmission of real power (or reactive power) from a seller to a buyer using a transmission network belonging to a third party [Muchayi *et al* 1995].

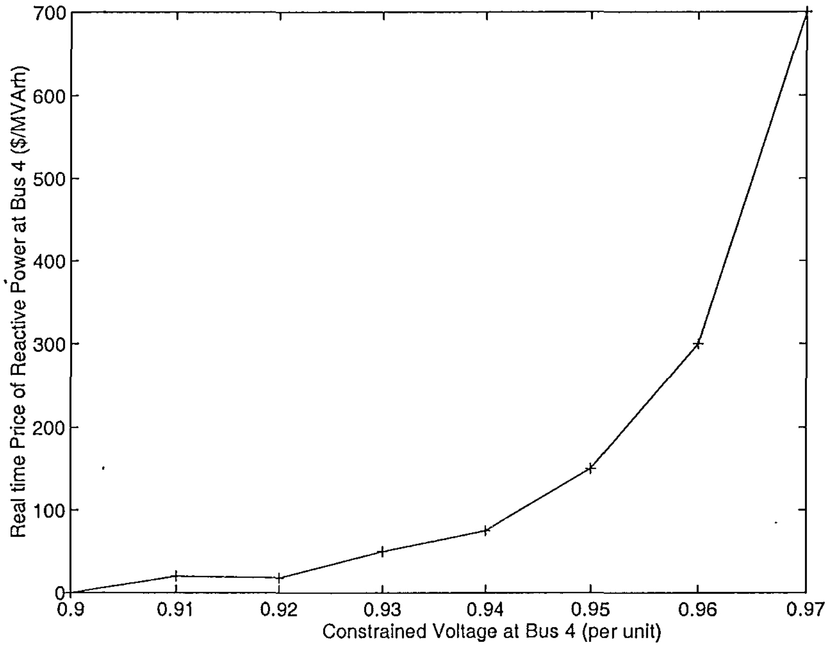


Figure 2.1 The real–time price of reactive power at a voltage constrained load bus increases as the lower voltage limit is tightened

The formulations have been designed either as planning tools or as tools for calculating prices in a real–time spot market. Therefore, these formulations calculate spot prices *ex ante*. Reactive power generation is still costless in most of these formulations. Only two known formulations accommodate reactive power cost functions to allow non–zero reactive power generation costs. These are proposed respectively by Dandachi *et al* [1996], and Read and Ring [1995d], and are discussed later in this chapter.

Baughman and Siddiqi [1991] highlighted one of the major concerns with reactive power spot prices – price volatility. Using a 4–bus test system Baughman and Siddiqi showed that a binding voltage limit can cause high reactive power prices. Figure 2.1 shows the effect of a binding voltage constraint on the cost of reactive power at a load bus. Note that the voltage at this bus was forced against a lower voltage limit. As the lower voltage limit is increased the voltage increases and the reactive power price increases from approximately $0/MVArhr to $700/MVArhr. The increase in spot price demonstrates that relieving the voltage constraint becomes more important to the power system as the voltage limit is raised.

Baughman and Siddiqi showed that spot pricing of reactive power can be a useful pricing mechanism despite the apparent volatility of reactive power prices under voltage constraints. This is because reactive power spot prices accurately represent the true state of reactive power in the power system. Baughman and Siddiqi also stated that penalties based on power–factors are not accurate representations of reactive power usage. For example, a large company may have an excellent power–factor and yet, draw more reactive power than a small company with a poor power–factor. Consequently, the small company will end up paying more than the large company even though the large company is putting a larger burden on the power system. However, spot pricing of reactive power will charge these companies for their actual reactive power usage, a fairer pricing scheme.

Muchayi *et al* [1995] provided a summary of formulations that include reactive power pricing. Their focus was on drawing attention to the importance of reactive power spot prices in wheeling transactions. They emphasised that the wheeling rate of reactive power is not a negligible factor, even though the marginal cost of reactive power may be much smaller than the marginal cost of real power,. Like Hogan [1996], and Baughman and Siddiqi [1991], they noted that dc OPF models fail to capture some of the pricing effects attributable to reactive power.

Chattopadhyay *et al* [1995] recognised that voltage constraints result in high reactive power prices, which in turn can cause stability problems in a spot market. To combat these problems a modified OPF was proposed. This OPF acted as a planning tool for the placement of capacitors to relieve voltage constraints.

One problem inherent in spot pricing models is that the high spot prices drop once a voltage constraint has been relieved through a fixed capacitor. This makes capacitor cost recovery impossible. In order to recover the installation and capital costs of capacitors placed by the OPF, Chattopadhyay *et al* added a fixed component to the variable spot price of reactive power.

Table 2.1 Results of spot pricing case studies for reactive power using the PTI optimal power flow program.

Scenario	Average Marginal Price per MVArhr	% Change from Base Case	% of System Average Marginal Price \$/MWhr[5]
Base Case:	\$ 0.45		3%
With no generator reactive limits	\$ 0.07	⇓ -85%	2%
With reactive capability curves	\$ 1.66	⇑ +272%	6%
With local area spinning reserve requirements	\$ 0.36	⇓ -19%	3%
With an interface constraint	\$ 0.40	⇓ -10%	3%
With load (P&Q) decrease 10%	\$ 0.05	⇓ -89%	2%
With load decrease 20%	\$ 0.05	⇓ -90%	2%

The behaviour of real power marginal costs and the behaviour of reactive power marginal costs were also investigated by Mitsche [1996]. Results were obtained using a 20–bus, 30–transmission line, 6–generator power system model. Two of the generating companies were represented as fixed output units. The other four were represented by a combination of linear or quadratic real power cost functions. The reactive power marginal price results are given in Table 2.1.

Mitsche confirmed the results of other authors. He showed that marginal prices for real power are closely linked to marginal prices for reactive power, thus highlight-

[5] The system average marginal price is obtained by taking the average of all real power spot prices.

ing the importance of reactive power spot prices. He also noted that reactive power spot prices are trivial if the generation units have not reached their respective reactive power generation limits, but are extremely volatile for changing load levels and generation patterns. Even so, his test results show that reactive power prices remain small compared to the system average marginal price.

El Keib and Ma [1997] produced experimental results confirming the behaviour of reactive power spot prices highlighted in previously reported papers. These results were obtained using a decoupled OPF formulation. This decoupled OPF was created by separating the real and reactive constraints in a coupled OPF formulation (such as the one used by Baughman and Siddiqi [1991]), to form P– and Q–subproblems. El Keib and Ma believed that this formulation shows the individual components that comprise the real and reactive marginal prices at each PQ node. This is in contrast to Baughman and Siddiqi's model which does not show the composition of each marginal price. The decoupled OPF was also chosen for its well-recognised superiority of speed. Reported results of OPF simulations for IEEE 30–bus and 118–bus power systems, showed small variations of less than $1/MWhr in the real power marginal cost profiles. Reactive power marginal cost profiles show variations of up to $1.5/MVArhr due to flow constraints and voltage violations, with reactive power spot prices being either positive or negative in the 118–bus profile.

One of the most comprehensive OPF spot pricing models formulated to date addresses the fact that most spot pricing models do not produce prices linked over time [Baughman *et al* 1997a, Baughman *et al* 1997b]. In their formulation, the authors identify the following ancillary services for which spot prices might be calculated:

1. voltage regulation;
2. maintenance of generation and transmission reserves;
3. regulation of frequency and tieline flows;
4. removal and/or control of power harmonics - that is the amount of harmonic distortion;
5. amount of spinning reserve necessary;
6. enviromental impact associated with producing and delivering electricity.

In order to calculate spot prices for ancillary services 3 and 4, dynamic constraints were included that relate one point in time to subsequent points in time. However, Baughman *et al* stated that prices must be published to the market participants several times a second for these constraints to be effective. These two papers are an excellent example how the accurate representation of the power system in a pricing model, reaches a limit. Although mathematically feasible, the authors recognised that, at that time, technology and other practical aspects prevented the pricing of rapidly changing aspects of the power system (such as system frequency). Nevertheless, this model can be used to produce real and reactive power prices without loss of accuracy, while ignoring the dynamic prices.

Baughman *et al* affirmed the need for reactive power pricing. In their formulation the price of reactive power at a node is equal to the marginal cost of producing any real power required by an increment of reactive power load at that node. This is in absence of binding power system constraints. Therefore, they concluded that the optimal transaction price in a wholesale electricity market should reflect the value of both real and reactive power.

Dandachi *et al* [1996] also reported a similar approach in the design of a security constrained OPF (SC–OPF) for the National Grid Company (NGC). This SC–OPF has pricing components based on both var–utilisation and var–capacity. Dandachi *et al* proposed that the utilisation payments should replace the fixed payments used by NGC to compensate generating companies for their reactive power contribution to the system. Part of the utilisation payments to be made to generating companies for reactive power support is based on reactive power generation cost functions. These were included in the SC–OPF objective function, and allow reactive power to be economically dispatched in the same way that real power is economically dispatched. It was planned that:

> "Under a market operation, these MVAr cost curves (*i.e. functions*), normally for individual generating units, would be offered by the generating companies for NGC to optimise the cost of reactive power."

Unlike the previous formulations, this SC–OPF implies that reactive power has a generation cost, as described by these reactive power cost functions. Thus, reactive power generation is no longer free.

Dandachi *et al* is one of two known publications where a spot market for reactive power is proposed that allows companies to offer a non–zero generation cost for reactive power. The other publication is that of Read and Ring [1995d] (the focus of this thesis).

Other features of the SC–OPF are limits on pre– and post–contingency voltages, local voltage control solutions, station var control and handling infeasible constraints. Two objective functions were used for different parts of the load profile: a cost–optimised, security constrained, reactive power dispatch objective function used only at the main load changes, and a minimum control action objective function used for reactive power refinement between major load changes.

Dandachi *et al* reported three cases solved by this security constrained OPF, using NGC's 713–bus system:

1. base case OPF with voltage limits

2. pre-contingency reactive power reserve constraints added to Case 1

3. contingency constraints added to Case 2

NGC's system was divided into eight zones of self-sufficient reactive power spinning reserve. For each case, the most expensive marginal price in each zone was reported. These cases highlighted an interesting behaviour of reactive power prices, namely the wide variation in reactive power prices across a power system:

- In Case 1, prices varied widely across the eight zones. Prices ranged from almost £0/MVArhr to approximately £60/MVArhr.

- Adding pre-contigency constraints altered the marginal prices of the zones dramatically. Some prices showed very little variation while others changed by almost 100%.

- Adding contingency constraints resulted in four zones experiencing price increases of up to 800%. The other zones saw very little price increase. Maximum prices across all zones ranged from £5/MVArhr to £400/MVArhr.

The main conclusion to be derived from these results is that reactive power prices are very sensitive to binding constraints. This is particularly true of contingency constraints, as these require dispatches that are conservative and not necessarily economic. Reactive power prices were also found to vary widely across the power system.

2.4.5 Ex post Pricing

OPFs are generally used either as planning tools or for real–time dispatch of power. Therefore prices from OPFs are usually calculated *ex ante*. However, Hogan *et al* [1996] stated that there is a general trend away from single organisations using centralised planning and *ex ante* prices to control the dispatch of power. The reason behind this trend is the belief that efficiencies in a power system can be gained through market and incentive mechanisms, rather than through top-down planning. Hogan *et al* said that this can be achieved by using *ex post* prices in a wholesale electricity market.

In an *ex post* pricing scheme, *ex ante* prices derived using supply and demand curves can be used "as forcasts of, or hedges against, *ex post* prices". Note that, it is possible to use a standard OPF to obtain *ex post* prices by tightening power system constraints. However, convergence to a feasible solution can be a problem if the optimisation algorithm is not carefully designed for optimal power flow applications. Such is the nature of the OPF from Cornell University, and used for this thesis (see Section 10.2).

The dc and ac spot pricing models proposed by Hogan [1992] calculate spot prices *ex post* (see Section 2.4.3). The approach used to calculate *ex post* spot prices is different to that used to calculate *ex ante* spot prices. *Ex ante* spot prices are calculated for a predicted dispatch. This predicted dispatch is generated with a standard OPF using bids and offers, prior to the actual dispatch. *Ex post* spot prices however, are calculated for a dispatch that has already occurred[6]. Thus, Hogan obtained his (*ex post*) ac spot pricing model by linearising an OPF formulation around an operating point (the observed dispatch). The equations of his dc pricing model were already linear.

In an OPF, the objective is often to minimise the total cost of generation. The OPF achieves this by redispatching the whole power system. In contrast, the dispatch cannot be changed with *ex post* pricing because the observed dispatch has already occurred. Therefore, the objective of the ac spot pricing model is to determine the least expensive source from which to generate the next unit of real or reactive power.

In both approaches, the most expensive generator available for generating power sets the marginal price of the next unit of power. This is because the most expensive generator currently generating is used to supply that next unit of power. Also, the behaviour of *ex ante* and *ex post* prices are the same. Only the point in time when the prices are calculated differs. Therefore, the price behaviour reported in the previous section applies equally to the behaviour of *ex post* prices.

Read and Ring [1995d] formalised Hogan's work, by creating an *ex post* pricing framework called 'Dispatch Based Pricing'. Read and Ring's model differs from Hogan's model in two important aspects:

1. Using power–flow terminology, Read and Ring modelled generator nodes as PV nodes, and all other nodes as PQ nodes. Hogan however, modelled all nodes as PQ nodes.

2. Read and Ring enabled marginal generation costs to be specified for reactive power, *viz.* the *ex post* equivalent of the SC–OPF proposed by Dandachi *et al* [1996].

The Dispatch Based Pricing philosophy and mathematics are presented in subsequent chapters.

The role of the objective function is to determine how the total wealth is distributed among the market participants when dispatching the power system [Ring 1995]. Since the dispatch already exists, both Hogan, and Read and Ring allow the objective function of the *ex post* model to have an arbitrary form (assuming that the dispatch is optimal). Hence, the choice of objective function is influenced by the goals of the spot market.

[6]Sometimes, not all the necessary dispatch data is available. For example, there may be unknown binding constraints, or details regarding some minor actions/decisions of the system operator may not have been recorded. In these cases, the data describes a dispatch that differs from the actual dispatch. Hence, this dispatch is called the "observed dispatch". It reflects the dispatch that is thought to have occurred.

2.4.6 Concerns Regarding Reactive Power Spot Prices

To date, three main criticisms of reactive power spot pricing have been identified in literature:

1. A sub–optimal dispatch may result in high reactive power prices [Kahn and Baldick 1994].

2. Reactive power spot prices are negligible compared to real power spot prices in an unconstrained, optimally dispatched power system [Hao and Papalexopoulos 1997].

3. Reactive power prices can be extremely volatile, causing unfavourable market conditions [Mitsche 1996].

Kahn and Baldick focussed on the effect of a sub–optimal reactive power dispatch on reactive power prices. They used the 3–bus examples from Hogan [1996] to show that artificially constraining the dispatch of reactive power can result in high reactive power prices. Further, this inflated reactive power spot price dropped from \$0.67/MVArhr to \$0.038/MVArhr when they removed the artificial constraint; the real power price at the same node dropped from \$1.427/MWhr to \$1.314/MWhr. Kahn and Baldick stated therefore, that reactive power prices are small for an optimal dispatch. However, they assumed a zero reactive power generation cost.

Spot pricing assumes an optimal dispatch. Kahn and Baldick questioned whether such a dispatch would be realised in practice. They also identified the potential for market participants to set artificial reactive power constraints. The objective being to manipulate reactive power prices for personal gain. Hence, they concluded that an audit function must be a necessary part of any spot pricing arrangement.

Chattopadhyay *et al*, Baughman and Siddiqi [1991], and Mitsche also demonstrated that spot prices of reactive power are trivial in an unconstrained (well designed) power system. Hao and Papalexopoulos opted for a different pricing system, having experienced reactive power spot prices to be 1% of real power spot prices. They reasoned that low reactive power spot prices represent only a small portion of the true cost of providing reactive power services. Moreover, they opined that the capital costs incurred as part of these reactive power services should be used in the reactive power price calculation in order to sufficiently compensate generators of reactive power. Consequently, they proposed two new pricing methodologies, both based on a unit cost measure of the total reactive power capacity of a generator rather than on reactive power utilisation.

NGC decided to solve the problem of negligible reactive power spot prices within a spot market framework, with payments for reactive power utilisation (i.e. using generation cost functions), and payments for reactive power capacity [Dandachi *et al* 1996].

Most literature recognise the volatility of spot prices for reactive power. The causes are shown to range from sub-optimal dispatch, to binding reactive power generator limits [Mitsche 1996], to binding voltage limits. A solution to this issue of volatility must be provided if a reactive power sub-market is to operate effectively[7]. This was another reason why Hao and Papalexopoulos proposed a new pricing system.

Chattopadhyay *et al* showed that price volatility can be removed through optimal placement of capacitors (or other reactive power sources) at load centers. Such placements were demonstrated to improve the voltage profile of the power system by relieving binding voltage constraints. In turn, this relief drops the reactive power spot prices by providing a cheap reactive power source. Chattopadhyay *et al* recovered the capital costs through fixed payments added to all reactive power spot prices.

2.5 ELECTRICITY PRICING IN NEW ZEALAND

2.5.1 An Historical Overview of Real Power Pricing

The structure of the real power pricing system in New Zealand has undergone a succession of changes. The most recent change was the inception of the Wholesale Electricity Market in 1996. Figure 2.2 highlights the main changes in the power sector that have influenced New Zealand's power system.

The changes to New Zealand's real power pricing system have been motivated by the need to more accurately emphasis and describe the costs of energy supply [Tariffs 1983]. These changes began in 1967 with the introduction of a 2–part bulk tariff reflecting energy usage and peak demand at the wholesale level. The Ministry of Energy changed this to a 4–part tariff in 1984, producing four time–of–use (TOU) charges. At the same time, a differential in wholesale electricity prices between the South and North Islands was introduced. The price differential described the marginal cost of transmission between the two islands.

Since 1987, major reforms have occurred. Corporatisation, deregulation and splits have occurred with the objective of returning competitive real power prices to the consumer. For example, the Electricity Division of the Ministry of Energy was corporatised to form the Electricity Corporation of New Zealand (ECNZ) in 1987.

The following year, ECNZ began signalling expected half–hourly spot prices to industry. Retail distribution customers and very large industrial customers announced to ECNZ, their expected half–hourly power demands for the following year. The customers then entered into a one year contract to purchase between 90% and 110% of their expected demand from ECNZ, at a TOU rate [Ring 1995]. Any shortfall or surplus in

[7]New Zealand's wholesale electricity market uses a system of *ex ante* bids, offers and financial hedges to help protect against spot price volatility [Alvey *et al* 1998].

Pre-1967 — Electricity Division recovers its operating costs by peak demand charges only. [Tariffs 1983]

1967 — 2-Part Bulk tariff introduced to charge for energy used and peak demand. [Tariffs 1983]

1976 — Greater emphasis placed on the energy charge and less on peak demand. [Tariffs 1983]

1st April 1984 — - 4-Part tariff introduced: Energy charge 7am - 11pm; 11pm - 7am
Peak Demand (7am - 11pm) Summer; Winter [Tariffs 1983]
- South Island - North Island price differential introduced. [Ring 1995]

1987 — Electricity Division of the Ministry of Energy, coporatised to ECNZ. [McLay 1996]

1988 — - Generation sector is deregulated. [McLay 1996]
- April - ECNZ begins signalling expected half-hourly marginal prices to industry. [Turner 1995]

1992 — A study to examine how a competitive WEM could operate.[EMCO 1996]

1993 — - Coporatisation of retail electricity distribution companies [McLay 1996]
- EMCO created to design the WEM [EMCO 1996]
- EMCO begins to facilitate contracts between ECNZ and retail distribution companies [Ring 1995]

1st July 1994 — Separation of the national grid operating company, Trans Power NZ Ltd, from ECNZ to form a separate State Owned Enterprise (SOE). [McLay 1996]

1st February 1996 — The NZ government, transfer significant generation assets from ECNZ to establish major new supply side competitor, Contact Energy Ltd. [McLay 1996]

1st October 1996 — Competitive wholesale electricity spot market begins; it is administered by EMCO. [Alvey *et al.* 1997]

Figure 2.2 Major transitions in wholesale pricing of real power within New Zealand.

these annual contracts was traded at the spot price. This pricing structure continued until the commencement of the Wholesale Electricity Market on 1st October 1996.

2.5.2 The New Zealand Wholesale Electricity Market

The New Zealand electricity market (NZEM) is designed for wholesale trade of real power. The market initially had two components: the "Day–Ahead Financial Commitment Market", and the "Real–Time Physical Spot Market" [NZEM 1996]. The Day–Ahead market was discontinued as at 30 September 1998, due to lack of participation from spot market participants.

2.5.2.1 The Day–Ahead Financial Commitment Market

This was a voluntary market. It provided a framework for spot market participants to establish financial hedge positions a day in advance of the actual, dispatch and transmission of real power. Such hedges protect participants against the volatility of the spot prices.

Participants in this market were to submit bids for specific quantities of real power at certain prices, or submit offers to sell financial hedges. The settlement of these bids and offers determined the price of power sold in this market. This market was similar to the TOU prices set by ECNZ between 1988 and 1996 (See Section 2.5.1).

2.5.2.2 Real–Time Physical Market

In this market, real power is sold at the spot price. A security–constrained, bid–clearing system (SC–BCS) is used in the scheduling process of this market [Alvey *et al* 1998].

Bids and offers (which reflect the unit costs of generation) are submitted daily by purchasers and generating companies respectively, for every half–hourly trading period in that day. Bids and offers reflect the expected quantity to be bought (or capacity to be sold) in each trading period. From these bids and offers, the SC–BCS determines an optimal generation dispatch schedule for that day. This schedule sets the provisional spot prices for real power. The grid dispatcher, Transpower New Zealand Ltd, is required to co-ordinate the dispatch of generation according to this schedule.

A clearing manager, the Electricity Market Company (EMCO[8]) calculates final spot prices, *ex post*. These are based on the provisional spot prices, but account for any deviations from the dispatch schedule. The final spot price at a node is the cost of any power not sold in the Day–Ahead market, at that node.

[8]EMCO is now known as M–co, The Marketplace Company, as at 16 November 1998.

The bid–clearing system is designed around a dc OPF–based algorithm, as it is required to optimise only the dispatch of real power generation. The dispatch is optimised subject to stringent constraints on:

- physical power system quantities;
- real power flow and generation;
- reserve requirements.

Other electricity services (frequency–keeping reserve and reactive and voltage support) are managed through side–contracts.

2.5.3 Reactive Power Pricing in New Zealand

In New Zealand, very little attention has been given to reactive power pricing at a wholesale level. Similarly, there is very little documentation on the history of any reactive power charges that may have been implemented.

Since the introduction of the NZEM however, New Zealand's high voltage network has been divided into four reactive power zones. These zones are used to pay for the different levels of voltage support required throughout the country. At each offtake, a fixed usage charge is added to the spot price[9]. Until mid–1998, the Zone 1 charge was 0.07 cents/kWhr/offtake/month. The charges for Zones 2, 3 and 4 were all 0.01 cents/kWhr/offtake/month. This pays the real power generation costs of stations dispatched only to provide reactive power generation, for voltage support. For example, part of these charges is used to finance the operation of Marsden and Otahuhu power stations. These stations must be dispatched to provide voltage support for the Auckland region, even when the NZEM does not want to purchase real power from these stations.

At a retail level (that is, between a retail power company and the end–users) reactive power is often incorporated in the price structure used for large business customers and industrial customers. This may take the form of a power demand charge based on the customer's maximum half–hourly kVA demand per month (cents/kVA/day). This is in addition to the usage charge for real power [Retail 1994].

During the summer, loads such as motors on irrigation equipment are common. These loads usually have a low power–factor. Having a low power–factor can cause an unacceptably low voltage at the point of load connection. Thus load owners are encouraged through incentives to install power–factor correction capacitors. An incentive may take the form of a rebate on the reactive power rating of connected capacitance, where the price of the rebate might have units of $/kVAr/year.

[9]An offtake is the point of connection in the high voltage network, from which a retail power company draws its power. 'Offtake' is also known as the 'supply–point' or 'point–of–connection'.

2.6 CONCLUSIONS.

Spot pricing has evolved greatly over the past three decades. Even though a spot pricing model for both real and reactive power was first introduced in 1982 by Caramanis *et al* [1982], spot pricing of reactive power has only received widespread attention since 1991. This attention has intensified with the need to find an acceptable pricing structure for reactive power ancillary services in a deregulated market.

As spot pricing models have developed, the importance of calculating spot prices for reactive power has become apparent, especially when binding voltage limits are present within the power system. Dc OPF models have been found to be highly inaccurate when reactive power flows and voltage limits are an issue. Consequently, ac OPF formulations have been extensively developed.

To date, literature have identified three main behavioural characteristics of *ex ante* reactive power spot prices, when reactive power generation is assumed as free. Reactive power spot prices are:

1. generally negligible for an unconstrained dispatch, representing only a portion of the true cost of providing ancillary services;
2. volatile in the presence of voltage constraints or reactive power generation constraints;
3. potentially volatile in a sub–optimal dispatch.

However, experimental results and the theory describing the mechanism behind this price behaviour are non–existent. Further, there is no literature demonstrating the behaviour of *ex post* reactive power spot prices for optimal and sub–optimal dispatches. Read and Ring [1995a] does however, use the behaviour of *ex post* real power spot prices to predict the behaviour of reactive power spot prices.

In response to the problem of negligibility, Dandachi *et al* has shown that the spot pricing methodology can be used to pay for reactive power services by using a form of reactive power generation cost functions. However, no adequate results are provided to demonstrate the effect of these functions on the behaviour of reactive power spot prices.

Read and Ring's *ex post* Dispatch Based Pricing model provides an excellent framework in which to investigate the behaviour of reactive power spot prices, because it allows non–zero generation costs to be specified for reactive power. Hence, this thesis uses a form of the Dispatch Based Pricing model to describe the behaviour of reactive power *ex post* spot prices. The emphasis is on price behaviour in a market where generation cost functions are used to economically dispatch reactive power, so as to fund the required ancillary services. Reactive power price behavioural descriptions are provided for optimal and sub–optimal, unconstrained and constrained dispatches of real and reactive power.

Chapter 3

POWER SYSTEM THEORY

3.1 INTRODUCTION

The previous chapter identified the need for further investigation into the behaviour of reactive power prices in the context of a spot market where reactive power is economically dispatched using reactive power generation cost functions. The Dispatch Based Pricing model of Ring [1995] (and reproduced in Read and Ring [1995d]) was identified as a suitable framework in which to conduct such an investigation. The Dispatch Based Pricing model has the equations necessary to calculate *ex post* prices for reactive power in a reactive power sub–market, in addition to the equations used to calculate *ex post* prices for real power. These reactive power equations are also necessary for correctly calculating real power marginal prices.

In this chapter, the philosophy behind the Dispatch Based Pricing model is applied to the economic dispatch of both real and reactive power. This is followed by the definition of power system nomenclature used in the context of the Dispatch Based Pricing framework.

Discussions with Read and Ring have revealed that it is not clear how the reactive power equations should be interpreted and applied in the context of a spot market having both real and reactive power sub–markets. Hence, the content of this chapter and Chapters 4 and 5 are used to form a theoretical foundation for the work presented in Chapters 6 to 8. In Chapters 6 to 8, an interpretation of the Dispatch Based Pricing model in the context of a spot market with real and reactive power sub–markets is proposed.

3.2 DISPATCH BASED PRICING

Dispatch Based Pricing enables the Clearing Manager (i.e. the Dispatch Based Pricing operator) to provide a rational economic explanation for an observed dispatch of real and reactive power generation [NZEM 1996]. Dispatch Based Pricing achieves this by providing a framework in which the Clearing Manager uses power system constraints to explain the observed dispatch. The Clearing Manager invokes constraints he/she perceives (correctly or incorrectly) as having restricted the dispatch when it occurred [Read and Ring 1995b, Ring 1995, Read and Ring 1996]. The Dispatch Based Pricing model then generates a set of real and reactive power marginal prices based on the marginal costs of these perceived constraints. Hence, the marginal costs of these constraints economically explain the observed dispatch.

It is not important that these perceived constraints do not accurately match reality, as long as all market participants agree with these perceived constraints. This is vital because, if the Clearing Manager were forced to include every binding constraint the pricing problem would not be solvable. The reason for this is because constraints as subtle as the Dispatcher adjusting reactive power generation to avoid a voltage limit are often overlooked.

In this thesis, a generic spot market is assumed for all discussions. This market is composed of real and reactive power sub–markets. The assumed goal for this generic market is to optimally (i.e. economically) dispatch real and reactive power by minimising the total cost of real and reactive power generation. The total cost is obtained by summing the real and reactive power generation cost functions of all the generators; this summation is the social welfare function (ref. Section 2.3). The cost functions are assumed to have been submitted by generating companies as offers in the 'Real–Time Physical Market' (described in Section 2.5.2.2). These offers, coupled with bids from consumers and binding constraints determine the actions of the dispatcher. They cause real and reactive power to be optimally dispatched.

After the fact, the Dispatch Based Pricing model is used by the Clearing Manager (see Section 2.5.2) to calculate *ex post* marginal prices for real and reactive power. These prices are based on the final dispatch, all bids and offers, all known binding constraints, and any known dispatcher actions that are not explained by the binding constraints.

3.3 REACTIVE POWER LOSSES

Marginal prices of real and reactive power are composed of similar marginal resource cost components within the Dispatch Based Pricing framework. Therefore, the reactive power marginal prices of the spot market described in the previous section are based on the definition of the real power marginal (spot) price in Section 2.3. Thus, the marginal price of reactive power at any node is defined to consist of the marginal costs of:

- the next unit of reactive power (defined by the generation cost functions);
- real power losses resulting from an incremental change in reactive power demand;

- reactive power losses resulting from an incremental change in reactive power demand;
- any constraints on the power system.

Defining the structure of reactive power marginal prices in this way implies that reactive power flows from the generator nodes (the producers) to the demand nodes (the consumers) when it is sold, just as real power does.

The 'reactive power loss' term is used to describe the cost to the system of 'transporting' reactive power from generator to demand. Physically, reactive power is described as the continual interchange of power between the generators and energy storage devices such as capacitors and inductors [Meadows 1972]. Furthermore, reactive power is never 'lost' from the power system in the form of 'work done', the case for real power. Therefore, to calculate the 'reactive power loss' term, the reactive power losses are defined for a pi–modelled branch (k) (see Figure 3.1) as being:

$$L_{Qk} = Q_{ki} + Q_{kj} \tag{3.1}$$

The total reactive power loss in the power system is therefore:

$$L_Q = \sum_{k \in K} L_{Qk} \tag{3.2}$$

K is the set of all branches in the network [Read and Ring 1995c].

The pi–model is used in Dispatch Based Pricing to model all transformers, transmission lines and shunt devices. Note that Read and Ring formulated the pi–model using shunt impedances instead of shunt admittances.

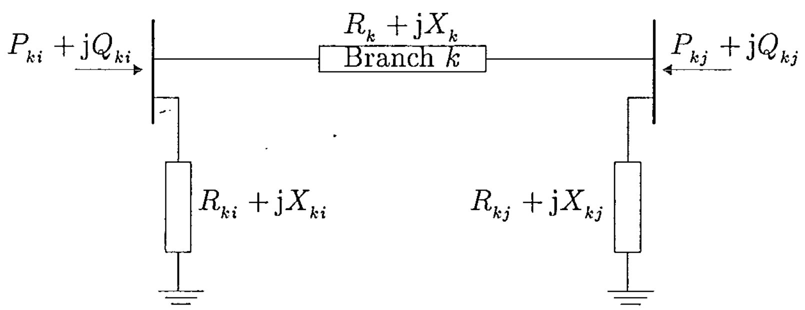


Figure 3.1 Pi-model representation of power system branches in the Dispatch Based Pricing model.

If Branch k is inductive, reactive power is drawn out of the network and L_{Qk} is positive, indicating that reactive power is 'lost'. If Branch k is capacitive, reactive power is injected into the network and L_{Qk} is negative, indicating that reactive power is being generated.

The average reactive power flow in a branch, k, is given by:

$$\bar{Q}_k = \frac{(Q_{ki} - Q_{kj})}{2} \tag{3.3}$$

3.4 OPTIMAL POWER FLOW TECHNOLOGY

In this thesis, the optimal power flow (OPF) has been divided into two parts for the purpose of discussion:

1. The OPF formulation, which is composed of an objective function and a set of constraints.

2. The OPF algorithm. This is the optimisation algorithm that finds optimal values for the power system resource variables. In doing this, the algorithm minimises (or maximises) the objective function within the feasible region defined by the power system constraints (see Figure 4.1).

These two parts are co–dependent.

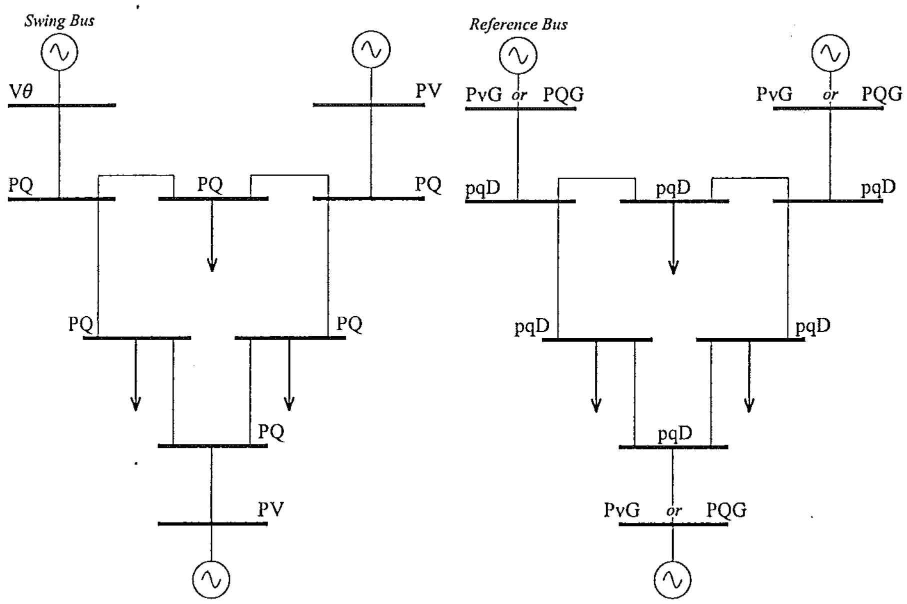
3.5 POWER SYSTEM VARIABLES

Four power system resources are the prime focus of power system dispatches within this thesis: real power injection (P); reactive power injection (Q); voltage magnitude (V); voltage angle (θ). The four variables representing these four resources apply to every node within the power system. Each node is usually classified by its two independent variables. The algorithm used determines which variables are independent, and hence used to classify nodes.

3.5.1 The Power–Flow Algorithm

Power–flow algorithms are generally used for planning purposes. Consequently, certain information is known prior to running the algorithm. All generator nodes (supply nodes) are classed as PV nodes because the real power generation and voltage magnitude are known. Hence, P and V are fixed at known levels, making these the independent variables. At load nodes (or demand nodes) the demand for real and reactive power is known. Hence, load nodes are classed as PQ nodes because P and Q are the fixed and independent variables.

The power–flow calculates the values of the remaining two power system variables at each node, with respect to the data describing the physical network and the values of the independent variables. These are the dependent power system variables.



(a) Power–flow (b) Optimal Power Flow

Figure 3.2 The classification of nodes is determined by the algorithm used.

There is a third type of node. This is the swing bus, also known as the slack bus. At this node the voltage magnitude and angle are fixed to provide a reference for all other nodes within the power system. Hence, this bus is classed as a $V\theta$ node.

The primary task of the generator at the swing bus is to modify its real and reactive power generation so that total generation matches total demand plus losses. Doing so maintains the frequency of the power system and ensures the conservation of real and reactive power in the system [Read and Ring 1995d, p.19]. Figure 3.2(a) illustrates the different power–flow, node classifications.

3.5.2 Optimal Power Flow Algorithm

Two node types exist in an OPF: generator nodes and load (demand) nodes. Often, power–flow classifications (PV and PQ) are used to describe these node types. However, this can create confusion, as the power system variables in an OPF algorithm behave differently to the independent variables in a power–flow algorithm.

Consider for example, an OPF generator node where the voltage (V) is fixed. At this node, the real power generation (P_G) is controlled by the OPF algorithm as it minimises the value of the objective function. Thus, V is fixed but the value of P_G changes from the start of the optimisation process to the finish of the process. By contrast, P_G and V at a power–flow PV node remain fixed throughout the power–flow solution process. Hence, this OPF generator node does not behave in the same way as a power–flow PV node. Therefore, to class this generator node as a PV node would be misleading. Below, an alternative node classification system is proposed to correctly describe the characteristics of OPF nodes in this thesis.

The power system resource variables are separated into three vectors in an optimal power flow algorithm [Wood and Wollenberg 1996]:

- state, or unknown, variables ($\tilde{x}$);
- control variables ($\tilde{u}$);
- fixed parameters ($\tilde{p}$).

Given these vectors, OPF nodes can be classified according to their known variables: upper case letters to indicate control variables; lower case letters to indicate fixed parameters. To identify generator nodes and demand nodes respectively, the letters 'G' and 'D' are used.

Two types of OPF algorithm are used in this thesis. The difference between the two types is the variables contained within each vector. In one OPF type, the vectors are:

$$\tilde{x} = \left[\begin{array}{c} \theta \\ V \\ \theta \end{array}\right] \begin{array}{l} \left.\begin{array}{l} \\ \\ \end{array}\right\}\text{Demand nodes} \\ \text{Generator nodes; no ref.} \end{array} \qquad \tilde{p} = \left[\begin{array}{c} \theta \\ P \\ Q \\ V \end{array}\right] \begin{array}{l} \text{Reference node} \\ \left.\begin{array}{l} \\ \\ \end{array}\right\}\text{Demand nodes} \\ \text{Generator nodes} \end{array}$$
$$\tilde{u} = \left[\, P \,\right] \text{Generator nodes} \tag{3.4}$$

where P $(= P_G - P_D)$ is the real power injection at each node, and where P_D is the real power demand (or load) at that node. The reactive power injections at all generator nodes ($Q = Q_G - Q_D$) do not feature in these vectors. This is because Q at each generator node is dependent on, and defined by, the values of P, V and θ at all power system nodes.

The vector contents of this type are typical of the commercially available OPFs considered during the research of this Master's thesis. Hence, the node classifications for this OPF type are:

- PvG for generator nodes, and
- pqD for demand nodes.

See Figure 3.2(b). An OPF with this vector–set (Vector–Set 3.4) is referred to as a 'PvG–type' OPF.

In the other OPF type, the vectors are:

$$\tilde{x} = \begin{bmatrix} \theta \\ V \\ \theta \\ V \end{bmatrix} \begin{matrix} \left.\vphantom{\begin{matrix} \theta \\ V \end{matrix}}\right\} \text{Demand nodes} \\ \left.\vphantom{\begin{matrix} \theta \\ V \end{matrix}}\right\} \text{Generator nodes; no ref.} \end{matrix} \qquad \tilde{p} = \begin{bmatrix} \theta \\ V \\ P \\ Q \end{bmatrix} \begin{matrix} \left.\vphantom{\begin{matrix} \theta \\ V \end{matrix}}\right\} \text{Reference node} \\ \left.\vphantom{\begin{matrix} P \\ Q \end{matrix}}\right\} \text{Demand nodes} \end{matrix} \tag{3.5}$$

$$\tilde{u} = \begin{bmatrix} P \\ Q \end{bmatrix} \left.\vphantom{\begin{matrix} P \\ Q \end{matrix}}\right\} \text{Generator nodes}$$

Here, the OPF algorithm directly controls the real and reactive power injections (P and Q) at generator nodes. Hence, the OPF algorithm also directly controls P_G and Q_G, as implied by the definitions of P and Q. Both V and θ at generator nodes are now dependent on P and Q because they are in $\tilde{x}$. The contents of these vectors were developed to minimise the costs of real and reactive power generation, specifically for the research work presented within this thesis.

The node classifications for this OPF type are:

- PQG for generator nodes, and
- pqD for demand nodes.

When referring to generator and demand nodes together, the term 'pq' is used:

$$\text{pq} = (\text{PQG} \cup \text{pqD}) \tag{3.6}$$

Hence, an OPF with Vector–Set 3.5 is referred to as a 'pq–type' OPF.

The OPF node classification system is therefore, capable of describing the characteristics of nodes in different OPF types.

The term 'swing bus' is not relevant when using an OPF. The dispatch produced by an OPF applies only to a single instant in time. Hence, it might be necessary to use an OPF for every trading period in a spot market to re–optimise the dispatch of real and reactive power generation. During this re–optimisation, the swing bus generator is dispatched in an identical manner to all other generators. Furthermore, the reference voltage magnitude and angle usually associated with the swing bus, can be assigned to any generator during optimisation. Consequently, no distinction is made between the swing bus and other generators. Therefore, the swing bus is a PQG node during the optimisation process (or PvG node depending on the OPF type). However, the swing bus operates as a power–flow Vθ bus (identified with 'S') between each dispatch optimisation, absorbing minor fluctuations of real power load in the power system to maintain the power system frequency.

3.5.3 OPF Independent and Dependent Variables

In a power–flow problem, the known, fixed variables are called the independent variables, and the other unknown variables are the dependent variables. For the OPF formulations in this thesis, the control variables and fixed parameters can be called the independent variables. The state variables (and Q_G in a PvG–type OPF) are therefore the dependent variables. To solve an OPF problem, the user sets the values of the fixed parameters. The OPF algorithm then determines the optimal values of the control parameters using an objective function. The values of these independent variables then determine the values of the dependent variables.

3.6 INTERACTIONS BETWEEN POWER SYSTEM VARIABLES

Figure 3.3 depicts the response (or sensitivity) of one power system variable to a change in another variable. These relationships only exist in networks where the reactance to resistance ratio (X/R) is high.

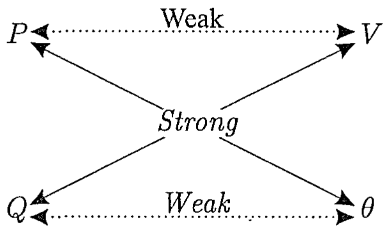


Figure 3.3 Strong responses exist between real power and voltage angle, and between reactive power and voltage magnitude.

3.7 CONCLUSIONS

In this chapter, the philosophy behind Dispatch Based Pricing has been presented. That is marginal prices can be calculated *ex post*, to provide a rational economic explanation for any observed dispatch of real and reactive power generation.

By defining the concept of reactive power losses Read and Ring [1995d] enable reactive power to be priced and traded within a spot pricing market.

Optimal power flow technology is used in this thesis to model the primary purpose of a spot market, to optimise the dispatch of real and reactive power. PvG–type and pq–type OPFs have been defined. These OPF types represent two types of spot market that both include a reactive power sub–market. The power system nodes in each spot market have therefore been classified to reflect their behaviour in the context of the relevant OPF type.

Chapter 4

THE PRIMAL OPF FORMULATION

4.1 INTRODUCTION

Read and Ring used Linear Programming Duality theory to derive the Dispatch Based Pricing model from their proposed non–linear OPF formulation. The stages in this derivation process are:

1. linearising the non–linear OPF formulation;
2. obtaining the dual formulation from this linearised OPF;
3. simplifying the dual formulation to obtain the Dispatch Based Pricing model.

This chapter is concerned with Stage 1.

In this chapter, Read and Ring's non–linear OPF formulation is reclassified to conform with the OPF nomenclature proposed in the previous chapter. This reclassified OPF formulation is then linearised.

The concept of linear programming is introduced using an example within this chapter. This example demonstrates how the linearised OPF formulation is a linear programming problem. Finally, a discussion on how the linearised OPF can be used to calculate marginal prices for real and reactive power is presented.

4.2 A NON–LINEAR OPF FORMULATION

Equations 4.1 to 4.13 describe the non–linear OPF formulation used to derive the pq/PvG Dispatch Based Pricing model used in this thesis. This OPF is a modified version of Read and Ring's OPF formulation (see Appendix C) and is referred to as the pq/PvG OPF formulation. The name indicates that the formulation is the combination of a pq–type OPF formulation and a PvG–type OPF formulation. This is discussed further in Chapters 6, 7 and 8.

The modifications consisted of reclassifying Read and Ring's formulation equations with respect to the pq and PvG node classifications defined in the previous chapter.

THE NON–LINEAR pq/PvG OPF FORMULATION

$$\underset{\mathbf{P}_D^{PX},\mathbf{Q}_D^{PX},\mathbf{P}_G^{PX},\mathbf{Q}_G^{PX},\mathbf{V}^{PX}}{\textbf{Minimise}} \qquad Costs(\mathbf{P}_G^{PX},\mathbf{Q}_G^{PX}) \tag{4.1}$$

subject to:

CONSERVATION OF POWER

$$\sum_{i\in PX}(P_{Gi}-P_{Di})-L_P\left(\mathbf{P}_G^{PX}-\mathbf{P}_D^{PX},\ \mathbf{Q}_G^{pq}-\mathbf{Q}_D^{pq},\mathbf{V}^{PvG}\right)=0 \tag{4.2}$$

$$\sum_{i\in PX}(Q_{Gi}-Q_{Di})-L_Q\left(\mathbf{P}_G^{PX}-\mathbf{P}_D^{PX},\ \mathbf{Q}_G^{pq}-\mathbf{Q}_D^{pq},\mathbf{V}^{PvG}\right)=0 \tag{4.3}$$

DEPENDENT REACTIVE POWER INJECTION AT PvG NODES

$$-Q_n\left(\mathbf{P}_G^{PX}-\mathbf{P}_D^{PX},\mathbf{Q}_G^{pq}-\mathbf{Q}_D^{pq},\mathbf{V}^{PvG}\right)+(Q_{Gn}-Q_{Dn})=0 \qquad \forall n\in \mathrm{PvG} \tag{4.4}$$

DEPENDENT VOLTAGE AT pq NODES

$$-V_n\left(\mathbf{P}_G^{PX}-\mathbf{P}_D^{PX},\mathbf{Q}_G^{pq}-\mathbf{Q}_D^{pq},\mathbf{V}^{PvG}\right)+V_n=0 \qquad \forall n\in \mathrm{pq} \tag{4.5}$$

TRANSMISSION LINE FLOWS

$$-\bar{P}_k\left(\mathbf{P}_G^{PX}-\mathbf{P}_D^{PX},\mathbf{Q}_G^{pq}-\mathbf{Q}_D^{pq},\mathbf{V}^{PvG}\right)+\bar{P}_k=0 \qquad \forall k\in \mathrm{K} \tag{4.6}$$

$$-\bar{Q}_k\left(\mathbf{P}_G^{PX}-\mathbf{P}_D^{PX},\mathbf{Q}_G^{pq}-\mathbf{Q}_D^{pq},\mathbf{V}^{PvG}\right)+\bar{Q}_k=0 \qquad \forall k\in \mathrm{K} \tag{4.7}$$

REAL AND REACTIVE GENERATION AND VOLTAGE SETTINGS

$$P_{Gi}^{min}\le P_{Gi}\le P_{Gi}^{max} \qquad \forall i\in \mathrm{PX} \tag{4.8}$$

$$Q_{Gi}^{min}\le Q_{Gi}\le Q_{Gi}^{max} \qquad \forall i\in \mathrm{PX} \tag{4.9}$$

$$V_i^{min}\le V_i\le V_i^{max} \qquad \forall i\in \mathrm{PX} \tag{4.10}$$

REAL AND REACTIVE POWER LOAD SETTINGS

$$P_{Di}=P_{Di}^{set} \qquad \forall i\in \mathrm{PX} \tag{4.11}$$

$$Q_{Di}=Q_{Di}^{set} \qquad \forall i\in \mathrm{PX} \tag{4.12}$$

TRANSMISSION LINE THERMAL LIMITS

$$\bar{P}_k^2+\bar{Q}_k^2\le T_k^{max} \qquad \forall k\in \mathrm{K} \tag{4.13}$$

Further, the swing bus in Read and Ring's OPF formulation was omitted from the pq/PvG OPF formulation since there is no swing bus in OPF technology.

The objective function of the pq/PvG OPF formulation (Equation 4.1) includes non–zero generation cost function for reactive power, as well as for real power. Accordingly, this pq/PvG OPF models a spot market where both real and reactive power are economically dispatched.

The goal of the pq/PvG OPF formulation is to optimise the usage of 'energy resources' within the power system, so as to minimise the objective function. Each resource is represented by a variable in this formulation. The behaviour of these resources are described by the equality constraints (Equations 4.2 to 4.7) and the availability of these resources are described by the inequality constraints (Equations 4.8 to 4.13).

4.2.1 The Equality Constraints

The equality constraints represent the physical characteristics of the power system variables. Hence, these constraints ensure that the OPF will calculate optimal values for the variables which represent an actual power system dispatch [Ring 1995, pp. 49-50].

4.2.1.1 Conservation of Power

Equations 4.2 and 4.3 are used to conserve real and reactive power within the system. For example, Equation 4.3 sums the nett reactive power injection at all nodes (generation minus demand) and ensures that this is equal to the reactive power losses in the system. The total loss of reactive power (L_Q, as defined in Section 3.3) is a function of the independent variables of the power system: $\mathbf{P}_G^{PX}$, $\mathbf{P}_D^{PX}$, $\mathbf{Q}_G^{pq}$, $\mathbf{Q}_D^{pq}$ and $\mathbf{V}^{PvG}$

In this thesis, PX is defined as the set of all nodes:

$$\mathrm{PX} = (\mathrm{PvG} \cup \mathrm{pq})$$

Note that Read and Ring use power–flow terminology, and thus use PXS to identify a set of all nodes that includes a swing bus:

$$\begin{aligned} \mathrm{PXS} &= (\mathrm{PV} \cup \mathrm{PQ} \cup \mathrm{S}) \\ &= (\mathrm{PX} \cup \mathrm{S}) \end{aligned}$$

4.2.1.2 Dependent Reactive Power Injection at PvG–Type Nodes

This equality constraint (Equation 4.4) is used to reflect the dependent nature of reactive power at PvG nodes. At a node n, the nett injection of reactive power into the network (Q_n) is equal to the difference between the generation and demand for reactive power at that node:

$$Q_n = Q_{Gn} - Q_{Dn}$$

Q_n represents a function, dependent on the independent power system variables.

4.2.1.3 Dependent Voltage at pq–Type Nodes

At a pq node n, the dependent nature of voltage magnitude (V_n) is described by Equation 4.5. At pq nodes, V_n is a function of the independent power system variables. This equation is valid for use with both PQG and pqD nodes (ref. Equation 3.6).

4.2.1.4 Transmission Line Flows

Equations 4.6 and 4.7 describe the dependency of average real and reactive power flow in a branch k on the independent power system variables.

4.2.2 The Inequality Constraints

Each inequality constraint (Equations 4.8 to 4.13) represents the finite availability of an energy resource. The limits that define the availability of each resource are:

- P_G^{min}: The minimum real power generation limit is defined by the physical capabilities of a generator. It need not be zero, and may be influenced by factors such as ramp rates [Ring 1995], or increased operating costs at low generation levels.

- P_G^{max},Q_G^{min},Q_G^{max}: These represent the generating limits for real and reactive power at each node. The maximum limits are usually set to zero for non–generator nodes. If Q_G^{min} is negative, an inductive power system component is present at that node. This may be a generator capable of absorbing reactive power.

- P_D^{set},Q_D^{set}: These are the demands for real and reactive power at each node, set by customers connected to the power system.

- V^{min},V^{max}: These limits restrict the range of the voltage magnitude at each node.

- T_k^{max}: This defines the square of the maximum MVA loading of Branch k. This is also referred to as the thermal limit, because exceeding the MVA limit causes excessive heating of the branch.

4.3 OPF FORMULATION MODIFICATIONS

Appendix C presents an OPF formulation taken from the work of Ring [1995], and Read and Ring [1995d]. In that OPF formulation, power–flow classifications are used to indicate whether each equation is to be applied to a generator node or a demand node.

In discussions with Read and Ring, these power–flow classifications caused confusion as to how their Dispatch Based Pricing equations should be interpreted and applied. Consequently, their OPF formulation has been modified according to the node classifications defined in Section 3.5.2, resulting in the pq/PvG OPF formulation (Equations 4.1 to 4.13). This has removed any confusion surrounding the interpretation and application of the resultant Dispatch Based Pricing equations (5.1 to 5.9), which are derived from the pq/PvG OPF formulation. Table 4.1 summarises the changes made to Read and Ring's OPF formulation.

Table 4.1 Summary of modifications made to the OPF formulation proposed by Read and Ring (ref. Appendix C).

	Read and Ring OPF	**pq/PvG OPF**
Independent Variables:	$Q_G^{PQ} - Q_D^{PQ}$ V^{PVS}	$Q_G^{pq} - Q_D^{pq}$ V^{PvG}
Dependent Voltage Constraint:	$\forall i \in \mathrm{PQ}$	$\forall i \in \mathrm{pq}$
Dependent Reactive Power Constraint:	$\forall i \in \mathrm{PV}$	$\forall i \in \mathrm{PvG}$
Resource Constraint Settings:	$\forall i \in \mathrm{PXS}$	$\forall i \in \mathrm{PX}$

The changes indicate that Read and Ring assumed a mixed PvG/pq–type OPF formulation. In addition however, they identified a swing bus (i.e. 'S') having assumed that the swing bus operates as a $V\theta$ node during the optimisation of the dispatch. Since the swing bus is just a normal generator node during optimisation, 'S' is not included in the modified OPF formulation (i.e. pq/PvG OPF).

4.4 LINEARISING THE OPF FORMULATION

4.4.1 An OPF Linear Program - The Primal Problem

The non–linear pq/PvG OPF optimises the usage of energy resources, to obtain a power system dispatch where the total generation cost is minimised. With *ex post* pricing however, a dispatch already exists. Given an observed dispatch therefore, the aim of an *ex post* OPF is to minimise the change in the total generation cost in response to any change in the real and/or reactive power demand. This is achieved by optimising the changes of all energy resources with respect to marginal increases in each of the following independent resource variables:

$$P_i \qquad \forall i \in \mathrm{PX}$$
$$Q_i \qquad \forall i \in \mathrm{pq}$$
$$V_i \qquad \forall i \in \mathrm{PvG}$$

An *ex post* OPF can be obtained by linearising all variables of the pq/PvG OPF equations around an operating point (the observed or existing dispatch). For example, the above independent variables become:

$$P_i^{\pm} \qquad \forall i \in \mathrm{PX}$$
$$Q_i^{\pm} \qquad \forall i \in \mathrm{pq}$$
$$V_i \qquad \forall i \in \mathrm{PvG}$$

where + and - indicate the next and last units of each resource. V_i remains unchanged because it is already linear. Appendix D summarises the process used to linearise the pq/PvG OPF formulation. The resultant equations (D.2–D.17) comprise a linear program, referred to throughout this thesis as the fundamental or 'primal' (linear programming) problem [Ring 1995]. A linear program is just a constrained optimisation problem composed of a linear objective function and linear constraints. Different OPF formulations result in different primal linear programming problems.

The variables of this primal problem are 'marginal variables' representing changes in the energy resources. The maximum amount by which each resource can change is defined by the linearised constraints. For example, Q_{Gi}^{+} indicates the marginal (incremental) increase in the amount of reactive power generation being injected into the power system by the generator at Node i, in response to a marginal increase in another power system variable. However, Equation D.11 restricts the amount by which Generator i can increase its generation in response to the other increased power system variable. The *ex post* objective function of Appendix D calculates the change in the total cost of generation, with respect to this change in generation.

With respect to an observed dispatch, solving this linear program determines which generators should be used to supply the next unit of real or reactive power, so as to minimise the *ex post* (i.e. linearised) objective function.

4.4.2 An Example of Linear Programming

Linear programming deals with the problem of maximising or minimising a linear function with respect to a set of linear constraints [Bazaraa and Jarvis 1977]. Below is an example of a simple, 2–variable, linear program:

$$\begin{array}{lll} \text{Minimise} & f(x_1, x_2) = 3x_1 + 5x_2 & \\ \text{subject to:} & -x_1 - 2x_2 \geq -12 & : \beta_1 \\ & x_1 + x_2 \geq 2 & : \beta_2 \\ & x_1, x_2 \geq 0 & \end{array}$$

This linear program is represented graphically in Figure 4.1.

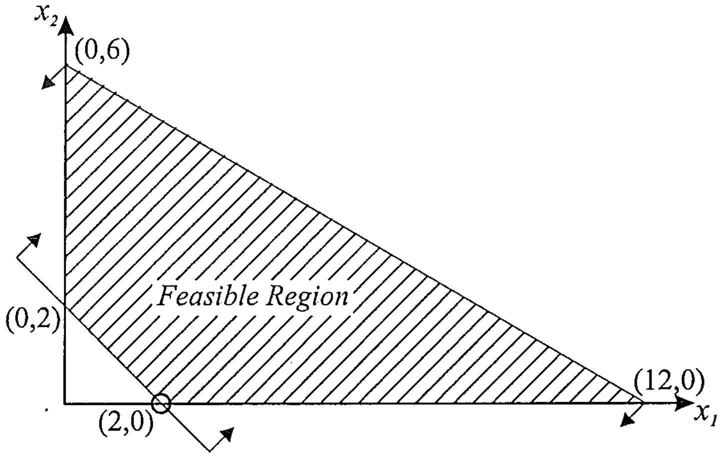


Figure 4.1 A graphical representation of the 2–variable linear program.

The set of all points that satisfies the constraints of the linear program is called the 'feasible region'. However, the point (2,0) is where the variables are optimised such that the objective function $f(x_1, x_2)$ is minimised. With respect to the linearised OPF therefore, each feasible point represents a valid dispatch, since the constraints define the physical properties of a power system. But, the vertex (2,0) represents the optimal dispatch. The optimal point always resides on a vertex of the feasible region.

The variables β_1, β_2 are the shadow prices (or LaGrange multipliers as described in Appendix B) corresponding to the constraints of this linear programming problem.

4.4.3 Marginal Prices From The Primal Problem

The solution to the primal linear programming problem is a set of optimal values for the marginal, power system variables. The goal of *ex post* pricing however, is to find a set of marginal prices for the observed dispatch (which is described by the observed values of the marginal power system variables).

The primal problem can be solved using the gradient method. The gradient method assigns a LaGrange multiplier to each resource constraint. Economically, these multipliers are known as 'shadow prices'. Read and Ring offer a description of the shadow price:

> "Each shadow price represents the change in primal objective function value which would be brought about if we could change the right hand side of the corresponding primal constraint by one unit. As the primal constraints effectively limit resource usage, the shadow prices define the value to the system of having one more (or less) unit of each resource."
>
> *Read and Ring [1995d, Appendix 1]*

That is, each shadow price is the marginal cost of a constrained resource. This is confirmed in Appendix B, where it is demonstrated that the value of each multiplier is equal to the marginal price of the corresponding power system resource. For example, the linearised OPF formulation (Equations D.2 to D.17) has 15 associated shadow prices. The shadow prices for Equations D.15 and D.16 are the marginal costs of real and reactive power demand at all nodes, and are the marginal prices used within a spot market to trade real and reactive power. The behaviour of these shadow prices are the focus of Chapters 9 and 10.

Read and Ring [1995d, Section 2.4] note that it is possible to obtain values for the shadow prices by perturbing each marginal resource variable in turn and resolving the primal problem with each perturbation. The corresponding change in the objective function is the value of the shadow price associated with the perturbed variable. In this regard, this perturbation method is analogous to the gradient method. Read and Ring however, rejected this approach in favour of linear programming duality theory. Duality enables the shadow prices to be solved directly, by forming a new linear program where the shadow prices become the variables to be optimised.

4.5 CONCLUSIONS

In this chapter a non–linear OPF formulation (pq/PvG OPF) has been introduced and described. The pq/PvG OPF formulation is a modified version of the OPF formulation taken from the work of Read and Ring [1995d] and of Ring [1995]. Both the pq/PvG OPF formulation and Read and Ring's OPF are a mixture of the pq–type and PvG–type OPFs described in the previous chapter. Hence, the classes PvG and pq have been used in the pq/PvG OPF formulation to identify power system nodes. Also, the swing bus has been omitted from the pq/PvG OPF formulation.

An *ex post* OPF was created by linearising the pq/PvG OPF formulation; this was Stage 1. The resulting equations (D.2 to D.17) form a primal linear program that can be solved to determine the most economic source from which to obtain the next unit of real or reactive power. However, Read and Ring only use this linear program as the first stage in deriving the Dispatch Based Pricing model.

In Stage 2, the primal linear program is used to derive a dual linear program, in which the shadow prices of the primal problem are solved directly. Stage 3 consists of simplifying this dual linear program to obtain the final pq/PvG Dispatch Based Pricing model. Stages 2 and 3 are the subject of the next chapter.

Chapter 5

DUALITY AND THE DISPATCH BASED PRICING MODEL

5.1 INTRODUCTION

The goal of *ex post* spot pricing is to obtain a set of shadow prices that provides an economic explanation for the observed dispatch. To achieve this goal, Read and Ring used Linear Programming Duality theory. They used duality theory to form a dual linear program, which was then rearranged to obtain the equations of their Dispatch Based Pricing model.

In this chapter, stages 2 and 3 in the derivation of the pq/PvG Dispatch Based Pricing model are presented. First though, the concept of duality is explained through a simple example. Duality is also explained in the context of a hypothetical competitive electricity market. The equations of the pq/PvG Dispatch Based Pricing model are then described with respect to the economic dispatch of real and reactive power.

Read and Ring used certain terms and symbols to describe the application of Dispatch Based Pricing to a real power spot market. These Dispatch Based Pricing terms (as presented in Ring [1995]) are redefined in this chapter for application to spot markets that include reactive power sub–markets. These markets are presented in subsequent chapters.

5.2 AN EXAMPLE OF A DUAL LINEAR PROGRAMMING PROBLEM

The variables in a dual linear program are the shadow prices (or LaGrange multipliers, or marginal prices) of the primal linear program. Take the linear programming example in Section 4.4.2. Given that the shadow prices of the constraints in that primal linear program are β_1 and β_2, the dual linear program is formulated as:

$$
\begin{array}{lll}
\text{Maximise} & w(\beta_1, \beta_2) = -12\beta_1 + 2\beta_2 & \\
\text{subject to:} & -\beta_1 + \beta_2 \leq 3 & : x_1 \\
 & -2\beta_1 + \beta_2 \leq 5 & : x_2 \\
 & \beta_1, \beta_2 \geq 0 &
\end{array}
$$

This dual linear program demonstrates that the shadow prices (or marginal prices) can be solved directly. Shadow prices are sometimes called dual variables because they are used to form the dual problem. Note that the shadow prices of these 'dual' constraints are the primal variables in the original linear programming example. Standard linear programming texts such as Bazaraa and Jarvis [1977] present duality theory in detail.

Read and Ring defined the relationship between a primal linear programming problem and its dual. For example, if β_1 and β_2 are actually marginal prices representing the value of two energy resources, they stated that:

> "If the primal objective is to minimise the costs (*of any resources used*), the dual objective is to maximise the value of the resources available, as defined by the constraints of the primal problem."
>
> *Read and Ring [1995d, Appendix 1]*

Read and Ring took the dual linear programmming approach, since their goal was to obtain a set of *ex post* marginal prices that represent the maximum value of the resources available to supply the next unit of real or reactive power. They did not need to solve the primal (*ex post* OPF) problem, which determines how to optimally dispatch the next unit of power so as to minimise the costs of resources used.

5.3 FORMING THE DUAL PRICING PROBLEM

The procedure used to obtain the example, dual linear program in Section 5.2 was applied to the *ex post* pricing, primal linear program (Equations D.2 to D.17). The result is the dual linear programming problem described by Equations D.18 to D.28. As in the examples above, the shadow (i.e. marginal) prices of the energy resources in the primal problem are now the variables of this dual problem. Hence, the marginal prices of the power system resources can be solved directly.

The marginal prices are optimised in order to maximise the objective function. However, each constraint equation in this dual problem prevents the sum of the shadow prices in that equation from exceeding the unit cost of the corresponding primal variable (resource). Note that the right hand side of a dual constraint defines the unit cost of the power system resource (i.e. the dual shadow price or primal variable) associated with that constraint.

5.4 AN ECONOMIC INTERPRETATION OF DUALITY

The competitive nature of participants in a decentralised power system exemplifies the concept of duality. In this discussion, the terms 'Demand–side' and 'Supply–side' are used to represent all consumers of electricity and all generators respectively. It

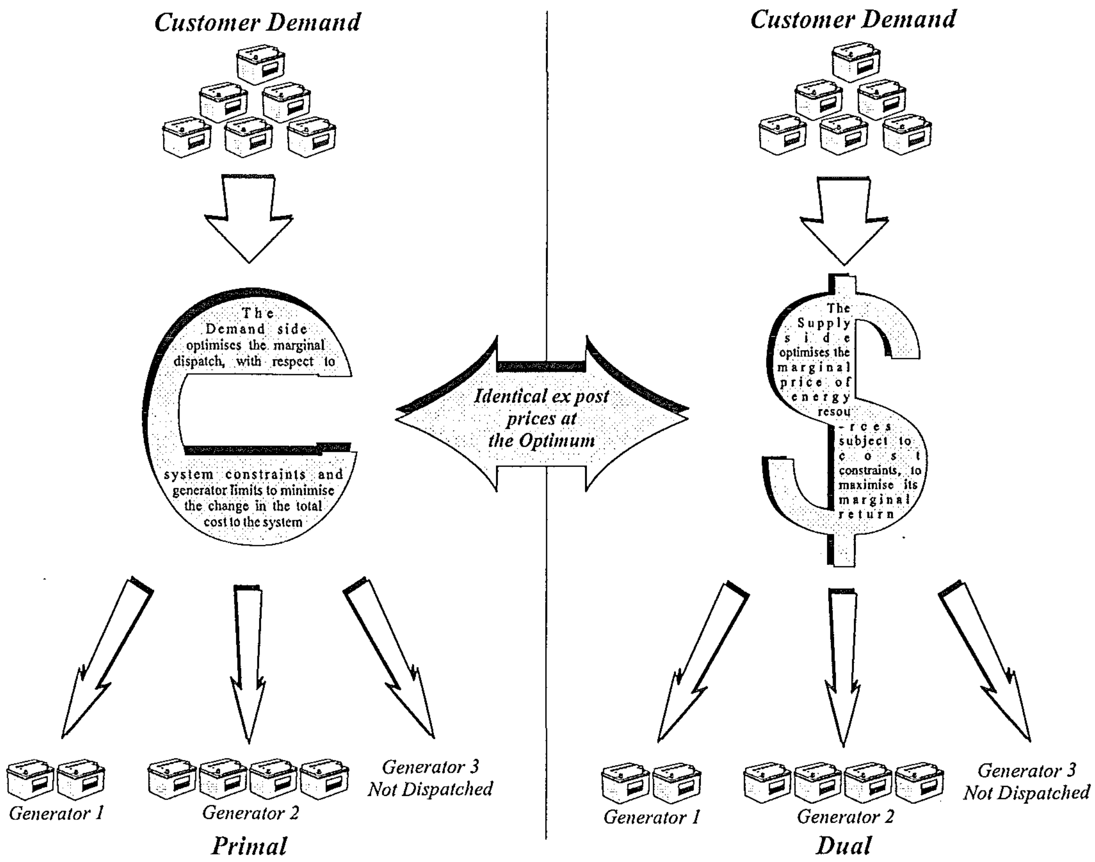


Figure 5.1 The optimal dispatch of real and reactive power using either cost considerations or pricing mechanisms.

is assumed that the primal objective function minimises the total cost of real and reactive power generation. The discussion utilises an example presented in Bazaraa and Jarvis [1977, pp. 249-250]

Marginal prices are only valid for a single instant in time, when the dispatch of real and reactive power is optimised. In the New Zealand spot market this instant is the start of each half–hourly trading period. At this point in time the Demand–side requires a certain amount of real and reactive power. Hence, the corresponding marginal prices are used to influence the behaviour of all market participants, to obtain the most economic way of dispatching the generators in order to supply this requirement.

5.4.1 The Primal Perspective

Consider the illustration in Figure 5.1. Each generator has its own unit generation costs, c_{P_i} and c_{Q_i}. These apply only to the current trading period, and reflect the offers submitted to the real–time physical market by the company owning that generator (see Section 2.5.2.2). From a primal perspective, the Demand–side agrees to pay the total cost of generation required to meet their load requirement (as defined by the unit generation costs). However, they request the right to optimise the generation profile so as to minimise the total cost. They achieve this by controlling which of the three generators are dispatched and quantity of power supplied by each.

The dispatch decisions made by the Demand–side are however, restricted by the physical properties of the power system. These properties are represented by the constraints of the primal problem. The minimum and maximum generation limits each company sets for its generating machines are included among these constraints.

Generation limits indicate the bounds on the amount of power each company is willing to let its machines generate at the unit generation cost it sets for that trading period. These may represent economic limits rather than the physical limits of the generator. For example, if the Supply–side companies are forced to reduce their unit generation cost of power, they might also reduce their generating limits for the next trading period to prevent the Demand–side taking advantage of the cheap power. This enables the Supply–side to influence the dispatch decisions of the Demand–side.

5.4.2 The Dual Perspective

From the dual perspective, the Demand–side agrees to pay the marginal prices β_P and β_Q for another unit of real or reactive power as set by the Supply–side, rather than controlling the generation output of these three generators to obtain an optimal dispatch. However, the Demand–side stipulates that these prices do not exceed the unit generation costs (c_{P_i} and c_{Q_i}) of the three generators. Such stipulations define the constraints in the dual problem.

The Supply–side will then choose a set of marginal prices to maximise their collective return given these (dual) price constraints. The constraints represent the limits of the marginal price at which the Demand–side is willing to buy power. Hence, the customer will refuse to use power from a generator if the marginal price of power at that generator node exceeds its unit generation cost (Generator 3, for example was not dispatched because either $c_{P3} > \beta_{P3}$ or $c_{Q3} > \beta_{Q3}$). In this way, the Demand–side influences the amount of power supplied by each generator and is able to influence the Supply–side regarding the setting of marginal prices.

5.4.3 The Optimal Solution

In a spot market, an equilibrium generator profile and an equilibrium set of prices are said to be obtained. The equilibrium is where the minimal generation cost is equal to the maximal collective return. It is a result of the influence each side has on the other. It is the 'constrained relative minimum' introduced in Appendix B.

Apply the example in Figure 5.1 to the *ex post* primal and dual problems in Appendix D. The dispatch already exists and is assumed to be optimal. The variables represent marginal (incremental) changes in power system resources (see Section 4.4.1). Therefore:

- the primal problem controls which generator(s) are used to supply the demand at each node for another unit of real or reactive power, so as to minimise the change in the total generation cost (the primal objective function) for supplying that unit. This change in the generation cost is the marginal price of power for that node.
- the dual problem determines the (marginal) prices of the power system resources that are used to charge for the next unit of real or reactive power at each node, so as maximise the change in the collective return (the dual objective function) for supplying that unit. This change in the collective return is the optimal contribution of power from the generator at that node.

The objective functions of these two problems are equal at the optimum. Thus, marginal prices for real and reactive power that are acceptable to both Supply and Demand sides, are delivered to all market participants. Bazaraa and Jarvis note the two possible approaches to obtaining these acceptable prices:

> "the primal objective function is delivered by cost considerations and the dual objective is arrived at by a pricing mechanism."
>
> *Bazaraa and Jarvis [1977, Section 6.3]*

5.5 ECONOMIC DISPATCH EXAMPLES

Two examples illustrating the marginal dispatch of reactive power are presented below, to aid the explanation of the pq/PvG Dispatch Based Pricing equations. These examples are based on the real power dispatch example presented in Read and Ring [1995b]. However, Read and Ring's example has been modified to demonstrate the dispatch of reactive power in a pq–type spot market, as proposed in Chapter 7. Thus, both generator nodes in these examples are PQG nodes, but with some local reactive power demand.

5.5.1 An Unconstrained Dispatch

Consider the power system in Figure 5.2. Generator 1 has a reactive power unit generation cost of \$9/MVAr and Generator 2 has a unit generation cost of \$12/MVAr[1].

[1]The 'unit generation cost' is also known as the 'offer price', where the 'offer price' = 'production (i.e. fuel) cost' + profit. The offer price is also the 'marginal fuel cost' in Equation 2.1. From a consumer frame of reference, the offer price is seen as a cost; hence the term 'unit generation cost'. In this thesis, 'unit generation cost' is used and represents the amount a generating company will be paid for producing an extra unit of power. It is represented by the symbol 'c'. The 'marginal price' for

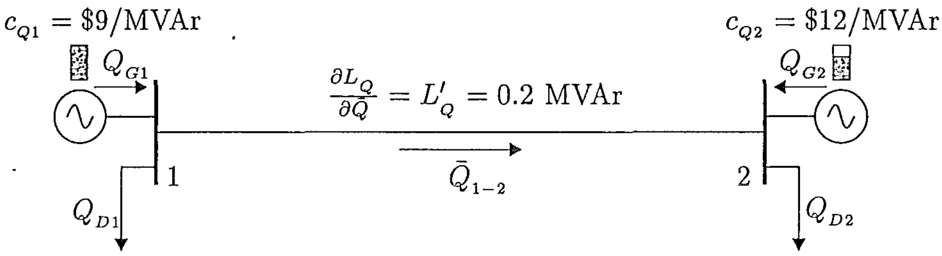


Figure 5.2 An unconstrained, economic dispatch of reactive power in a simple two–node power system.

The impedance in Transmission Line 1–2 is such, that the reactive power losses will increase by 0.2 MVAr for a 1 MVAr increase in the average flow of power along the line ($\bar{Q}_{1-2}$). Also, Generator 1 is fully dispatched. It is supplying the reactive power demand at Node 1 (Q_{D1}) and part of the demand at Node 2 (Q_{D2}). Generator 2 supplies the remainder of Q_{D2}.

The next incremental increase in the demand for reactive power at Node 2 (Q_{D2}) will be supplied by Generator 2, since it is partially loaded. The marginal price (i.e. the market price[2]) of reactive power at Node 2 (β_{Q2}) is equal to (i.e. fixed at) the unit generation cost of Generator 2 (c_{Q2}). That is, $\beta_{Q2} = c_{Q2} = \$12/\mathrm{MVAr}$. If this were not so, the market would not have used Generator 2 to supply the incremental increase. This is physically consistent in that supplying a change in Q_{D2} from Generator 2 does not change the real or reactive power losses, resulting in a zero cost for marginal losses. If the losses changed, the cost of marginal losses would be non–zero, making $\beta_{Q2} \neq c_{Q2}$.

This marginal price can be used to find the marginal price of reactive power at Node 1 (β_{Q1}). If the customer at Node 1 decreases its reactive power demand (Q_{D1}) by 1 MVAr, Generator 2 will decrease its reactive power output by $(1 - L'_Q) = 0.8$ MVAr. Therefore, the marginal price that a customer at Node 1 will pay for reactive power is:

$$\begin{aligned} \beta_{Q1} &= (1 - L'_Q)\beta_{Q2} \\ &= 0.8 \times 12 \\ &= \$9.6/\mathrm{MVAr} \end{aligned}$$

This marginal price is greater than the unit generation cost of Generator 1 (i.e. $\beta_{Q1} > c_{Q1}$). This indicates that, at Node 1, the market is willing to pay \$0.6/MVAr to relax the upper generation limit on Generator 1 in order to gain another 1 MVAr of reactive power from this cheaper generator. Note that Generator 1 is making a profit of \$0.6/MVAr.

If instead, the reactive power unit generation cost was $c_{Q1} = \$11/\mathrm{MVAr}$, then

supplying an incremental demand for power at a node, equals the unit generation cost plus the other components of Equation 2.1.

[2]The marginal price is the price the market is willing to pay for another unit of power.

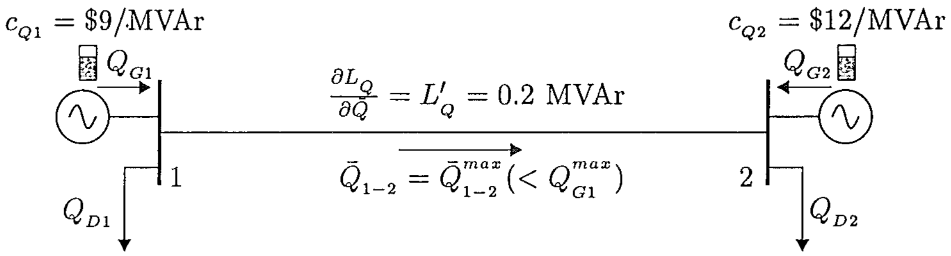


Figure 5.3 An economic dispatch of reactive power in a simple two–node power system. A binding reactive power, flow limit exists on this transmission line.

Generator 1 would not have been dispatched. This is because β_{Q1} is now less that c_{Q1}, and Generator 1 would be losing \$1.4/MVAr if it dispatched.

5.5.2 A Constrained Dispatch

Consider again the case where $c_{Q1} = \$9/\text{MVAr}$. However, the average flow of reactive power in Transmission Line 1–2 ($\bar{Q}_{1-2}$) is now equal to the maximum capacity of the line ($\bar{Q}^{max}_{1-2}$). Figure 5.3 shows that Generator 1 is only partially loaded. This is because the flow limit is preventing the demand at Node 2 from utilising the full reactive power capacity of Generator 1.

The marginal price of reactive power at Node 1 is now $\beta_{Q1} = \$9/\text{MVAr}$, because Generator 1 can still generate more reactive power to meet the local demand, Q_{D1}. However, β_{Q1} was previously calculated at \$9.6/MVAr. This apparent price inconsistency of \$0.6/MVAr is the cost to the system of relieving the flow constraint, in order to allow an extra unit of reactive power to be transmitted through the transmission line. This cost on the power flow constraint is calculated thus:

- Increasing the flow by 1 MVAr decreases the total generation cost to the power system by \$12/MVAr, but
- the total generation cost simultaneously increases by \$9/MVAr. Also,
- the cost of marginal losses (supplied by Generator 2) is $\$12 \times 0.2 = \$2.4/\text{MVAr}$.
- Hence, the cost of the flow constraint is $9 + 2.4 - 12 = -\$0.6/\text{MVAr}$, or $\eta_{Q1-2} = \$0.6/\text{MVAr}$.

That is:

$$\begin{aligned}
\beta_{Q1} &= (1 - L'_Q)\beta_{Q2} - \eta_{Q1-2} \\
&= \beta_{Q2} - L'_Q\beta_{Q2} - \eta_{Q1-2} \\
&= 12 - 2.4 - 0.6 \\
&= \$9/\text{MVAr}
\end{aligned}$$

The marginal prices at Nodes 1 and 2 are therefore economically consistent with each other in both the unconstrained dispatch example and this constrained dispatch example.

5.6 DISPATCH BASED PRICING DEFINITIONS

From the above examples, definitions can be created for certain Dispatch Based Pricing terms used throughout this thesis.

> **DBP 5.1 Price Consistency:** *The set of marginal prices corresponding to all nodes in a power system is defined to be consistent, when the differences between the marginal prices of connected nodes are explained by the cost components of the marginal prices. The marginal cost components include the marginal costs of generation, total marginal losses and binding constraints (see Figure 5.4).*

In the 'Constrained Dispatch' example, β_{Q1} and β_{Q2} are consistent with each other through the cost of total reactive power marginal losses $\left(\frac{\partial L_Q}{\partial Q}\right)$ and the marginal cost of the flow constraint (η_{Q1-2}). Marginal prices are inconsistent with each other when there is no feasible solution to the dual pricing problem.

> **DBP 5.2 Marginal Generator:** *A generator whose upper and lower generation limits are not binding and whose marginal price is equal to its unit generation cost is defined to be marginal for the current time period (e.g. 1/2 hour). Only this generator adjusts it output for each marginal change in demand for power during that time period. A marginal generator can be marginal for either real or reactive power, or both.*

The marginal generator is different to the swing bus generator defined in Section 3.5.2. A generator can be identified as being marginal when the marginal price (or market price) at that node is equal to (or fixed at) the generation cost of the next unit of power to be produced by that generator (see Figure 5.4). If this were not so, the market would not call on this generator to supply any marginal change in demand. Each generator can be associated with up to two marginal prices. One for real power, and one for reactive power. The two generators in the 'Constrained Dispatch' example are both marginal.

> **DBP 5.3 Non–marginal Generator:** *A generator forced against a generation limit is non–marginal. Such a generator cannot respond to an incremental demand for power that requires it to violate this limit.*

A non–marginal generator is either fully loaded or not dispatched (Generator 1 in Figure 5.2 for example, is non–marginal). The marginal price at the node of a non–marginal generator is always different to the generator's unit generation cost. This deters the market from using this generator to supply any marginal change in demand. The generation limits can be either physical or economic as stated in Section 5.4.

DBP 5.4 Merit Order Dispatch: *An OPF dispatches the set of generators in a power system with respect to a merit order. This merit order is defined by the unit generation costs of the generators. The dispatch of generators starts with the cheapest and ends with the most expensive. In an unconstrained power system the next cheapest generator in the merit order will be dispatched only when the current marginal generator becomes fully loaded.*

A merit order minimises the change in the value of the primal objective function where there is an incremental change in demand.

DBP 5.5 Out of Merit Order Dispatch: *When a power system is constrained, generators are dispatched out of merit order to work around the constraints, so as to minimise the marginal change in the objective function, for an incremental change in demand.*

There are always at least two fixed marginal prices when an 'Out of Merit Order Dispatch' occurs. The relationship between the number of fixed prices and binding primal constraints is defined by DBP 5.6.

DBP 5.6 Marginal Price Criterion: *The number of binding primal constraints equals the number of fixed dual marginal prices minus 1:*

$$(\ \#\mathit{constraints} = \#\mathit{prices} - 1\ )$$

The '-1' in DBP 5.6 indicates there is only one fixed marginal price when the power system is totally unconstrained. This is the unit generation cost at the marginal generator. This fixed marginal price can be for either real or reactive power because this generator can be marginal for either real or reactive power. Each binding constraint necessitates that a generator become marginal for either real or reactive power, so as to fix another marginal price.

The dual pricing problem has a unique solution when DBP 5.6 holds. Too many fixed dual prices result in an over–defined dual problem and an infeasible solution. Too many primal constraints results in an under–defined dual problem and multiple solutions.

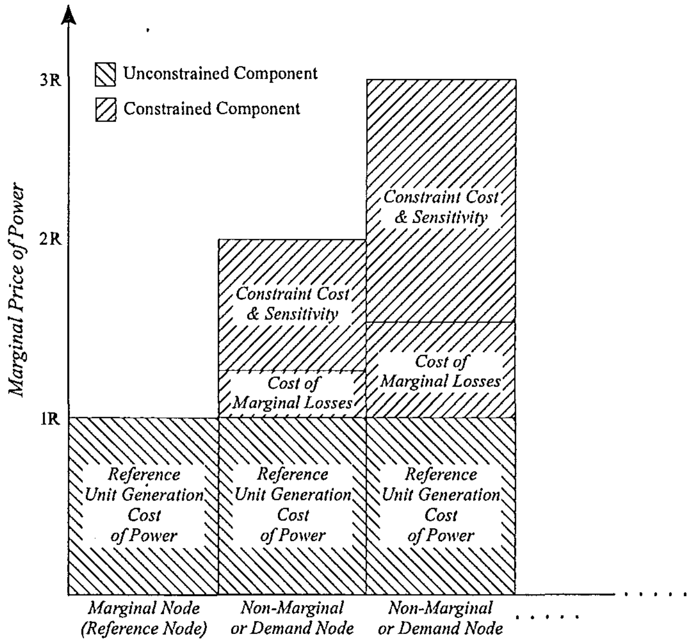


Figure 5.4 Cost components comprising the marginal price at each power system node. This applies to marginal prices for both real and reactive power.

5.6.1 Price Inconsistencies

DBP 5.6 implies that a constraint cost component can be added to the marginal price at each node to explain the inconsistencies between the unit generation costs at the marginal generator nodes. Thus, invoking primal constraints (economic or actual) to explain multiple marginal generators is equivalent to making another generator marginal to transmit power round the constraint.

5.6.2 A Reference Node

Supplying the demand for real or reactive power at a demand Node i (or a non-marginal generator Node i) directly from the marginal generator is economically the same as exporting that demand from the marginal generator to a reference node, and then supplying the demand from there [Ring 1995]. The 'Constrained Dispatch' example however, demonstrates that multiple marginal generators can be collectively used to supply any marginal change in demand in a constrained power system. In this scenario, the generation contributions from all the marginal generators are conceptually accumulated at the reference node, and then exported to Node i as a single block.

The unit generation cost component at demand Node i is therefore the unit generation cost of the reference generator (see Figure 5.4). However, this unit generation cost is the same as the sum of the unit generation costs of every marginal generator, multiplied by the contribution of those generators.

The reference generator has no special status over other generators. It is dispatched by the OPF according to the merit order. Hence, this reference node is only a concept, used to simplify the mathematics

5.7 DISPATCHING REACTIVE POWER GENERATION

Reactive power generation can be dispatched as either an independent or dependent resource. The examples in Section 5.5 represent a pq–type spot market where reactive power generation is independently dispatched. To illustrate, consider the demand for an extra unit of reactive power (e.g. 1 MVAr) in a pq–type spot market. Physically, the system operator of this market would independently dispatch reactive power to meet that demand by instructing the marginal power station to generate an extra 1 MVAr (assuming no losses). In response, the control system of the station would adjust the voltage set–points of the station AVR's until the desired 1 MVAr output is obtained.

Consider the alternative, a demand for an extra 1 MVAr in a PvG–type spot market. In this market every power station is operated by voltage set–point. Consequently, the system operator will need to estimate the voltage set–point of the marginal station (as well as the levels of all other independent power system resources) necessary to obtain the required 1 MVAr output. In this way, reactive power is dependently dispatched. If however, the voltage profile must remain constant, then the 1 MVAr can only be dispatched by changing the independent resources, such as P_G.

5.8 SOME RULES GOVERNING DUALITY THEORY

In Dispatch Based Pricing there are five main rules that govern the relationship between the primal and dual pricing problems.

Duality 5.1 *For every binding primal constraint there is a free dual variable.*

Duality 5.2 *For every free primal variable there is a binding dual constraint.*

Duality 5.3 *For every non-binding primal constraint there is a fixed dual variable.*

Duality 5.4 *For every fixed primal variable there is a non–binding dual constraint.*

Duality 5.5 Complimentary Slackness: *For a non–binding constraint, the corresponding dual variable is zero.*

These rules are explained below in the context of the pq/PvG Dispatch Based Pricing equations.

5.9 THE pq/PvG DISPATCH BASED PRICING MODEL

The final formulation of the pq/PvG Dispatch Based Pricing model is obtained by simplifying and rearranging the dual linear programming problem presented in Appendix D. This process is described in detail by Read and Ring [1995d]. This Dispatch Based Pricing model is described by Equations 5.1 to 5.9. The changes made in Chapter 4 to Read and Ring's OPF are evident in these equations. An explanation of these equations in the context of Dispatch Based Pricing and duality theory is presented in the remainder of this chapter.

5.9.1 A Description of Nomenclature

The following list describes the main symbolic conventions used in the pq/PvG Dispatch Based Pricing formulation (Equations 5.1 to 5.9):

- $\frac{\partial V_n}{\partial Q_i}$ All partial derivatives perform similar functions. Such derivatives are called 'linear sensitivity coefficients' [Wood and Wollenberg 1996]. For example, this coefficient describes the sensitivity (or change) in voltage magnitude at Node n, due to an incremental increase in reactive power injection at Node i.

  Each derivative is multiplied by a shadow price. Therefore, these derivatives can also been seen as conversion factors, converting the units of the shadow price into the units of the variable on the left–hand side of the corresponding Dispatch Based Pricing equation.

- P_i and Q_i These represent real and reactive power injection at Node i. For example, $Q_i = Q_{Gi} - Q_{Di}$. Hence, the partial derivative described above will be positive if Q_i is incrementally increased, assuming a corresponding positive change in V_n. An incremental increase in demand for power is equivalent to a negative increase in power injection.

- $\langle z \rangle$ This notation says that the term represented by z only appears in the Dispatch Based Pricing equations if the corresponding primal constraint is binding (see Duality 5.5).

- c_P, c_Q These are used to simplify the Dispatch Based Pricing notation, because the generation costs of the next (c_P^+, c_Q^+) and last (c_P^-, c_Q^-) units of real and reactive power are assumed to be equal in this thesis. For example, the unit generation cost of reactive power at Node i is c_{Qi} $\left(= c_{Qi}^+ = c_{Qi}^-\right)$.

THE pq/PvG DISPATCH BASED PRICING MODEL

$$\underset{\substack{\chi^{K},\upsilon_P^{+PX},\upsilon_P^{-PX};\upsilon_Q^{+PX},\upsilon_Q^{-PX},\upsilon_V^{+PX},\upsilon_V^{-PX}\ \geq 0\\ \lambda_P,\lambda_Q,\beta_P^{PX},\beta_Q^{PX},\mu_Q^{PvG},\mu_V^{pq}}}{\text{MAXIMISE}} \quad \textit{The pq/PvG, Dual Linear Program Objective Function (Equation D.18)} \tag{5.1}$$

subject to:

Marginal Price Equations

$$\beta_{Pi} = \lambda_P\left(1-\frac{\partial L_P}{\partial P_i}\right) - \lambda_Q\frac{\partial L_Q}{\partial P_i} - \sum_{n\in \mathrm{PvG}}\mu_{Qn}\frac{\partial Q_n}{\partial P_i} - \sum_{n\in \mathrm{pq}}\langle\mu_{Vn}\rangle\left\langle\frac{\partial V_n}{\partial P_i}\right\rangle - 2\sum_{k\in \mathrm{K}}\langle\chi_k\rangle\left(\left\langle\bar{P}_k^*\frac{\partial\bar{P}_k}{\partial P_i}\right\rangle+\left\langle\bar{Q}_k^*\frac{\partial\bar{Q}_k}{\partial P_i}\right\rangle\right) \quad \forall i\in \mathrm{PX} \tag{5.2}$$

$$\beta_{Qi} = -\lambda_P\frac{\partial L_P}{\partial Q_i} + \lambda_Q\left(1-\frac{\partial L_Q}{\partial Q_i}\right) - \sum_{n\in \mathrm{PvG}}\mu_{Qn}\frac{\partial Q_n}{\partial Q_i} - \sum_{n\in \mathrm{pq}}\langle\mu_{Vn}\rangle\left\langle\frac{\partial V_n}{\partial Q_i}\right\rangle - 2\sum_{k\in \mathrm{K}}\langle\chi_k\rangle\left(\left\langle\bar{P}_k^*\frac{\partial\bar{P}_k}{\partial Q_i}\right\rangle+\left\langle\bar{Q}_k^*\frac{\partial\bar{Q}_k}{\partial Q_i}\right\rangle\right) \quad \forall i\in \mathrm{pq} \tag{5.3}$$

$$\beta_{Vi} = -\lambda_P\frac{\partial L_P}{\partial V_i} - \lambda_Q\frac{\partial L_Q}{\partial V_i} - \sum_{n\in \mathrm{PvG}}\mu_{Qn}\frac{\partial Q_n}{\partial V_i} - \sum_{n\in \mathrm{pq}}\langle\mu_{Vn}\rangle\left\langle\frac{\partial V_n}{\partial V_i}\right\rangle - 2\sum_{k\in \mathrm{K}}\langle\chi_k\rangle\left(\left\langle\bar{P}_k^*\frac{\partial\bar{P}_k}{\partial V_i}\right\rangle+\left\langle\bar{Q}_k^*\frac{\partial\bar{Q}_k}{\partial V_i}\right\rangle\right) \quad \forall i\in \mathrm{PvG} \tag{5.4}$$

$$\mu_{Qn} = \beta_{Qn} - \lambda_Q \quad \forall n\in \mathrm{PvG} \tag{5.5}$$

$$\mu_{Vn} = \beta_{Vn} \quad \forall n\in \mathrm{pq} \tag{5.6}$$

Price Bound Equations

$$\bar{c}_{Pi} - \left\langle\bar{\upsilon}_{Pi}\right\rangle \leq \beta_{Pi} \leq c^{+}_{Pi} + \left\langle\upsilon^{+}_{Pi}\right\rangle \quad \forall i\in \mathrm{PX} \tag{5.7}$$

$$\bar{c}_{Qi} - \left\langle\bar{\upsilon}_{Qi}\right\rangle \leq \beta_{Qi} \leq c^{+}_{Qi} + \left\langle\upsilon^{+}_{Qi}\right\rangle \quad \forall i\in \mathrm{PX} \tag{5.8}$$

$$\beta_{Vi} = \left\langle\bar{\upsilon}_{Vi}\right\rangle - \left\langle\upsilon^{+}_{Vi}\right\rangle \quad \forall i\in \mathrm{PX} \tag{5.9}$$

5.9.2 A Description of the pq/PvG Dispatch Based Pricing Equations

This section is based on Section 4.3 of Ring [1995]. However, the emphasis is on describing the equations of the pq/PvG Dispatch Based Pricing model, with respect to the previously stated definitions and rules stated in previous sections.

Three marginal prices are associated with each node, one for real power, one for reactive power and one for voltage. Accordingly, each node is represented by three marginal price equations when using the pq/PvG Dispatch Based Pricing model. The node's type determines the three equations used for that node.

5.9.2.1 Real Power Prices at PX Nodes

Equation 5.2 is used to calculate the marginal price of real power at every node within the power system. That is, real power prices are calculated for the PX set of nodes. β_{P_i} is the marginal price of real power demand at Node i because it is the shadow price of Equation D.15.

The first term of this equation can be rewritten as:

$$\lambda_P - \lambda_P \frac{\partial L_P}{\partial P_i}$$

The marginal generator generates sufficient power to supply the required marginal demand, plus marginal real power losses, so as to conserve real power. λ_P is the shadow price of the energy conservation equation constraint equation for real power (D.3). Therefore, λ_P is also the marginal price of real power at the marginal generator.

Ring [1995] stated that λ_P is the cost of supplying the marginal demand from the swing bus generator. This is technically incorrect, as there is no swing bus in an OPF algorithm. However, it is feasible for λ_P to be the marginal price of real power at some arbitrary reference node that has a fixed location. This is convenient, since the marginal generator node can be different from one trading period to the next (see Section 5.6.2). Hence, the pq/PvG Dispatch Based Pricing model assumes that the generator at the reference node supplies every demand for power. Thus, λ_P (and λ_Q) is known as the marginal price for real (and reactive) power at the reference node. λ_P (and λ_Q) is depicted by the 'Reference Unit Generation Cost of Power' in Figure 5.4.

The second part of the term $\left(-\lambda_P \frac{\partial L_P}{\partial P_i}\right)$ is the cost of the total marginal real power losses (supplied by the marginal generator via the reference generator) resulting from an incremental increase in real power injection. If the reference generator is marginal, it supplies any demand at its own node without changing L_P. Therefore, this term is zero for the marginal reference node. Physically, not changing L_P when supplying local demand is true also of all other marginal generators. Mathematically though, this term is non–zero since the local demand is still supplied by these generators via the reference generator. Despite the difference between reality and the pricing model, the customer at a marginal generator node will still only get charged the unit cost of generation of that local marginal generator, and not the costs of transporting that power via the reference generator.

Rewriting this term with respect to demand produces:

$$\lambda_P + \lambda_P \frac{\partial L_P}{\partial P_{Di}} \tag{5.10}$$

Note that an injection is just a negative demand.

- The second term of Equation 5.2, $-\lambda_Q \frac{\partial L_Q}{\partial P_i}$, is the cost of the change in total marginal reactive power losses resulting from an increment in real power injection at PX Node i. These losses are supplied by the marginal generator(s) via the reference generator.
- The third term, $-\sum_{n \in \mathrm{PvG}} \mu_{Qn} \frac{\partial Q_n}{\partial P_i}$, describes the cost of the change in reactive power generation at each PvG node, as a result of an increment in real power injection at PX Node i.
- The fourth term, $-\sum_{n \in \mathrm{pq}} \langle \mu_{Vn} \rangle \left\langle \frac{\partial V_n}{\partial P_i} \right\rangle$, appears for any pq nodes where there is a binding primal voltage constraint. It is the dual cost of the change in pressure on (i.e. tightening or relieving) each constraint due to an increment in real power injection at PX Node i.
- The final term, $-2\sum_{k \in K} \langle \chi_k \rangle \left(\left\langle \bar{P}_k^* \frac{\partial \bar{P}_k}{\partial P_i} \right\rangle + \left\langle \bar{Q}_k^* \frac{\partial \bar{Q}_k}{\partial P_i} \right\rangle \right)$, is only non–zero for thermally constrained branches. It is the dual cost of the change in pressure on any binding primal thermal limits, as a result of an increment in real power injection at PX Node i.

Equation 5.7 sets bounds on the marginal price of real power demand at every node. When a generator at Node i is marginal for real power, P_{Gi} is a free variable and the upper and lower primal generation constraint equations (D.9 and D.10) are not binding. By complimentary slackness (see Duality 5.5), the corresponding shadow prices, υ_{Pi}^+ and υ_{Pi}^-, equal zero. As a result, Equation 5.7 fixes the marginal price of real power at Node i between the generation costs of the next and last units of real power:

$$c_{Pi}^- \leq \beta_{Pi} \leq c_{Pi}^+ \qquad \text{where} \qquad c_{Pi}^- = c_{Pi}^+ \tag{5.11}$$

That is, $\beta_{Pi} = c_{Pi}$. Hence, the marginal price (or market price) for real power at Node i is equal to the unit generation cost of Generator i, as discussed in Section 5.6.

A generator is non–marginal if P_G is forced against a generation limit so that either Equation D.9 or Equation D.10 becomes binding. For example, if a generator is fully loaded the upper generation constraint (D.9) is binding and $\upsilon^+_{Pi} \geq 0$. Thus, Equation 5.7 becomes:

$$c^-_{Pi} \leq \beta_{Pi} \leq c^+_{Pi} + \upsilon^+_{Pi} \tag{5.12}$$

The shadow price (υ^+_{Pi}) is equal to the change in the total generation cost when the primal upper generation limit is relaxed by one unit. υ^+_{Pi} acts as a dual slack variable, allowing the marginal price (β_{Pi}) to move outside the range defined by c^-_{Pi} and c^+_{Pi}. If β_{Pi} moves outside this range, this indicates that Generator i is no longer marginal for real power. The slack variable assumes the difference between the upper unit generation cost (c^+_{Pi}) and the marginal price (β_{Pi}). The value of this marginal price is set by the terms on the right hand side of Equation 5.2. Thus, the marginal price of real power varies freely at generator nodes that are non–marginal for real power. The Dualty 5.1 and Duality 5.4 rules apply here. If instead, a lower generation limit was to become binding (that is, $P_G = P^{min}_G$), complimentary slackness requires that $\upsilon^+_{Pi} = 0$ and $\upsilon^-_{Pi} \geq 0$.

This discussion on non–marginal generator nodes also applies to pqD nodes where the real power injection is fixed. For these nodes, either υ^+_{Pi} or υ^-_{Pi} becomes non–zero depending on whether the market attempts to force P_{Di} up or down when searching for an optimal dispatch.

5.9.2.2 Reactive Power Prices at pq Nodes

The function of Equation 5.3 is the same as Equation 5.2, except that it calculates the marginal price for reactive power demand (β_{Qi}) at pq nodes. Read and Ring allowed for reactive power generation at pq nodes through the definition of reactive power injection: $Q_i = Q_{Gi} - Q_{Di}$. However, they generally treated pq nodes as pqD nodes by assuming no generation at these nodes ($Q_{Gi} = 0$).

λ_Q is the marginal price of reactive power at the marginal generator node. Alternatively, it is the marginal price at the reference node, especially when there are multiple marginal generators.

- The first term, $-\lambda_P \frac{\partial L_P}{\partial Q_i}$, is the cost of the change in total real power losses caused by an incremental increase in reactive power injection (decrease in demand) at pq Node i. These losses are supplied by th al generator(s) via the reference generator.
- $\lambda_Q \left(1 - \frac{\partial L_Q}{\partial Q_i}\right)$ is the cost of using the marginal generator(s) to supply the marginal reactive power injection at pq Node i and the resultant losses, via the reference generator.

- The term $-\sum_{n\in\text{PvG}} \mu_{Qn} \frac{\partial Q_n}{\partial Q_i}$ is the cost of the change in reactive power generation at PvG nodes as Q_i is incremented at pq Node i.

- The fourth term, $-\sum_{n\in\text{pq}} \langle\mu_{Vn}\rangle \left\langle \frac{\partial V_n}{\partial Q_i} \right\rangle$, is the dual cost of the change in pressure on the binding primal voltage limits at pq nodes (n) as Q_i is incremented at pq Node i.

- The final term, $-2\sum_{k\in\text{K}} \langle\chi_k\rangle \left(\left\langle \bar{P}_k^* \frac{\partial \bar{P}_k}{\partial Q_i} \right\rangle + \left\langle \bar{Q}_k^* \frac{\partial \bar{Q}_k}{\partial Q_i} \right\rangle \right)$, is only non–zero for thermally constrained branches. It is the dual cost of the change in pressure on any binding primal thermal limits, incurred by incrementing Q_i at pq Node i.

Equation 5.8 works to constrain the marginal price of reactive power (β_{Qi}) in the same way that Equation 5.7 constrains the marginal price of real power. Consider a pq Node i where Q_i is fixed, because either $Q_{Gi} = 0$, or because $Q_{Gi} = Q_{Gi}^{max}$ or $Q_{Gi} = Q_{Gi}^{min}$. By complimentary slackness, β_{Qi} is allowed to vary at Node i because either $\upsilon_{Qi}^{+} \geq 0$ or $\upsilon_{Qi}^{-} \geq 0$.

Q_{Gi} is an unrestricted variable if reactive power generation is freely available at a pq node. Consequently, $\upsilon_{Qi}^{+} = \upsilon_{Qi}^{-} = 0$ and the dual marginal price of reactive power demand is fixed to c_{Qi} (as required by DBP 5.2). That is:

$$\beta_{Qi} = c_{Qi} \tag{5.13}$$

5.9.2.3 Reactive Power Prices at PvG Nodes

Reactive power generation is a dependent variable at PvG nodes. Rearranging the corresponding Dispatch Based Pricing equation (5.5) gives:

$$\beta_{Qn} = \mu_{Qn} + \lambda_Q \qquad \forall n \in \text{PvG} \tag{5.14}$$

μ_{Qn} is the cost, reflecting the dependent nature of reactive power generation at PvG nodes.

Q_G is free to vary when a generator at a PvG node is marginal for reactive power. By complimentary slackness, Equation 5.8 fixes the marginal price of reactive power at Node n to the unit cost of generation, c_{Qn}:

$$c_{Qn}^{-} \leq \beta_{Qn} \leq c_{Qn}^{+} \qquad \forall n \in \text{PvG} \tag{5.15}$$

Either Equation D.11 or Equation D.12 becomes binding when the generator at PvG Node n reaches a reactive power generation limit . Accordingly, Generator n is non–marginal for reactive power. Intuitively, it appears that either υ_{Qn}^{+} or υ_{Qn}^{-} allows the dual marginal price (β_{Qn}) to vary. However, Section 8.4.2 shows that when a PvG

Node n becomes non–marginal for reactive power generation, a PvG–type OPF actually treats Node n as a PqG node. Therefore, Equations 5.3 and 5.6 must be used.

5.9.2.4 Voltage Prices at PvG Nodes

The voltage magnitude at every PvG node (V_i) is an independent variable. Ring [1995] stated that voltage magnitude is a resource that can be optimised to minimise the total generation cost, as calculated by the primal objective function. Hence, Equation 5.4 calculates the marginal price of this resource.

- β_{Vi} is the price of demanding another unit of voltage.
- There is no '1' in this equation since there is no unit generation cost associated with voltage.
- The first term, $-\lambda_P \frac{\partial L_P}{\partial V_i}$, is the cost of the change in total real power losses for an incremental increase in voltage magnitude at PvG Node i. The real power associated with the change in losses are supplied by the marginal generator(s) via the reference generator.
- The second term, $-\lambda_Q \frac{\partial L_Q}{\partial V_i}$, is the cost of the change in the total reactive power losses for an incremental increase in voltage magnitude at PvG Node i. The reactive power for this change in losses is supplied by the marginal generator(s), via the reference generator.
- The third term, $-\sum_{n\in \mathrm{PvG}} \mu_{Qn} \frac{\partial Q_n}{\partial V_i}$, is the cost of the change in reactive power supplied by generators at PvG nodes, for an incremental increase in voltage magnitude at PvG Node i.
- The fourth term, $-\sum_{n\in \mathrm{pq}} \langle \mu_{Vn} \rangle \left\langle \frac{\partial V_n}{\partial V_i} \right\rangle$, is only non–zero for pq nodes where the voltage is constrained. Hence, it is the dual cost of the change in pressure on each primal voltage constraint due to an incremental increase in voltage magnitude at PvG Node i.
- The final term, $-2\sum_{k\in \mathrm{K}} \langle \chi_k \rangle \left(\left\langle \bar{P}_k^* \frac{\partial \bar{P}_k}{\partial V_i} \right\rangle + \left\langle \bar{Q}_k^* \frac{\partial \bar{Q}_k}{\partial V_i} \right\rangle \right)$, is only non–zero for thermally constrained branches. It is the dual cost of the change in pressure on each binding primal thermal constraint resulting from an incremental increase in voltage magnitude at PvG Node i.

Equation 5.9 defines the bounds on the marginal price of voltage magnitude at every node. V_i is an independent variable at PvG nodes. Therefore, it is possible for an OPF to optimise the value of V_i in the same way as P_i at PX nodes or Q_i at pq nodes. Thus, when V_i is unconstrained, the shadow prices of primal Equations D.13

and D.14 are zero: $\upsilon^+_{Vi} = \upsilon^-_{Vi} = 0$. Accordingly, Equation 5.9 forces the marginal cost of voltage to equal zero:

$$\beta_{Vi} = 0$$

However, V_i is constrained at every PvG node because it is held to a voltage set–point, as the definition of 'PvG' implies. This is equivalent to:

$$V_i = V_i^{min} = V_i^{max} \qquad \forall i \in \text{PvG} \quad (5.16)$$

So, by Equations D.13 and D.14, an OPF (i.e. the market) will force V_i against the upper limit:

$$-V_i = -V_i^{max} \qquad (5.17)$$

Or, the OPF will force V_i against the lower limit:

$$V_i = V_i^{min} \qquad (5.18)$$

As a result, Equation 5.9 allows marginal voltage price (β_{Vi}) to vary freely because either $\upsilon^+_{Vi} \geq 0$, or $\upsilon^-_{Vi} \geq 0$, depending on whether Equation 5.17 or Equation 5.18 is binding. Therefore, β_{Vi} is only non–zero at a PvG Node i when a voltage set–point (voltage constraint) is enforced. β_{Vi} is the cost the market is willing to pay in order to relieve this constraint by 'one per–unit voltage'.

5.9.2.5 Voltage Prices at pq Nodes

The marginal price of voltage magnitude at pq nodes is described by Equation 5.6, reproduced here:

$$\mu_{Vn} = \beta_{Vn}$$

β_{Vn} is subject to the price bounds defined by Equation 5.9:

$$\beta_{Vn} = \left\langle \upsilon^-_{Vn} \right\rangle - \left\langle \upsilon^+_{Vn} \right\rangle \qquad \forall n \in \text{pq} \quad (5.19)$$

This means that the voltage price at a pq Node n will only be non–zero if the voltage magnitude is constrained at that node, by either Equation D.13 or D.14. Therefore, the shadow price μ_{Qn} only appears in Equations 5.2, 5.3 and 5.4 when there is a binding voltage constraint at Node n.

5.9.3 The Marginal Price Criterion

The need to satisfy the Marginal Price Criterion (DBP 5.6) when using the equations of the pq/PvG Dispatch Based Pricing model, is explained here with an illustration. When a generator is marginal for reactive power at Node n, the marginal price β_{Qn} is fixed with Equation 5.13. Thus, a degree of freedom is removed from the Dispatch Based Pricing problem. This degree of freedom must be reintroduced to be able to solve the problem. That is, the problem is infeasible without that freedom. This might be achieved by assuming that the voltage magnitude at pq Node i has reached an upper limit, V_i^{max}. To reflect this limit in the pq/PvG Dispatch Based Pricing model, β_{Vi} is allowed to vary. β_{Vi} varies when the corresponding shadow price is no longer fixed to zero (i.e. $\upsilon_{Vi}^{+} \geq 0$). This shadow price provides the required degree of freedom. Hence, DBP 5.6 ensures that there are sufficient degrees of freedom so that a feasible solution can be found.

5.9.4 Simultaneous Equations

The Dispatch Based Pricing model is a set of simultaneous equations. Therefore, each price must be consistent with all other prices if a feasible solution is to be found. For example, the following marginal cost variables are present in every marginal price equation: λ_P, λ_Q, μ_{Qn}, μ_{Vn} and χ_k. Hence, the values of these variables are the same for every node (each node is represented by three marginal price equations). However, the values of the corresponding partial derivatives are node specific.

Consider therefore, a Node i and a Node n. When a feasible solution is obtained, β_{Pi} and β_{Pn} of Equation 5.2 are consistent with each other through the marginal cost variables. Furthermore, β_{Pi} is consistent with β_{Qi} and β_{Qn} of Equation 5.3 through the same marginal cost variables.

5.10 CONCLUSIONS

The primal and dual problems have been compared to the operation of two competitive markets. The primal market is optimised through cost considerations, the dual by a pricing mechanism. Both markets however, arrive at the same optimal result.

The comparison has been used to explain the derivation of the pq/PvG Dispatch Based Pricing model equations. The model is the simplified dual equivalent of the primal linear OPF formulation (composed of Equations D.2 to D.17). This pq/PvG Dispatch Based Pricing model can be used to find a consistent set of *ex post* marginal prices for any observed dispatch from a spot market, as long as the user adheres to DBP 5.6.

Chapter 6

DEVELOPMENT OF SOFTWARE PRICING TOOLS

6.1 INTRODUCTION

Two new Dispatch Based Pricing models are presented in Chapters 7 and 8 respectively. This chapter describes the 'nodal pricing' software used to implement these two pricing models. The software programs used as benchmarks to validate this nodal pricing software are also described. A pq–type OPF is developed as part of this benchmark software. It is capable of accepting non–zero unit generation costs for reactive power.

6.2 TWO NEW DISPATCH BASED PRICING MODELS

Two OPF types were defined when the OPF node classification system was proposed in Chapter 3. These were the pq–type OPF in which reactive power is dispatched as an independent variable, and the PvG–type OPF in which reactive power is dispatched as a dependent variable. The pq–type OPF is named thus, because only PQG and pqD nodes are used in its formulation. Similarly, the PvG–type OPF is named because only PvG and pqD nodes are used in its formulation.

The pq/PvG OPF formulation presented in Chapter 4 contains primal constraint equations for PvG nodes, PQG nodes and pqD nodes. Therefore, this pq/PvG OPF is the combination of a pq–type OPF and a PvG–type OPF (hence the name 'pq/PvG OPF'). These two constituent OPFs are referred to as 'pq–OPF' and 'PvG–OPF' respectively. It follows therefore, that the pq/PvG Dispatch Based Pricing model presented in Chapter 5 is the combination of a pq–type pricing model and a PvG–type pricing model. Respectively, these two Dispatch Based Pricing models are referred to as the 'pq pricing model' and the 'PvG pricing model'. The pq–OPF formulation and the pq pricing model are presented in Chapter 7, whereas the PvG–OPF formulation and the PvG pricing model are presented in Chapter 8.

6.3 NODAL PRICING SOFTWARE

Prior to the start of the research for this thesis, software was developed to implement the Dispatch Based Pricing equations in Read and Ring [1995d]. These equations are repeated in Appendix C for clarity. This software was developed by the Energy Modelling Research Group at Canterbury University in collaboration with Core Management Services Ltd, and funded by Transpower New Zealand Ltd[1]. Two software programs were the result.

The first program is NODAL1, which implements a simplified Dispatch Based Pricing model. NODAL1 calculates marginal prices for real power at all nodes and reactive power prices at pqD nodes. It does not calculate reactive power marginal prices PvG nodes, and cannot accept non–zero reactive power unit generation costs.

The second is NODAL2. This program implements the pq/PvG Dispatch Based Pricing model described by Equations 5.1 to 5.9. These equations are the only equations needed when constructing the pq and PvG pricing models proposed in Chapters 7 and 8 respectively. By default therefore, NODAL2 implements these two pricing models.

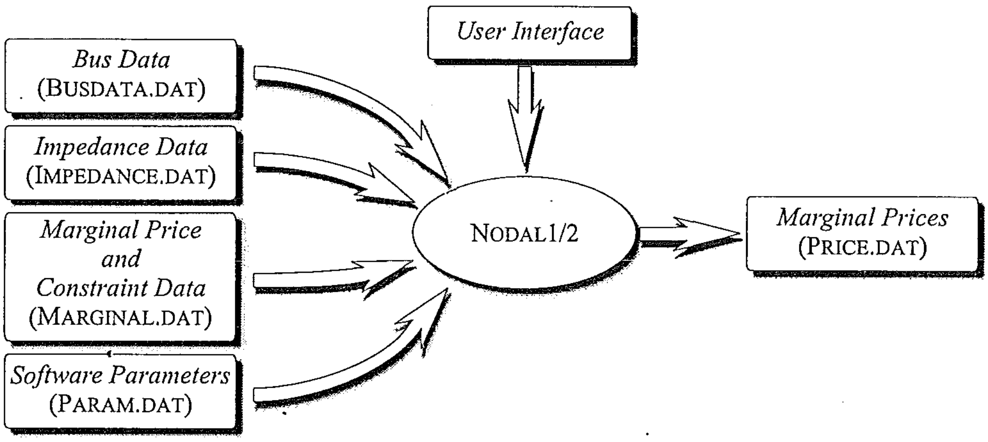


Figure 6.1 Software components comprising the nodal pricing software, which is used to implement the Dispatch Based Pricing equations.

NODAL1 and NODAL2 have five main software components. These components are depicted in Figure 6.1, and are as follows:

- **Bus Data** - This contains the following data for every node: real and reactive power generation, voltage magnitudes and voltage angles, and real and reactive power demand. This data describes the observed dispatch for which *ex post* marginal prices are to be calculated.

- **Impedance Data** - The impedance of every network component is described in this software component using pi–models [Read and Ring 1995c]. See Figure 3.1.

[1]This software is known within Transpower New Zealand Ltd. as 'nodal pricing software', since marginal prices are calculated for all nodes in a power system.

- **Marginal and Constraint Data** - This component contains marginal data identifing the generators that are marginal for real power, and the generators that are marginal for reactive power. The unit generation cost for each of these marginal generators is also stated. Any binding constraints are also identified herein. According to Dispatch Based Pricing philosophy, the constraint data are reflections of what the Clearing Manager perceived to be the reason for dispatching the marginal generators, identified by the marginal data.

- **Software Parameters** - This component contains miscellaneous parameters controlling the operation of the nodal pricing software. Some parameters control the terms that are to appear in the objective function.

- **Marginal Prices** - The output of both NODAL1 and NODAL2 is the set of *ex post* marginal prices for real and reactive power for the observed dispatch. This component also contains other relevant information. See Appendix F for an example.

6.4 BENCHMARK SOFTWARE

6.4.1 A Reactive Power Optimal Power Flow

Mathematically, the equations of the pq/PvG Dispatch Based Pricing model are correct. However, the real and reactive power marginal prices from NODAL2 had to be validated to:

- determine how to correctly apply the pq/PvG Dispatch Based Pricing equations (i.e. correctly operate NODAL2) in order to form the exact dual of the OPF used to optimise any dispatch from a spot market. If incorrectly applied, the set of prices calculated is not the correct price–set for the optimised dispatch and therefore meaningless. For example, the prices will not correctly identify the most economic generators from which to obtain the next unit of real or reactive power;

- ensure that the NODAL2 source code correctly implements the pq/PvG Dispatch Based Pricing equations.

Benchmark prices are required to validate the NODAL2 prices. OPF software can be used to generate the required benchmark prices by exploiting the primal–dual relationship. These benchmark prices are the, easily accessible, LaGrange multipliers used by the OPF to obtain an optimal dispatch.

6.4.1.1 The Primal–Dual Relationship

An ideal spot market can be operated using cost considerations (the primal perspective) or by using price mechanisms (the dual perspective); see Figure 5.1. Both approaches

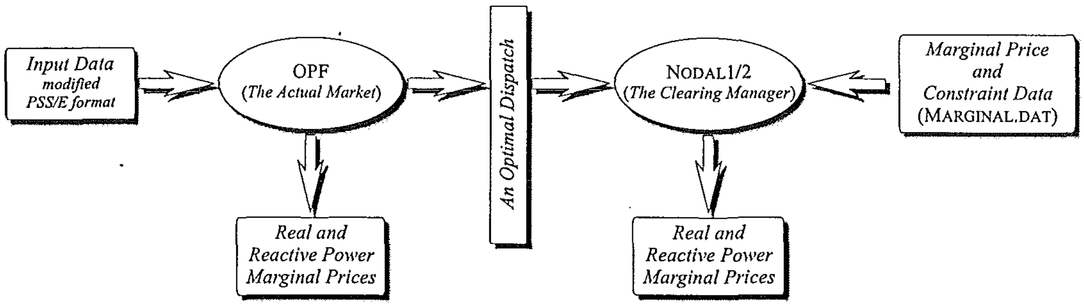


Figure 6.2 An OPF and NODAL2 will produce identical marginal prices under certain conditions.

result in the same optimal dispatch, and hence the same set of real and reactive power marginal prices. Figure 6.2 portrays this relationship from a software perspective.

The OPF models the optimising influence of a spot market on the dispatch of real and reactive power. NODAL2 is used by the Clearing Manager to calculate *ex post* marginal prices for the observed optimal dispatch, while accounting for any binding constraints and marginal generators. Once an OPF has found an optimal dispatch, the OPF's marginal real and reactive power prices for this dispatch identify the most economic generator(s) from which to supply the next unit of real or reactive power. NODAL2 produces a set of *ex post* marginal prices identifying the most economic generator(s) that will supply the next unit of power for the observed dispatch.

The OPF marginal prices will be identical to the NODAL2 marginal prices given the following assumptions:

- NODAL2 is told by the Clearing Manager, of all binding constraints that QOPF had to work around (i.e. constraint data), and the resultant marginal generators (i.e. marginal data),
- the market (i.e. the OPF) optimises the dispatch with a formulation identical to the non–linear pq/PvG OPF formulation. That is, the linearised OPF formulation is the dual of the pq/PvG Dispatch Based Pricing model (i.e. NODAL2), and
- the optimal dispatch from the OPF is used as the observed dispatch given to NODAL2.

The optimal dispatch from the OPF represents an operating point, to which the next unit of power and the OPF marginal prices are referenced. The observed dispatch is the operating point used by NODAL2. NODAL2 is a simplified version of the dual problem in Appendix D, which is linearised around the observed dispatch. The OPF and NODAL2 will produce identical prices if the two operating points are the same. The assumptions ensure identical operating points (i.e. that the optimal dispatch is the observed dispatch). They also ensure the NODAL2 formulation is the dual of the OPF formulation.

6.4.1.2 A Developmental Overview

The discussions in the previous sections identify the need for pq–type OPF software and PvG–type OPF software to validate the pq and PvG pricing models in Chapters 7 and 8 respectively. The formulations of these two OPFs must implement Equations 7.1 to 7.12 and Equations 8.1 to 8.13, if their marginal prices are to serve as benchmarks for the two pricing models.

Equations 7.1 and 8.1 show that these two OPFs must be capable of minimising the total cost of real and reactive power generation. However, commercial OPF software that will accept cost functions for reactive power generation (as well as cost functions for real power generation) could not be found. It is evident from the publications surveyed in Chapter 2 that this absence is a result of a common opinion: 'the cost of reactive power generation cannot be minimised because reactive power generation has a zero unit generation cost'. Thus, OPF software capable of accommodating reactive power generation cost functions has been developed for this thesis.

An OPF, known in this thesis as Matpower, was used as the starting point from which to develop the required reactive power OPF software. Matpower was written by PSERC at Cornell University, for use in the Matlab environment with the Optimisation Toolbox (by Mathworks Inc.)[2]. Matpower only accepts real power cost functions, and is therefore only capable of minimising the cost of real power generation.

Time restrictions have meant that only the pq–type reactive power OPF software has been developed. The development of PvG–type reactive power OPF software is the subject of future work.

6.4.1.3 Development of the Reactive Power OPF

The objective function of Matpower is:

$$f\,(cost)_P = \sum_{i \in Gen} M_P\,(P_G)_i \tag{6.1}$$

Gen is the set of all generator nodes. $M_P\,(P_G)_i$ is a polynomial describing the cost of real power generation at Node i:

$$M_P\,(P_G)_i = a + bP_{Gi} + cP_{Gi}^2 + \cdots \tag{6.2}$$

[2]PSERC is an acronym for the Power System Engineering Research Center.

Matpower is a PvG–type OPF, optimising the power system variables in Vector–Set 3.4 so as to minimise the objective function. To ensure that the final values of these variables describe a valid dispatch, the power system is described within Matpower as the set of power injections at all nodes:

$$\tilde{P}_{mismatch} + \mathrm{j}\,\tilde{Q}_{mismatch} = \tilde{V}\tilde{V}^{*}Y^{*} - \frac{\left(\tilde{P} + \mathrm{j}\,\tilde{Q}\right)}{S_{base}} \tag{6.3}$$

Y^* is the conjugate of the admittance matrix. Equation 6.3 appears in the Matpower formulation as three equality constraints:

$$P_{mismatch_i} = 0 \qquad \forall i \in Gen \tag{6.4}$$
$$P_{mismatch_n} = 0 \qquad \forall n \in Dem \tag{6.5}$$
$$Q_{mismatch_n} = 0 \qquad \forall n \in Dem \tag{6.6}$$

Dem is the set of all non-generator nodes. The mismatch equation for reactive power injection is not required at any generator node (i), as the value of 'Q' is defined by the values of the variables in vectors $\tilde{u}$ and $\tilde{p}$. To use this fourth reactive power mismatch equation is to over–define the optimisation problem and make the problem infeasible.

A gradient vector is formed by Matpower when composing the LaGrange equation used to find the optimal dispatch (ref. Appendix B). It is obtained by taking partial derivatives of the objective function (Equation 6.1) with respect to the elements in the $\tilde{x}$ and $\tilde{u}$ vectors:

$$\nabla f\,(cost)_P = \left[\frac{\partial f}{\partial \theta_i}, \frac{\partial f}{\partial \theta_n}, \frac{\partial f}{\partial V_n}, \frac{\partial f}{\partial P_i}\right] \qquad \begin{array}{l} \forall i \in Gen \\ \forall n \in Dem \end{array} \tag{6.7}$$

Equations 6.1 to 6.7 were modified to obtain the required pq–type reactive power OPF. First, to minimise the cost of reactive power generation, the objective function (Equation 6.1) becomes:

$$f\,(cost)_{P,Q} = \sum_{i \in Gen} M_P\,(P_G)_i + \sum_{i \in Gen} M_Q\,(Q_G)_i \tag{6.8}$$

That is, the objective function now includes cost functions for reactive power generation:

$$M_Q\,(Q_G)_i = a + b\,Q_{Gi} + c\,Q_{Gi}^2 + \cdots \tag{6.9}$$

Second, to convert Matpower into a pq–type algorithm, Vector–Set 3.4 was replaced with Vector–Set 3.5. This means that Q is now an element of $\tilde{u}$. V has shifted from the fixed parameter vector ($\tilde{p}$) to the state vector ($\tilde{x}$). This only shows that the modified Matpower is capable of dispatching a power system with either fixed or variable generator voltages. V is now a dependent variable, dependent on the P and Q injections at generator nodes.

Q is now an independent variable, able to assume any value. Thus, an extra equality (mismatch) constraint is required in Matpower's formulation to ensure that Q takes on values consistent with a power system dispatch. It ensures that the power system is still sufficiently described. Wherefore, the required mismatch equations are:

$$P_{mismatch_i} = 0 \qquad \forall i \in Gen \tag{6.10}$$

$$P_{mismatch_n} = 0 \qquad \forall n \in Dem \tag{6.11}$$

$$Q_{mismatch_i} = 0 \qquad \forall i \in Gen \tag{6.12}$$

$$Q_{mismatch_n} = 0 \qquad \forall n \in Dem \tag{6.13}$$

The gradient vector for Matpower was modified accordingly:

$$\nabla f\,(cost)_{P,Q} = \left[\frac{\partial f}{\partial \theta_i}, \frac{\partial f}{\partial V_i}, \frac{\partial f}{\partial \theta_n}, \frac{\partial f}{\partial V_n}, \frac{\partial f}{\partial P_i}, \frac{\partial f}{\partial Q_i}\right] \qquad \begin{array}{l}\forall i \in Gen\\ \forall n \in Dem\end{array} \tag{6.14}$$

These modifications converted Matpower (a PvG–type OPF) into a pq–type reactive power OPF. To avoid confusion, this new pq–type OPF is called Matpower^{pq}.

The development of Matpower^{pq} was paralleled by the development of a similar pq–type reactive power OPF by PSERC. Collaboration with PSERC was undertaken to ensure that Matpower^{pq} and this PSERC OPF produced identical dispatches and marginal prices.

The PSERC OPF has been used instead of Matpower^{pq} because its professional design and additional features make it easier to use than Matpower^{pq}. In this thesis, the PSERC OPF is called QOPF where 'Q' indicates the ability to accept reactive power generation cost functions.

6.4.2 The PSERC Power–Flow

Power–flow software was used as benchmark software for the Matpower, Matpower^{pq} and QOPF software. It is also used as part of the perturbation process that generates sub–optimal dispatches (see Section 10.2). This power–flow was developed by PSERC and accompanied the Matpower OPF.

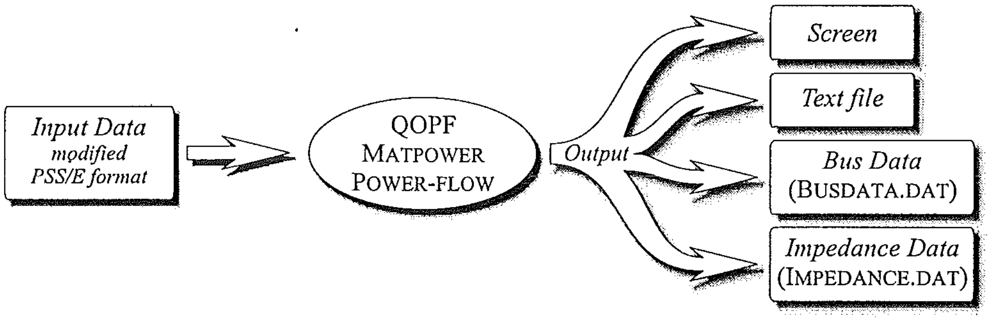


Figure 6.3 Software components that comprise the OPF and power–flow software.

6.4.3 Structure of the Power System Software

Matpower, QOPF and the PSERC power–flow have similar software structures. These structures are illustrated in Figure 6.3.

The main components are:

- **Input Data** - This component contains bus data including: voltages; real and reactive power demand; generator data; impedance data; operating limits on power system resources; real and reactive power generation cost function data.

- **Output Results** - The output of each OPF and the power–flow describes a dispatch based on the input data. In the case of the OPFs, the output also includes marginal prices for real and reactive power (Appendix F contains an example of the QOPF output). The two OPFs and the power–flow were modified to enable the dispatch output to be passed directly to NODAL2 for calculation of *ex post* prices. That is, the dispatch information is saved to file in the NODAL2 BUSDATA.DAT and LINKDATA.DAT formats. All shunt susceptances and tap information are converted to resistance and reactance, so as to conform to the standard pi–model depicted in Figure 3.1.

6.5 SOFTWARE TESTING METHODOLGY

6.5.1 Stages of Software Testing

Figure 6.4 depicts the stages in the software testing process. The real power prices generated by NODAL1 were verified prior to this Masters degree, using a commercial OPF called PSS/E. PSS/E is developed by Power Technologies Inc. The PSERC power–flow was verified with a power–flow package called LFH, developed by the Electrical Engineering Department, University of Canterbury.

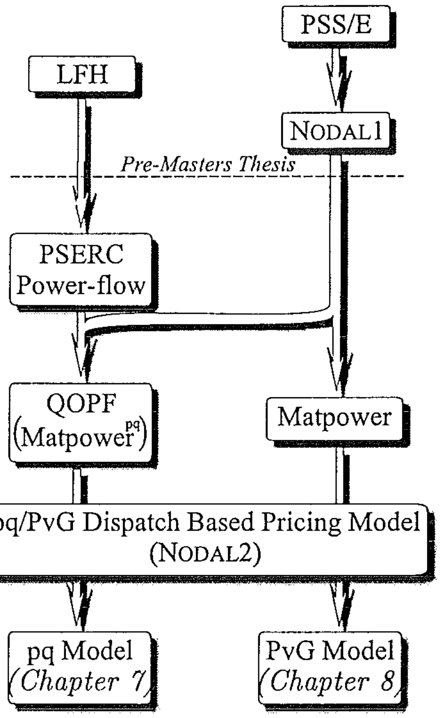


Figure 6.4 Successive stages in the software testing process.

6.5.2 Validating Matpower^{pq} and Matpower

During the development of Matpower^{pq} (and QOPF), the PSERC power–flow and NODAL1 were used as validation tools. The PSERC power–flow was used to ensure that Matpower^{pq} generates valid dispatches. A Matpower^{pq} dispatch was deemed to be valid once it could be passed through the power–flow without the swing bus generator changing its real and reactive output (to 10 decimal places, in units of MW and MVAr).

NODAL1 was used to verify the real power marginal prices calculated by Matpower, Matpower^{pq} and QOPF. The bus, marginal and binding constraint data describing the optimal dispatches generated by each OPF were passed through NODAL1. The OPF prices were then compared with the resultant NODAL1 prices. This comparison utilises the primal–dual relationship. OPF prices could only be verified to an accuracy of 4 d.p. because the nodal pricing software only reports marginal prices to 4 d.p.

Unconstrained, voltage constrained, MVA constrained and reactive power generation constrained dispatches were used to verify marginal prices. However, the objective functions used for these dispatches had the form of Equation 6.1, in which the unit generation cost of reactive power is zero. This was because NODAL1 and Matpower do not allow the use of reactive power cost functions, having been developed under the assumption that reactive power generation has no cost. Note that QOPF and Matpower^{pq} were configured to optimise dispatches using only real power cost functions by setting $M_Q\left(Q_G\right)_i = 0$ in Equation 6.8 for all generators.

Some reverse checking occurred, using NODAL2 as a benchmark for Matpower^{pq} (and QOPF). This was done to check the integrity of reactive power prices generated by Matpower^{pq} and QOPF. This revealed a problem with the "Constrained Optimisation" algorithm from the Matlab Optimisation Toolbox, which Mathworks duly corrected.

6.5.3 Validating NODAL2

Black box testing was used to find errors in the operation of NODAL2, because the source code was unavailable for reasons of commercial sensitivity. Transpower New Zealand Ltd corrected errors when advised of these.

At the beginning of this work, NODAL2 had not been fully tested because an OPF capable of accepting reactive power cost functions was not available. However, once Matpower, QOPF and the PSERC power–flow became available, NODAL2 was tested in two parts: by testing the equations of the pq pricing model, and then by testing the equations of the PvG pricing model. The test experiments for the two parts are discussed in Chapter 7 and Chapter 8 respectively.

The set of marginal real and reactive power prices from each validated OPF was used as a benchmark for the set of marginal prices from NODAL2. It was reasoned that, when the NODAL2 prices matched the OPF prices, NODAL2 would be free of source code errors. This price match was also used to demonstrate that the Dispatch Based Pricing equations were being correctly applied, so as to obtain the dual formulation of the OPF used to optimise the dispatch.

The validation process at this stage was similar to that in the Matpower^{pq}–NODAL1 stage, with optimal dispatches from each OPF being passed through NODAL2. However, non–zero reactive power generation cost functions were used when using the price set from QOPF. NODAL2 were deemed to be validated when the real and reactive power prices reported by NODAL2 were identical to the marginal prices from each OPF (accurate to 4 d.p.). An example of identical marginal price sets from QOPF and NODAL2 is presented in Appendix F.

6.6 TEST POWER SYSTEMS

Three test power systems were used to generate all software–validating dispatches. They are:

- the 9–bus power system from Cornell University; this accompanied the Matpower software;
- the IEEE 14–bus power system representing the American electric power system;
- the IEEE 30–bus power system representing the American electric power system.

The schematics and raw data for each power system are presented in Appendix E. The IEEE power systems were used because of their wide availability.

6.7 CONCLUSIONS

Two OPFs have been reliably tested during the stages of software testing:

- **Matpower:** a PvG–type OPF, only capable of minimising the total cost of real power generation. Matpower models a traditional spot market composed only of a real power sub–market. It is similar in formulation to most of the OPFs discussed in Chapter 2;
- **QOPF:** a pq–type OPF, capable of minimising the total cost of real and reactive power generation. QOPF models one form of reactive power spot market, composed of real and reactive power sub–markets.

The validation process of NODAL2 has been outlined in this chapter. However, the results of this validation process are detailed in Chapters 7 and 8. In those chapters, two new Dispatch Based Pricing models are validated using Matpower and QOPF. Each new model describes a form of spot pricing market having both real and reactive power sub–markets. Both models are implemented in software by NODAL2.

Chapter 7

A pq–TYPE DISPATCH BASED PRICING MODEL

7.1 INTRODUCTION

The previous chapter established that the pq/PvG OPF (Equations 4.1 to 4.13) is the combination of two other OPFs, and that the pq/PvG Dispatch Based Pricing model (Equations 5.1 to 5.9) is the combination of two other Dispatch Based Pricing models. Furthermore, it was argued that the software which implementes the pq/PvG pricing model (i.e. NODAL2), also implements these two constituent pricing models.

This chapter is concerned with the first of the constituent Dispatch Based Pricing models. This model can be used to calculate *ex post* marginal prices for a pq–type spot market. Accordingly, it is referred to as the 'pq pricing model'.

The equations of this pq pricing model are presented in this chapter, together with the non–linear OPF formulation from which the pricing model is derived. Initially, the model is validated. Then examples are provided, demonstrating the use the validated pq pricing equations in order to calculate *ex post* marginal prices for different observed dispatches.

A pq–type spot market is assumed for the examples. In this type of spot market, reactive power is optimally dispatched as an independent resource, just as real power is dispatched as an independent resource. It is also assumed that this spot market accommodates cost functions for both real and reactive power, to facilitate real and reactive power sub–markets.

7.2 THE pq PRICING MODEL

7.2.1 The Formulations

All generator nodes are modelled as PQG nodes because reactive power generation is an independent resource in a pq–type spot market. This PQG classification indicates that a pq–type OPF must be used to model the optimisation process of this pq–type spot market. The pq/PvG OPF of Chapter 4 can be converted into a pq–type OPF by removing all terms used for PvG nodes. The result is Equations 7.1 to 7.12.

THE NON–LINEAR pq–OPF FORMULATION

$$\underset{\mathbf{P}_{\mathbf{D}}^{\mathrm{pq}},\mathbf{Q}_{\mathbf{D}}^{\mathrm{pq}},\mathbf{P}_{\mathbf{G}}^{\mathrm{pq}},\mathbf{Q}_{\mathbf{G}}^{\mathrm{pq}},\mathbf{V}^{\mathrm{pq}}}{\textbf{Minimise}} \qquad Costs(\mathbf{P}_{\mathbf{G}}^{\mathrm{pq}},\mathbf{Q}_{\mathbf{G}}^{\mathrm{pq}}) \tag{7.1}$$

subject to:

CONSERVATION OF POWER

$$\sum_{i\in \mathrm{pq}} (P_{Gi} - P_{Di}) - L_P\left(\mathbf{P}_{\mathbf{G}}^{\mathrm{pq}} - \mathbf{P}_{\mathbf{D}}^{\mathrm{pq}}, \mathbf{Q}_{\mathbf{G}}^{\mathrm{pq}} - \mathbf{Q}_{\mathbf{D}}^{\mathrm{pq}}\right) = 0 \tag{7.2}$$

$$\sum_{i\in \mathrm{pq}} (Q_{Gi} - Q_{Di}) - L_Q\left(\mathbf{P}_{\mathbf{G}}^{\mathrm{pq}} - \mathbf{P}_{\mathbf{D}}^{\mathrm{pq}}, \mathbf{Q}_{\mathbf{G}}^{\mathrm{pq}} - \mathbf{Q}_{\mathbf{D}}^{\mathrm{pq}}\right) = 0 \tag{7.3}$$

DEPENDENT VOLTAGE AT pq NODES

$$-V_n\left(\mathbf{P}_{\mathbf{G}}^{\mathrm{pq}} - \mathbf{P}_{\mathbf{D}}^{\mathrm{pq}}, \mathbf{Q}_{\mathbf{G}}^{\mathrm{pq}} - \mathbf{Q}_{\mathbf{D}}^{\mathrm{pq}}\right) + V_n = 0 \qquad \forall n \in \mathrm{pq} \tag{7.4}$$

TRANSMISSION LINE FLOWS

$$-\bar{P}_k\left(\mathbf{P}_{\mathbf{G}}^{\mathrm{pq}} - \mathbf{P}_{\mathbf{D}}^{\mathrm{pq}}, \mathbf{Q}_{\mathbf{G}}^{\mathrm{pq}} - \mathbf{Q}_{\mathbf{D}}^{\mathrm{pq}}\right) + \bar{P}_k = 0 \qquad \forall k \in \mathrm{K} \tag{7.5}$$

$$-\bar{Q}_k\left(\mathbf{P}_{\mathbf{G}}^{\mathrm{pq}} - \mathbf{P}_{\mathbf{D}}^{\mathrm{pq}}, \mathbf{Q}_{\mathbf{G}}^{\mathrm{pq}} - \mathbf{Q}_{\mathbf{D}}^{\mathrm{pq}}\right) + \bar{Q}_k = 0 \qquad \forall k \in \mathrm{K} \tag{7.6}$$

REAL AND REACTIVE GENERATION AND VOLTAGE SETTINGS

$$P_{Gi}^{min} \le P_{Gi} \le P_{Gi}^{max} \qquad \forall i \in \mathrm{pq} \tag{7.7}$$

$$Q_{Gi}^{min} \le Q_{Gi} \le Q_{Gi}^{max} \qquad \forall i \in \mathrm{pq} \tag{7.8}$$

$$V_i^{min} \le V_i \le V_i^{max} \qquad \forall i \in \mathrm{pq} \tag{7.9}$$

REAL AND REACTIVE POWER LOAD SETTINGS

$$P_{Di} = P_{Di}^{set} \qquad \forall i \in \mathrm{pq} \tag{7.10}$$

$$Q_{Di} = Q_{Di}^{set} \qquad \forall i \in \mathrm{pq} \tag{7.11}$$

TRANSMISSION LINE THERMAL LIMITS

$$\bar{P}_k^2 + \bar{Q}_k^2 \le T_k^{max} \qquad \forall k \in \mathrm{K} \tag{7.12}$$

THE pq PRICING MODEL

$$\underset{\substack{\chi^{K},\, \upsilon_P^{+pq},\, \upsilon_P^{-pq},\, \upsilon_Q^{+pq},\, \upsilon_Q^{-pq},\, \upsilon_V^{+pq},\, \upsilon_V^{-pq} \geq 0 \\ \lambda_P,\, \lambda_Q,\, \beta_P^{pq},\, \beta_Q^{pq},\, \mu_V^{pq}}}{\text{MAXIMISE}} \quad \textit{The pq, Dual Linear Program Objective Function (A modified version of Equation D.18)} \tag{7.13}$$

subject to:

<u>*Marginal Price Equations*</u>

$$\beta_{Pi} = \lambda_P \left(1 - \frac{\partial L_P}{\partial P_i}\right) - \lambda_Q \frac{\partial L_Q}{\partial P_i} - \sum_{n \in pq} \langle \mu_{Vn} \rangle \left\langle \frac{\partial V_n}{\partial P_i} \right\rangle - 2 \sum_{k \in K} \langle \chi_k \rangle \left(\left\langle \bar{P}_k^* \frac{\partial \bar{P}_k}{\partial P_i} \right\rangle + \left\langle \bar{Q}_k^* \frac{\partial \bar{Q}_k}{\partial P_i} \right\rangle \right) \qquad \forall i \in pq \tag{7.14}$$

$$\beta_{Qi} = -\lambda_P \frac{\partial L_P}{\partial Q_i} + \lambda_Q \left(1 - \frac{\partial L_Q}{\partial Q_i}\right) - \sum_{n \in pq} \langle \mu_{Vn} \rangle \left\langle \frac{\partial V_n}{\partial Q_i} \right\rangle - 2 \sum_{k \in K} \langle \chi_k \rangle \left(\left\langle \bar{P}_k^* \frac{\partial \bar{P}_k}{\partial Q_i} \right\rangle + \left\langle \bar{Q}_k^* \frac{\partial \bar{Q}_k}{\partial Q_i} \right\rangle \right) \qquad \forall i \in pq \tag{7.15}$$

$$\mu_{Vn} = \beta_{Vn} \qquad \forall n \in pq \tag{7.16}$$

<u>*Price Bound Equations*</u>

$$\bar{c}_{Pi}^{-} - \left\langle \upsilon_{Pi}^{-} \right\rangle \leq \beta_{Pi} \leq c_{Pi}^{+} + \left\langle \upsilon_{Pi}^{+} \right\rangle \qquad \forall i \in pq \tag{7.17}$$

$$\bar{c}_{Qi}^{-} - \left\langle \upsilon_{Qi}^{-} \right\rangle \leq \beta_{Qi} \leq c_{Qi}^{+} + \left\langle \upsilon_{Qi}^{+} \right\rangle \qquad \forall i \in pq \tag{7.18}$$

$$\beta_{Vi} = \left\langle \upsilon_{Vi}^{-} \right\rangle - \left\langle \upsilon_{Vi}^{+} \right\rangle \qquad \forall i \in pq \tag{7.19}$$

The pq pricing model (Equations 7.13 to 7.19) can be derived from the pq–OPF formulation by following the derivation process outlined in Chapters 4 and 5. Deriving the pq pricing model in this way is equivalent to removing all PvG terms from the pq/PvG Dispatch Based Pricing model of Chapter 5. This pq pricing model must be used to calculate *ex post* marginal prices only for pq–type spot markets. This is because all terms apply only to pqD and PQG nodes.

In Equations 7.1 to 7.19, 'pq' is used to indicate those equations that apply to both PQG nodes and pqD nodes. For example, Equations 7.15 and 7.18 are used together to calculate a reactive power marginal price for every PQG (i.e. generator) node and every pqD (i.e. demand) node. It can be seen these two equations correspond to Equations 5.3 and 5.8.

The dual objective function (7.13) is the same as the objective function of the dual problem (D.18), notwithstanding the following changes:

- PX has been changed to pq;
- all V^* terms have been removed;
- all η_P and η_Q terms have been substituted out;
- the following term has been removed.

$$\sum_{n\in \mathrm{PvG}} \mu_{Qn} \left(-Q^*_{Dn} + \sum_{i\in \mathrm{PX}} \frac{\partial Q_n}{\partial P_i} P^*_{Di} + \sum_{i\in \mathrm{pq}} \frac{\partial Q_n}{\partial Q_i} Q^*_{Di} - \sum_{i\in \mathrm{PvG}} \frac{\partial Q_n}{\partial V_i} V^*_i \right)$$

7.2.2 Interpretation Experiments

In Section 6.4 it was stated that pq–type OPF software can be used as benchmark to validate this pq pricing model (which is implemented by NODAL2). In that section, the development of QOPF (a pq–type OPF) was presented. Hence, QOPF has been used to generate the required optimal benchmark dispatches.

Dispatches were generated for the following power system conditions: unconstrained; voltage constrained; thermally constrained; reactive power generation constrained. These benchmark dispatches were passed through NODAL2 and the resultant prices compared with the QOPF benchmark marginal prices (see Section 6.5.3). The comparisons revealed that the NODAL2 marginal prices matched the QOPF benchmark marginal prices to 4 d.p., in all but the thermally constrained dispatches. From the successful experiments, it can be concluded that ' is the software implementation of Equations 7.1 to 7.12 and is therefore the primal equivalent of the 'pq' part of NODAL2 (as depicted in Figure 6.2). Therefore, the experiments demonstrate the pq pricing model can calculate correct marginal prices for a pq–type spot market. Also, the experiments imply that the NODAL2 source code correctly implements the pq pricing model equations in all but the thermal case. The unsuccessful thermal case indicates a possible error in the code used to implement the χ_k terms.

The voltage constrained dispatch of the IEEE 14 bus power system, given in Appendix F, is a typical example of the successful validation experiments.

7.3 APPLICATION: AN UNCONSTRAINED DISPATCH

Examples of how the Clearing Manager (introduced in Section 2.5.2) should use the pq pricing model to calculate *ex post* real and reactive power prices for an observed dispatch are presented in this section. The observed dispatch is assumed to have been optimal. That is, an ideal pq–type spot market (modelled by QOPF) is assumed to have produced this dispatch. It is also assumed the dispatch is unconstrained.

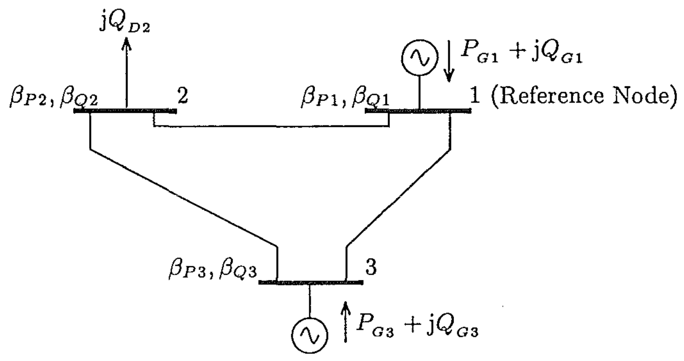


Figure 7.1 A 3–node power system.

7.3.1 The Dispatch and the Equations

The 3–node power system, illustrated in Figure 7.1, is used to demonstrate the application of the *ex post* pq pricing model. The power system is assumed to have been dispatched with variable generator voltages. The resultant unconstrained optimal dispatch is assumed to have the following characteristics:

- Generator 1 was marginal for real and reactive power;
- Generator 3 was not dispatched (i.e. not marginal) for either real or reactive power: $P_{G3} = P_{G3}^{min} = 0$ and $Q_{G3} = Q_{G3}^{min} = 0$;
- Node 1 is the Dispatch Based Pricing reference node;
- Nodes 1 and 3 are PQG nodes;
- Node 2 is a pqD node;
- The voltage magnitude at Node 1 was fixed, providing a reference voltage for the rest of the power system.

The resultant pq pricing equations for this observed dispatch are:

<u>Real power marginal price equations</u>

$$\beta_{P1} = \lambda_P \left(1 - \frac{\partial L_P}{\partial P_1}\right) - \lambda_Q \frac{\partial L_Q}{\partial P_1} - \mu_{V1} \frac{\partial V_1}{\partial P_1} \tag{7.20}$$

$$\beta_{P2} = \lambda_P \left(1 - \frac{\partial L_P}{\partial P_2}\right) - \lambda_Q \frac{\partial L_Q}{\partial P_2} - \mu_{V1} \frac{\partial V_1}{\partial P_2} \tag{7.21}$$

$$\beta_{P3} = \lambda_P \left(1 - \frac{\partial L_P}{\partial P_3}\right) - \lambda_Q \frac{\partial L_Q}{\partial P_3} - \mu_{V1} \frac{\partial V_1}{\partial P_3} \tag{7.22}$$

<u>Reactive power marginal price equations</u>

$$\beta_{Q1} = -\lambda_P \frac{\partial L_P}{\partial Q_1} + \lambda_Q \left(1 - \frac{\partial L_Q}{\partial Q_1}\right) - \mu_{V1} \frac{\partial V_1}{\partial Q_1} \tag{7.23}$$

$$\beta_{Q2} = -\lambda_P \frac{\partial L_P}{\partial Q_2} + \lambda_Q \left(1 - \frac{\partial L_Q}{\partial Q_2}\right) - \mu_{V1} \frac{\partial V_1}{\partial Q_2} \tag{7.24}$$

$$\beta_{Q3} = -\lambda_P \frac{\partial L_P}{\partial Q_3} + \lambda_Q \left(1 - \frac{\partial L_Q}{\partial Q_3}\right) - \mu_{V1} \frac{\partial V_1}{\partial Q_3} \tag{7.25}$$

<u>Voltage marginal price equations</u>

$$\mu_{V1} = \beta_{V1} \tag{7.26}$$

$$\mu_{V2} = \beta_{V2} \tag{7.27}$$

$$\mu_{V3} = \beta_{V3} \tag{7.28}$$

<u>Price bound equations</u> (which determine the range of values for each marginal price):

$$\bar{c}_{P1} \leq \beta_{P1} \leq \overset{+}{c}_{P1} \tag{7.29}$$

$$\bar{c}_{P2} - \left\langle \bar{v}_{P2} \right\rangle \leq \beta_{P2} \leq \overset{+}{c}_{P2} + \left\langle \overset{+}{v}_{P2} \right\rangle \tag{7.30}$$

$$\bar{c}_{P3} - \bar{v}_{P3} \leq \beta_{P3} \leq \overset{+}{c}_{P3} \tag{7.31}$$

$$\bar{c}_{Q1} \leq \beta_{Q1} \leq \overset{+}{c}_{Q1} \tag{7.32}$$

$$\bar{c}_{Q2} - \left\langle \bar{v}_{Q2} \right\rangle \leq \beta_{Q2} \leq \overset{+}{c}_{Q2} + \left\langle \overset{+}{v}_{Q2} \right\rangle \tag{7.33}$$

$$\bar{c}_{Q3} - \bar{v}_{Q3} \leq \beta_{Q3} \leq \overset{+}{c}_{Q3} \tag{7.34}$$

$$\beta_{V1} = \left\langle \bar{v}_{V1} \right\rangle - \left\langle \overset{+}{v}_{V1} \right\rangle \tag{7.35}$$

$$\beta_{V2} = 0 \tag{7.36}$$

$$\beta_{V3} = 0 \tag{7.37}$$

7.3.2 A Merit Order Dispatch of Reactive Power

Generator nodes are classed as PQG nodes in the pq–OPF formulation. This implies that Q_G is an independent primal variable in the pq–OPF formulation, and is an element in the OPF control vector ($\tilde{u}$). The independent nature of Q_G implies that reactive power generation is dispatched in a manner identical to that of real power generation. This means that pq–OPF (i.e QOPF or any other pq–type OPF) will dispatch reactive power according to a merit order. This is verified in Appendix G.

The merit order dispatch of reactive power is also evident in the structure of the pq pricing model equations. The reactive power price equations that correspond to the PQG nodes (i.e. Equations 7.23 and 7.25) are identical in form to the real power price equations (7.20 and 7.22). Since real power is always dispatched according to a merit order in an unconstrained power system, the identical form of the equations implies reactive power will also be dispatched according to a merit order.

7.3.3 Marginal Generators

Generator 1 was marginal for both real and reactive power. Therefore, Equations 7.29 and 7.32 are used to fix the real and reactive power marginal prices at Node 1 (in Equations 7.20 and 7.23). These equations make the marginal prices equal to the unit generation costs of Generator 1:

$$\beta_{P1} = c_{P1} \qquad \text{and} \qquad \beta_{Q1} = c_{Q1} \tag{7.38}$$

The corresponding dual slack variables are set to zero because the primal, Equations 7.7 and 7.8 are not binding (see Duality 5.5):

$$\overset{+}{v}_{P1} = \overset{-}{v}_{P1} = \overset{+}{v}_{Q1} = \overset{-}{v}_{Q1} = 0$$

7.3.4 Non–Marginal Generators

In this 3–node dispatch, Generator 3 was non–marginal for real and reactive power because it had not been dispatched. Accordingly, the primal real and reactive power lower generation limits (Equations 7.7 and 7.8 or Equations D.10 and D.12) were binding:

$$P_{G3}^{min} = P_{G3} = 0 \qquad \text{and} \qquad Q_{G3}^{min} = Q_{G3} = 0$$

By complimentary slackness, the corresponding dual slack variables are non–zero and appear in Equations 7.31 and 7.34 (ref. Duality 5.5). That is:

$$\overset{-}{v}_{P3} \geq 0 \qquad \text{and} \qquad \overset{-}{v}_{Q3} \geq 0$$

These slack variables allow the marginal price variables of real and reactive power (β_{P3} and β_{Q3} in Equations 7.22 and 7.25) to drop to values below the unit generation costs (c_{P3} and c_{Q3}). The fact that β_{P3} and β_{Q3} drop below c_{P3} and c_{Q3} implies the market was not willing to pay for power at the unit generation costs of Generator 3. v^-_{P3} and v^-_{Q3} allow the Node 3 prices (β_{P3} and β_{Q3}) to be consistent with the unit generation costs (c_{P3} and c_{Q3}).

Constraint Equations D.9 and D.11 were not binding. Therefore, v^+_{P3} and v^+_{Q3} do not feature in Equations 7.31 and 7.34 respectively.

A pqD demand node is mathematically equivalent to a PQG generator node that is non–marginal for both real and reactive power. At such a non-marginal PQG node, the real and reactive power injections are fixed due to binding generation limits. At every pqD Node i, the real and reactive power injections are similarly fixed. Consider reactive power as an example. Reactive power generation is equal to zero ($Q_{Gi} = 0$). Reactive power demand is defined by a set–point. This set–point is set by the customer who is external to the pricing model (ref. Equation 7.11). That is:

$$Q_{Di}^{min} = Q_{Di}^{max} = Q_{Di}^{set}$$

Thus, the injection of reactive power at pqD nodes (Q_i) is fixed because:

$$\begin{aligned} & Q_i^{min} \le Q_i \le Q_i^{max} \\ \Rightarrow \quad & Q_{Gi}^{min} + Q_{Di}^{min} \le Q_{Di} \le Q_{Gi}^{max} + Q_{Di}^{max} \\ \Rightarrow \quad & Q_{Di}^{min} \le Q_{Di} \le Q_{Di}^{max} \end{aligned}$$

Since the real and reactive power injections are fixed at pqD nodes, the corresponding real and reactive power marginal prices must be allowed to vary, just as with non–marginal PQG nodes. This is why the price equations of demand Node 2 (i.e. Equations 7.21, 7.24, 7.27, 7.30, 7.33 and 7.36) have the same form as the price equations of the non–marginal, generator Node 3 (i.e. Equations 7.22, 7.25, 7.28, 7.31, 7.34 and 7.37). The only difference is, the pqD marginal prices (β_{P3} and β_{Q3}) can vary freely in both directions, since either the upper or lower slack variable (v^+_P or v^-_P, and v^+_Q or v^-_Q) can be non–zero. The slack variable that becomes non–zero is determined by whether the OPF attempts to force the P_i and Q_i injections against the upper or lower limits.

Note that there is no generation at pqD nodes, and hence, no unit generation costs. For pqD Node 2 therefore:

$$c_{P2} = 0 \qquad \text{and} \qquad c_{Q2} = 0$$

7.3.5 Marginal Voltage Prices

In a power–flow problem, the voltage is fixed at every generator node. In the New Zealand power system however, the terminal voltage of each generator is usually allowed to vary between upper and lower operating limits. So, for OPF runs in this thesis where the generator terminal voltages are allowed to vary, the voltage at one bus is fixed. This fixed voltage serves as a reference voltage, preventing the drift of the voltages at all other power system nodes.

The voltage at any node can be used as a reference. In this 3–node example the Node 1 voltage is used. A reference voltage implies that constraint Equation 7.9 is binding:

$$V_{1(ref)}^{min} = V_{1(ref)} = V_{1(ref)}^{max}$$

Since V_1 was constrained as the reference voltage, complimentary slackness (i.e. Duality 5.5) requires the marginal voltage price (β_{V1}) to be unrestricted in value. This is achieved by including slack variables in Equation 7.35.

β_{V1} is the cost imposed by this reference voltage constraint on the system. Equation 7.16 is used to make it appear as a cost component in every real power and reactive power marginal price. The resultant cost components are the μ_{V1} terms present in Equations 7.20 to 7.25.

The reference voltage is the only constraint. Therefore, DBP 5.6 is satisfied by fixing two marginal prices at marginal generator nodes. Hence, the prices of marginal Generator 1, β_{P1} and β_{Q1}, have been fixed (see Equation 7.38). In an 'unconstrained' power system therefore, the reference voltage ensures there is always a minimum of two constraints.

The voltages at Nodes 2 and 3 were unconstrained for this dispatch. Hence, the marginal voltage prices, β_{V2} and β_{V3}, are fixed to zero (ref. Equations 7.36 and 7.37). This is because complimentary slackness requires the slack variables to be zero:

$$\overset{+}{v}_{V2} = \overset{-}{v}_{V2} = \overset{+}{v}_{V3} = \overset{-}{v}_{V3} = 0$$

7.3.6 The Reference Node

In this example, the marginal Node 1 has been designated as the Dispatch Based Pricing reference node (see Section 5.9.2.1). Read and Ring [1995d] note that, for a marginal reference node, all partial derivatives in the relevant Dispatch Based Pricing equations are zero. Therefore, Equations 7.20 and 7.23 can be simplified to:

$$\beta_{P1} = \lambda_P \, (= c_{P1}) \qquad \text{and} \qquad \beta_{Q1} = \lambda_Q \, (= c_{Q1})$$

This simplification is made within the source code of NODAL2, and is validated by the fact that QOPF and NODAL2 produce identical marginal prices. These simplified equations say that when the reference generator is marginal for real and reactive power, the reference prices are equal to the unit generation costs of the reference generator.

7.4 APPLICATION: MULTIPLE MARGINAL GENERATORS

Multiple marginal generators indicate an out-of-merit-order-dispatch. Binding constraints or marginal losses are the causes of these deviations from the merit order if the dispatch was optimal. The following discussions show how the pq pricing model should be used to calculate *ex post* marginal prices for observed out-of-merit-order-dispatches.

Only voltage constraints are considered in this section. However, the Dispatch Based Pricing framework allows for other constraints to be added to the pq pricing model. These constraints are formulated and implemented using terms similar to the voltage constraint terms in Equations 7.14 and 7.15 (the thermal constraint terms for example). Hence, the methods described herein can be applied to other constraint types.

In this section, Generator 3 is assumed to be marginal for reactive power (i.e. $Q_{G3}^{min} < Q_{G3} < Q_{G3}^{max}$).

7.4.1 Voltage Constraints

For every binding voltage constraint, a term will appear in Equation 7.14 and Equation 7.15. These terms take the form:

$$\mu_{Vi} \frac{\partial Vi}{\partial z}$$

z is used to represent P_i and Q_i. The appearance of this cost term is not dependent on whether the binding voltage constraint is at a PQG node or at a pqD node. This because the voltage pricing equations (7.16 and 7.19) are used for both PQG and pqD nodes.

One example of binding voltage constraints is where all generators in a pq-type spot market operate to a voltage set-point. Consider the generator voltages in the 3-node power system. They are fixed as follows:

$$V_1 = V_1^{min} = V_1^{max} \quad \text{and} \quad V_3 = V_3^{min} = V_3^{max}$$

To reflect these fixed generator voltages, Equations 7.20 to 7.25 need to include two μ_{Vi} terms. For example, Equation 7.24 would become:

$$\beta_{Q2} = -\lambda_P \frac{\partial L_P}{\partial Q_2} + \lambda_Q \left(1 - \frac{\partial L_Q}{\partial Q_2}\right) - \mu_{V1} \frac{\partial V_1}{\partial Q_2} - \mu_{V3} \frac{\partial V_3}{\partial Q_2} \tag{7.39}$$

For each fixed generator voltage, an extra marginal price must be fixed to satisfy DBP 5.6. Therefore, three fixed marginal prices are required. There are only three marginal generators: Generator 1 and 3 for reactive power and Generator 1 for real power. Hence, the only option is to fix the marginal prices for these generators:

$$\beta_{P1} = c_{P1} \, (= \lambda_P) \, , \; \beta_{Q1} = c_{Q1} \, (= \lambda_Q) \; \text{ and } \beta_{Q3} = c_{Q3}$$

In this example, Node 3 was marginal for reactive power and non–marginal for real power. If this dispatch was reversed so that Node 3 was instead marginal for real power and non–marginal for reactive power, the fixed marginal prices will be:

$$\beta_{P1} = c_{P1}, \; \beta_{Q1} = c_{Q1} \text{ and } \beta_{P3} = c_{P3}$$

It should be noted that fixed marginal prices have been used here to economically explain the presence of the binding voltage constraints. However, when calculating *ex post* prices for an observed dispatch, a dual perspective is often encountered. That is, binding constraints can be used to economically explain the presence of multiple marginal generators. As with the original perspective, this is achieved by adding constraint terms to the marginal price equations (7.14 and 7.15).

As another example, if the voltage at Node 2 was constrained $\mu_{V2} \frac{\partial V_2}{\partial Q_2}$ would need to be added to Equation 7.39.

7.4.2 Implicit Loss Constraints

It is demonstrated in Appendix G that marginal losses for real and/or reactive power can act as implicit constraints when finding an optimal dispatch. These implicit constraints cause multiple generators to be marginal for real and/or reactive power in an unconstrained optimal dispatch[1,2]. When losses act as implicit constraints, the cost of these marginal losses ensures that the prices of the multiple marginal generators are consistent with each other. Marginal losses are described as implicit constraints because all other types of binding constraints must be explicitly identified by the Clearing Manager when using the pq pricing model to calculate marginal prices.

Consider the constrained dispatch example in Section 5.5. If the MVAr limit is removed, Generator 1 and Generator 2 can both still be marginal. This will occur if

[1]Definition DBP 5.2 states that a generator is marginal if the marginal price at that generator's node is equal to the unit generation cost of the generator.

[2]Ring [1995] mentioned that losses can cause multiple marginal generators for real power.

the cost of marginal reactive power losses is equal to the difference in reactive power unit generation costs of the two generators. That is:

$$\begin{aligned} L'_Q \beta_{Q2} &= \beta_{Q2} - \beta_{Q1} \\ &= \$12/\text{MVAr} - \$9/\text{MVAr} \\ &= \$3/\text{MVAr} \end{aligned}$$

Therefore, when the marginal losses (L'_Q) equal 0.25 MVAr, the price of supplying the demand at Node 2 using Generator 2 is the same as the price of supplying the demand at Node 2 using Generator 1. Hence, β_{Q1} and β_{Q2} are consistent with each other, and β_{Q1} is the 'loss–adjusted' equivalent of β_{Q2}.

The pq pricing model can cope with multiple marginal generators that result from implicit marginal loss constraints. However, the number of fixed marginal prices must always be one more than the number of binding explicit constraints (DBP 5.6). Therefore, all generators that became marginal due to the losses must be formulated as non–marginal. This is achieved by using slack variables to allow their prices to vary. If the dispatch was optimal, these loss-adjusted marginal prices will equal the unit generation cost, forcing the slack variables to equal zero.

Consider the 'unconstrained' dispatch of the 3–node power system (ref. Section 7.3.1). Assume however, that the marginal losses acted as constraints, causing Generator 3 to be marginal for both real and reactive power (as well as Generator 1). That is:

$$P_{G3} > 0 \qquad \text{and} \qquad Q_{G3} > 0$$

The real and reactive power marginal prices of the marginal, Generator 1 have already fixed to account for the voltage constraint at Node 1 (i.e. $\beta_{P1} = c_{P1}$ and $\beta_{Q1} = c_{Q1}$). Since there are no other binding explicit constraints, Generator 3 must be formulated as non–marginal for both real and reactive power, so as to satisfy DBP 5.6. This is achieved by inserting $\overset{+}{v}_{P3}$ and $\overset{+}{v}_{Q3}$ into Equations 7.31 and 7.34 respectively, thus allowing β_{P3} and β_{Q3} to be unrestricted. For example, Equation 7.34 becomes:

$$\overset{-}{c}_{Q3} - \left\langle \overset{-}{v}_{Q3} \right\rangle \leq \beta_{Q3} \leq \overset{+}{c}_{Q3} + \left\langle \overset{+}{v}_{Q3} \right\rangle$$

This can be simplified to:

$$\beta_{Q3} = c_{Q3} + v_{Q3} \tag{7.40}$$

where either $v_{Q3} = \overset{+}{v}_{Q3}$ or $v_{Q3} = -\overset{-}{v}_{Q3}$. Thus, Equation 7.25 becomes:

$$\beta_{Q3} = c_{Q3} + v_{Q3} \left[= -\lambda_P \frac{\partial L_P}{\partial Q_3} + \lambda_Q \left(1 - \frac{\partial L_Q}{\partial Q_3}\right) - \mu_{V1} \frac{\partial V_1}{\partial Q_3} \right] \tag{7.41}$$

The real and reactive power marginal prices however, must be equal to the real and reactive power unit generation costs, because Generator 3 is marginal for real and reactive power. This only occurs when the slack variables are equal to zero. That is:

$$\beta_{P3} = c_{P3} + 0 \qquad \text{and} \qquad \beta_{Q3} = c_{Q3} + 0$$

because:

$$\overset{+}{v}_{P3} = \overset{-}{v}_{P3} = \overset{+}{v}_{Q3} = \overset{-}{v}_{Q3} = 0$$

The slack variables will only be zero when the cost of using the reference generator (Generator 1) to supply a demand for reactive power at Node 3 is equal to the unit generation cost of the marginal, Generator 3. That is, when the marginal losses are acting as implicit constraints.

In summary, all marginal prices are consistent and DBP 5.6 is satisfied. The marginal losses have acted as implicit constraints to justify $\beta_{P3} = c_{P3}$ and $\beta_{Q3} = c_{Q3}$, in the same way that the voltage constraint was used to justify $\beta_{P1} = c_{P1}$ and $\beta_{Q1} = c_{Q1}$.

The example in Appendix F verifies this discussion. The loss–adjusted marginal prices of every marginal generator node (β_{Pn} and β_{Qn}) are equal to the unit generation costs (c_{P1n} and c_{Qn}), in the marginal price set generated by the pq pricing model. These prices indicate that all generators were marginal for real and reactive power, even though only six generators were formulated as marginal.

7.5 CONCLUSIONS

In this chapter, a pq pricing model has been proposed that allows non–zero reactive power unit generation costs to be specified. This pricing model can therefore calculate *ex post* marginal prices for a spot market where real and reactive power are independently controlled, and where reactive power is optimally dispatched using generator cost functions.

Examples have been given, describing how the pq pricing model can be used to calculate *ex post* prices for dispatches resulting from different power system conditions. These examples exemplify the way in which the pq pricing model is used in Chapters 9 to 10 to investigate the behaviour of reactive power prices.

Chapter 8

A PvG DISPATCH BASED PRICING MODEL

8.1 INTRODUCTION

This chapter is concerned with a PvG–type Dispatch Based Pricing model. It is the second of the two constituent Dispatch Based Pricing models that combine to form the pq/PvG Dispatch Based Pricing model presented in Chapter 5. It is also the other pricing model that is implemented by the nodal pricing software, called NODAL2.

This pricing model is referred to as the 'PvG pricing model' because it is used to calculate *ex post* marginal prices for PvG–type spot markets. In this spot market only real power generation is optimally dispatched as an independent resource. Reactive power is optimally dispatched as a dependent resource, by optimising the independent resources of real power generation and voltage magnitude (as described in Section 5.7). This spot market is assumed to accommodate generation cost functions for both real and reactive power, to facilitate real and reactive power sub–markets. In this respect, this PvG–type spot market is identical to the pq–type spot market proposed in Chapter 7.

The equations of this PvG pricing model are presented in this chapter, together with the non–linear OPF formulation from which it is derived (referred to as PvG–OPF). The pricing model is validated first. Then examples are provided to demonstrate how to use the validated PvG pricing equations when calculating *ex post* marginal prices for different observed dispatches.

8.2 THE PvG PRICING MODEL

8.2.1 The Formulations

In a PvG–type spot market, generation nodes are modelled as PvG nodes because reactive power generation is a dependent resource. Thus a PvG–type OPF is used to model the optimisation process of this spot market. The pq/PvG OPF of Chapter 4 can be converted into a PvG–type OPF by restricting the usage of all terms used for pq nodes to usage only with pqD nodes (Note that pq=PQG$\cup$pqD). The result is Equations 8.1 to 8.13. By following the derivation of Chapters 4 and 5, the PvG pricing

THE NON–LINEAR PvG–OPF FORMULATION

$$\underset{\mathbf{P}_{\mathrm{D}}^{\mathrm{PY}},\mathbf{Q}_{\mathrm{D}}^{\mathrm{PY}},\mathbf{P}_{\mathrm{G}}^{\mathrm{PvG}},\mathbf{Q}_{\mathrm{G}}^{\mathrm{PY}},\mathbf{V}^{\mathrm{PY}}}{\text{Minimise}} \quad Costs(\mathbf{P}_{\mathrm{G}}^{\mathrm{PvG}},\mathbf{Q}_{\mathrm{G}}^{\mathrm{PvG}}) \tag{8.1}$$

subject to:

CONSERVATION OF POWER

$$\sum_{i\in\mathrm{PvG}}(P_{Gi}-P_{Di})-\sum_{i\in\mathrm{pqD}}(P_{Di})-L_P\left(\mathbf{P}_{\mathrm{G}}^{\mathrm{PvG}}-\mathbf{P}_{\mathrm{D}}^{\mathrm{PvG}},-\mathbf{P}_{\mathrm{D}}^{\mathrm{pqD}},-\mathbf{Q}_{\mathrm{D}}^{\mathrm{pqD}},\mathbf{V}^{\mathrm{PvG}}\right)=0 \tag{8.2}$$

$$\sum_{i\in\mathrm{PvG}}(Q_{Gi}-Q_{Di})-\sum_{i\in\mathrm{pqD}}(Q_{Di})-L_P\left(\mathbf{P}_{\mathrm{G}}^{\mathrm{PvG}}-\mathbf{P}_{\mathrm{D}}^{\mathrm{PvG}},-\mathbf{P}_{\mathrm{D}}^{\mathrm{pqD}},-\mathbf{Q}_{\mathrm{D}}^{\mathrm{pqD}},\mathbf{V}^{\mathrm{PvG}}\right)=0 \tag{8.3}$$

DEPENDENT REACTIVE POWER INJECTION AT PvG NODES

$$-Q_n\left(\mathbf{P}_{\mathrm{G}}^{\mathrm{PvG}}-\mathbf{P}_{\mathrm{D}}^{\mathrm{PvG}},-\mathbf{P}_{\mathrm{D}}^{\mathrm{pqD}},-\mathbf{Q}_{\mathrm{D}}^{\mathrm{pqD}},\ \mathbf{V}^{\mathrm{PvG}}\right)+(Q_{Gn}-Q_{Dn})=0 \quad \forall n\in\mathrm{PvG} \tag{8.4}$$

DEPENDENT VOLTAGE AT pq NODES

$$-V_n\left(\mathbf{P}_{\mathrm{G}}^{\mathrm{PvG}}-\mathbf{P}_{\mathrm{D}}^{\mathrm{PvG}},-\mathbf{P}_{\mathrm{D}}^{\mathrm{pqD}},-\ \mathbf{Q}_{\mathrm{D}}^{\mathrm{pqD}},\mathbf{V}^{\mathrm{PvG}}\right)+V_n=0 \qquad \forall n\in\mathrm{pqD} \tag{8.5}$$

TRANSMISSION LINE FLOWS

$$-\bar{P}_k\left(\mathbf{P}_{\mathrm{G}}^{\mathrm{PvG}}-\mathbf{P}_{\mathrm{D}}^{\mathrm{PvG}},-\mathbf{P}_{\mathrm{D}}^{\mathrm{pqD}},-\ \mathbf{Q}_{\mathrm{D}}^{\mathrm{pqD}},\mathbf{V}^{\mathrm{PvG}}\right)+\bar{P}_k=0 \qquad \forall k\in\mathrm{K} \tag{8.6}$$

$$-\bar{Q}_k\left(\mathbf{P}_{\mathrm{G}}^{\mathrm{PvG}}-\mathbf{P}_{\mathrm{D}}^{\mathrm{PvG}},-\mathbf{P}_{\mathrm{D}}^{\mathrm{pqD}},-\ \mathbf{Q}_{\mathrm{D}}^{\mathrm{pqD}},\mathbf{V}^{\mathrm{PvG}}\right)+\bar{Q}_k=0 \qquad \forall k\in\mathrm{K} \tag{8.7}$$

REAL AND REACTIVE GENERATION AND VOLTAGE SETTINGS

$$P_{Gi}^{min}\le P_{Gi}\le P_{Gi}^{max} \qquad \forall i\in\mathrm{PvG} \tag{8.8}$$

$$Q_{Gi}^{min}\le Q_{Gi}\le Q_{Gi}^{max} \qquad \forall i\in\mathrm{PvG} \tag{8.9}$$

$$V_i^{min}\le V_i\le V_i^{max} \qquad \forall i\in\mathrm{PY} \tag{8.10}$$

REAL AND REACTIVE POWER LOAD SETTINGS

$$P_{Di}=P_{Di}^{set} \qquad \forall i\in\mathrm{PY} \tag{8.11}$$

$$Q_{Di}=Q_{Di}^{set} \qquad \forall i\in\mathrm{PY} \tag{8.12}$$

TRANSMISSION LINE THERMAL LIMITS

$$\bar{P}_k^2+\bar{Q}_k^2\le T_k^{max} \qquad \forall k\in\mathrm{K} \tag{8.13}$$

The Objective Function of the PvG Pricing Model

$$
\begin{aligned}
&\underset{\substack{\chi^{K},\, \upsilon_P^{+PvG},\, \upsilon_P^{-PvG},\, \upsilon_Q^{+PvG},\, \upsilon_Q^{-PvG},\, \upsilon_V^{+PY},\, \upsilon_V^{-PY} \geq 0 \\ \lambda_P,\, \lambda_Q,\, \beta_P^{PY},\, \beta_Q^{PY},\, \mu_Q^{PvG},\, \mu_V^{pqD}}}{\text{MAXIMISE}} \quad \lambda_P \left(L_P^{*} + \sum_{i \in PY} \frac{\partial L_P}{\partial P_i} P_{Di}^{*} + \sum_{i \in pqD} \frac{\partial L_P}{\partial Q_i} Q_{Di}^{*} - \sum_{i \in PvG} \frac{\partial L_P}{\partial V_i} V_i^{*} - \sum_{i \in PvG} P_{Gi}^{*} \right) \\
&+ \lambda_Q \left(L_Q^{*} + \sum_{i \in PY} \frac{\partial L_Q}{\partial P_i} P_{Di}^{*} + \sum_{i \in pqD} \frac{\partial L_Q}{\partial Q_i} Q_{Di}^{*} - \sum_{i \in PvG} \frac{\partial L_Q}{\partial V_i} V_i^{*} - \sum_{i \in PvG} Q_{Gi}^{*} \right) \\
&+ \sum_{n \in PvG} \mu_{Qn} \left(-Q_{Dn}^{*} + \sum_{i \in PY} \frac{\partial Q_n}{\partial P_i} P_{Di}^{*} + \sum_{i \in pqD} \frac{\partial Q_n}{\partial Q_i} Q_{Di}^{*} - \sum_{i \in PvG} \frac{\partial Q_n}{\partial V_i} V_i^{*} \right) \\
&+ \sum_{n \in pqD} \mu_{Vn} \left(V_n^{*} + \sum_{i \in PY} \frac{\partial V_n}{\partial P_i} P_{Di}^{*} + \sum_{i \in pqD} \frac{\partial V_n}{\partial Q_i} Q_{Di}^{*} - \sum_{i \in PvG} \frac{\partial V_n}{\partial V_i} V_i^{*} \right) \\
&+ \sum_{i \in PvG} \left(\overset{+}{\upsilon}_{Pi} \left(-P_{Gi}^{max} + P_{Gi}^{*} \right) + \overset{-}{\upsilon}_{Pi} \left(P_{Gi}^{min} - P_{Gi}^{*} \right) \right) \qquad (8.14) \\
&+ \sum_{i \in PvG} \left(\overset{+}{\upsilon}_{Qi} \left(-Q_{Gi}^{max} + Q_{Gi}^{*} \right) + \overset{-}{\upsilon}_{Qi} \left(Q_{Gi}^{min} - Q_{Gi}^{*} \right) \right) \\
&+ \sum_{i \in PY} \left(-\overset{+}{\upsilon}_{Vi} V_i^{max} + \overset{-}{\upsilon}_{Vi} V_i^{min} \right) \\
&+ \sum_{i \in PY} \beta_{Pi} P_{Di}^{*} + \sum_{i \in PY} \beta_{Qi} Q_{Di}^{*} \\
&+ \sum_{k \in K} \chi_k \left(-T_k^{max} - \bar{P}_k^{*2} - \bar{Q}_k^{*2} \right)
\end{aligned}
$$

Continued overleaf

THE CONSTRAINTS OF THE PvG PRICING MODEL

subject to:

Marginal Price Equations

$$\beta_{Pi} = \lambda_P\left(1-\frac{\partial L_P}{\partial P_i}\right) - \lambda_Q\frac{\partial L_Q}{\partial P_i} - \sum_{n\in \mathrm{PvG}}\mu_{Qn}\frac{\partial Q_n}{\partial P_i} - \sum_{n\in \mathrm{pqD}}\langle\mu_{Vn}\rangle\left\langle\frac{\partial V_n}{\partial P_i}\right\rangle - 2\sum_{k\in \mathrm{K}}\langle\chi_k\rangle\left(\left\langle\bar{P}_k^*\frac{\partial\bar{P}_k}{\partial P_i}\right\rangle+\left\langle\bar{Q}_k^*\frac{\partial\bar{Q}_k}{\partial P_i}\right\rangle\right) \qquad \forall i\in \mathrm{PY} \qquad (8.15)$$

$$\beta_{Qi} = -\lambda_P\frac{\partial L_P}{\partial Q_i} + \lambda_Q\left(1-\frac{\partial L_Q}{\partial Q_i}\right) - \sum_{n\in \mathrm{PvG}}\mu_{Qn}\frac{\partial Q_n}{\partial Q_i} - \sum_{n\in \mathrm{pqD}}\langle\mu_{Vn}\rangle\left\langle\frac{\partial V_n}{\partial Q_i}\right\rangle - 2\sum_{k\in \mathrm{K}}\langle\chi_k\rangle\left(\left\langle\bar{P}_k^*\frac{\partial\bar{P}_k}{\partial Q_i}\right\rangle+\left\langle\bar{Q}_k^*\frac{\partial\bar{Q}_k}{\partial Q_i}\right\rangle\right) \qquad \forall i\in \mathrm{pqD} \qquad (8.16)$$

$$\beta_{Vi} = -\lambda_P\frac{\partial L_P}{\partial V_i} - \lambda_Q\frac{\partial L_Q}{\partial V_i} - \sum_{n\in \mathrm{PvG}}\mu_{Qn}\frac{\partial Q_n}{\partial V_i} - \sum_{n\in \mathrm{pqD}}\langle\mu_{Vn}\rangle\left\langle\frac{\partial V_n}{\partial V_i}\right\rangle - 2\sum_{k\in \mathrm{K}}\langle\chi_k\rangle\left(\left\langle\bar{P}_k^*\frac{\partial\bar{P}_k}{\partial V_i}\right\rangle+\left\langle\bar{Q}_k^*\frac{\partial\bar{Q}_k}{\partial V_i}\right\rangle\right) \qquad \forall i\in \mathrm{PvG} \qquad (8.17)$$

$$\mu_{Qn} = \beta_{Qn} - \lambda_Q \qquad \forall n\in \mathrm{PvG} \qquad (8.18)$$

$$\mu_{Vn} = \beta_{Vn} \qquad \forall n\in \mathrm{pqD} \qquad (8.19)$$

Price Bound Equations

$$c^{-}_{Pi} - \left\langle v^{-}_{Pi}\right\rangle \le \beta_{Pi} \le c^{+}_{Pi} + \left\langle v^{+}_{Pi}\right\rangle \qquad \forall i\in \mathrm{PvG} \qquad (8.20)$$

$$c^{-}_{Qi} - \left\langle v^{-}_{Qi}\right\rangle \le \beta_{Qi} \le c^{+}_{Qi} + \left\langle v^{+}_{Qi}\right\rangle \qquad \forall i\in \mathrm{PvG} \qquad (8.21)$$

$$\beta_{Vi} = \left\langle v^{-}_{Vi}\right\rangle - \left\langle v^{+}_{Vi}\right\rangle \qquad \forall i\in \mathrm{PY} \qquad (8.22)$$

model (Equations 8.14 to 8.22) can be obtained. The derivation process is equivalent to restricting the use of all pq terms to pqD nodes. Hence, this PvG pricing model can be used to calculate *ex post* marginal prices for this PvG–type spot market.

P_G is independently controlled and V is fixed at all PvG nodes in the formulations of this PvG–OPF and the PvG pricing model. It follows that Q_G is a dependent variable at PvG nodes and is optimised by optimising the independent variables:

$$\mathbf{P}_{\mathbf{G}}^{\mathrm{PvG}}, \mathbf{P}_{\mathbf{D}}^{\mathrm{PvG}}, \mathbf{P}_{\mathbf{D}}^{\mathrm{pqD}}, \mathbf{Q}_{\mathbf{D}}^{\mathrm{pqD}}, \mathbf{V}^{\mathrm{PvG}} \tag{8.23}$$

Equation 8.4 describes the dependence of Q_G on these variables.

The main difference between the PvG pricing model and the pq/PvG Dispatch Based Pricing model in Chapter 5 comes from the PvG–OPF formulation. PvG–OPF is formulated so that there is no real or reactive power generation at non–generator nodes:

$$P_{Gi} = Q_{Gi} = 0 \qquad \forall i \in \textit{non–generator nodes} \tag{8.24}$$

The real and reactive power injections therefore, are fixed at non–generator nodes because there is no generation and any power demand is fixed by Equations 8.11 and 8.12. Therefore, all non–generator nodes are classed as pqD nodes.

Equation 8.24 implies that the generation limits described by Equations 8.8 and 8.9 apply only to PvG nodes. Whereas, in the pq/PvG OPF the equivalent equations (i.e Equations 4.8 and 4.9) apply to all nodes. By duality theory, the absence of generation limits at pqD nodes in the OPF formulation means there are no 'Price Bound Equation' constraints in the PvG pricing model for pqD nodes. This means the marginal prices of real and reactive power at pqD nodes are unrestricted. That is, β_{Pi} and β_{Qi} are unrestricted variables at pqD nodes because the Price Bound Equations 8.20 and 8.21 only apply to PvG nodes.

In PvG–OPF and the PvG pricing model, the set of all nodes is defined as:

$$\mathrm{PY} = (\mathrm{PvG} \cup \mathrm{pqD})$$

8.2.2 Reactive Power Generation at Non–Generator Nodes

The PvG pricing model is a limiting case of a model where it is possible to have reactive power generation at non–generator nodes. That is

$$P_{Gi} = 0, \text{ but } Q_{Gi} \neq 0 \qquad \forall i \in \textit{non–generator nodes}$$

Using the node classification system in Section 3.5.2, these nodes are classed as pQG nodes. The uppercase 'Q' indicates that Q_i is an independent variable for these nodes, and must therefore be included as an element in control vector $\tilde{u}$.

Including pQG nodes in the PvG formulation results in changes to:

- the objective function,

$$Costs(\mathbf{P}_{G}^{PvG}, \mathbf{Q}_{G}^{PvG}, \mathbf{Q}_{G}^{pQG})$$

- the independent variables,

$$\left(\mathbf{P}_{G}^{PvG} - \mathbf{P}_{D}^{PvG}, -\mathbf{P}_{D}^{pQG \cup pqD}, \mathbf{Q}_{G}^{pQG} - \mathbf{Q}_{D}^{pQG}, -\mathbf{Q}_{D}^{pqD}, \mathbf{V}^{PvG}\right)$$

- Equation 8.9,

$$\forall i \in (\mathrm{PvG} \cup \mathrm{pQG})$$

- and the set of all nodes.

$$\mathrm{PY} = (\mathrm{PvG} \cup \mathrm{pQG} \cup \mathrm{pqD})$$

A Dispatch Based Pricing model can be derived from this modified OPF, which incorporates unit generation costs for devices providing reactive power support at non–generator nodes. Examples of such devices are static VAr compensators and capacitor banks. An investigation of this pricing model is outside the scope of this thesis and has been left as future work.

8.2.3 Validation of the PvG Pricing Model

8.2.3.1 Background

The validation of the PvG pricing model was not straightforward. A software implementation does not exist for PvG–OPF, unlike QOPF which implements pq–OPF of Chapter 7. However, it is possible to use Matpower and QOPF to establish a certain amount of confidence in the marginal price output of NODAL2. Thus, most terms of the PvG pricing model are validated, indicating that these Dispatch Based Pricing equations are being applied correctly to describe the optimisation process of this PvG–type spot market. The confidence also implies that, possibly, the NODAL2 source code correctly implements the PvG pricing model.

An early assumption was that the PvG pricing model and pq pricing model were equivalent, and would produce identical sets of real and reactive power marginal prices. This is because the PvG–type and pq–type OPFs from which these pricing models are derived were assumed to be equivalent, and assumed to produce identical optimal dispatches. However, this assumption is incorrect.

The assumption was founded on the apparent similarities between the formulations of the two OPF types. Their similarities are described below. One point of clarification, this assumption requires that all generator voltages are fixed using voltage constraints when considering the pq–type OPF.

Consider the formulations of the pq–type and PvG–type OPFs. Both have the same objective function:

$$f\left(cost\right)_{P,Q} = \sum_{i \in Gen} M_P\left(P_G\right)_i + \sum_{i \in Gen} M_Q\left(Q_G\right)_i \tag{8.25}$$

For both OPF types, $M_P\left(P_G\right)_i$ and $M_Q\left(Q_G\right)_i$ are functions only of P_G and Q_G respectively. This is illustrated by Equations 6.2 and 6.9.

Both OPF types are assumed to use mismatch Equation 6.3 to describe the power system. This equation is incorporated into the formulation of the pq–type OPF using Equations 6.10 to 6.13, which are repeated here for clarity:

$$P_{mismatch_i} = 0 \qquad \forall i \in Gen \tag{8.26}$$

$$P_{mismatch_n} = 0 \qquad \forall n \in Dem \tag{8.27}$$

$$Q_{mismatch_i} = 0 \qquad \forall i \in Gen \tag{8.28}$$

$$Q_{mismatch_n} = 0 \qquad \forall n \in Dem \tag{8.29}$$

For reasons discussed in Section 6.4.1.3, only constraint Equations 8.26, 8.27 and 8.29 (ref. Equations 6.4 to 6.6) are used in the formulation of a PvG–type OPF.

In the pq–type OPF algorithm, Vector–Set 3.5 applies, except that V_i (for all generator nodes) has been transferred from $\tilde{x}$ to $\tilde{p}$. This indicates that all generator voltages are fixed. The gradient vector used by this pq–type OPF algorithm to minimise the objective function is:

$$\nabla f\left(cost\right)_{P,Q} = \left[\underbrace{\frac{\partial M_P}{\partial \theta_i}}_{=0}+\underbrace{\frac{\partial M_P}{\partial \theta_i}}_{=0}, \underbrace{\frac{\partial M_P}{\partial V_i}}_{=0}+\underbrace{\frac{\partial M_Q}{\partial V_i}}_{=0}, \underbrace{\frac{\partial M_P}{\partial \theta_n}}_{=0}+\underbrace{\frac{\partial M_Q}{\partial \theta_n}}_{=0}, \underbrace{\frac{\partial M_P}{\partial P_i}}_{\neq 0}+\underbrace{\frac{\partial M_P}{\partial P_i}}_{=0}, \underbrace{\frac{\partial M_P}{\partial Q_i}}_{=0}+\underbrace{\frac{\partial M_P}{\partial Q_i}}_{\neq 0}\right] \begin{array}{l}\forall i \in Gen\\ \forall n \in Dem\end{array} \tag{8.30}$$

The gradient vector used by a PvG–type OPF algorithm is:

$$\nabla f\left(cost\right)_{P,Q} = \left[\underbrace{\frac{\partial M_P}{\partial \theta_i}}_{=0}+\underbrace{\frac{\partial M_P}{\partial \theta_i}}_{\neq 0}, \underbrace{\frac{\partial M_P}{\partial V_i}}_{=0}+\underbrace{\frac{\partial M_Q}{\partial V_i}}_{\neq 0}, \underbrace{\frac{\partial M_P}{\partial \theta_n}}_{=0}+\underbrace{\frac{\partial M_Q}{\partial \theta_n}}_{\neq 0}, \underbrace{\frac{\partial M_P}{\partial P_i}}_{\neq 0}+\underbrace{\frac{\partial M_P}{\partial P_i}}_{\neq 0}\right] \begin{array}{l}\forall i \in Gen\\ \forall n \in Dem\end{array} \tag{8.31}$$

Q_{Gi} is an independent variable in a pq–type OPF, so its value is known at all times. Note that $Q_i = Q_{Gi} - Q_{Di}$, and that Q_{Di} is fixed. Thus, $\frac{\partial M_Q}{\partial Q_{Gi}}$ of Equation 6.9 is actually $\frac{\partial M_Q}{\partial Q_i}$ expressed only in terms of the variable Q_{Gi}. This is also true for P_{Gi} and

$\frac{\partial M_P}{\partial P_{Gi}}$. Hence, only these two partial derivatives are not equal zero in Equation 8.30. For a PvG–type OPF however, Q_G is only calculated by the algorithm at the end of each iteration. Consequently, all partial derivatives of Equation 6.9 (i.e. $\frac{\partial M_Q}{\partial z}$) must be expressed in terms of all other variables. For example:

$$\frac{\partial M_Q}{\partial \theta_n} = f\left(\theta^{\mathrm{PX}}, \mathbf{V}^{\mathrm{PX}}, \mathbf{P}^{\mathrm{PX}}, \mathbf{Q}^{\mathrm{PX}}\right)$$

As a result, all partial derivatives of M_Q do not equal zero in gradient vector 8.31.

The reasoning behind the early assumption is that the same OPF formulation is being solved using different algorithms. That is, the PvG-type OPF formulation and the pq–type OPF formulation describe the same problem. The only difference is, for the pq–type OPF reactive power conservation at generator nodes is included in the form of Equality Constraint 8.29. Whereas, reactive power conservation is included in the definition of the variable Q_G for the PvG–type OPF.

8.2.3.2 Dispatch Experiments

Experiments were used to validate the PvG pricing model, and to test the assumption. The results of the experiments indicate that: the PvG pricing model is mathematically correct; that NODAL2 correctly implements the model; and that the initial assumption was incorrect.

The validation experiments summarised in Section 7.2.2 minimise objective function 8.25. Since these experiments validated the pq pricing model and the 'pq' part of NODAL2, the following conclusions can be drawn:

- $\sum_{n \in \mathrm{pq}} \mu_{V_n} \frac{\partial V_n}{\partial z_i}$ has been correctly implemented in software, and
- Equation 8.17 (i.e. Equation 5.4) has been correctly implemented.

Further, Equation 8.18 (i.e. Equation 5.5) has been checked and found to be correctly implemented. Therefore, it can be concluded (with some caution) that the term unique to the PvG pricing model, $\sum_{n \in \mathrm{PvG}} \mu_{Q_n} \frac{\partial Q_n}{\partial z_i}$, has also been correctly implemented. This implies that the marginal prices from the PvG pricing model are correct.

Matpower and QOPF were used to compare the PvG pricing model to the pq pricing model. Three dispatches of the IEEE 14 bus power system were considered, the data of which is in Appendix E. The three dispatches were optimised using Objective Function 6.1 where the unit generation cost of reactive power is zero. Hence, the real power generator cost data in the appendix is unmodified, but the reactive power generator cost data is set to zero: $M_{Q_i} = 0$ for every generator Node i. The three dispatches were:

1. Unconstrained. The data is unchanged.
2. Q constrained. Generator 8 is forced to be non–marginal for reactive power by setting: $Q_{G8}^{max} = 15.85$ MVAr.
3. V constrained. A voltage constraint is made binding at Node 13 by setting: $V_{13}^{max} = 1.049$ pu.

Table 8.1 summarises the results.

Table 8.1 A comparison of marginal price profiles between Matpower and the PvG and pq models, and between QOPF and the PvG and pq models. Reactive power has a zero unit generation cost.

	Matpower			*QOPF*	
Dispatch	PvG	pq	Generation Profile	PvG	pq
Unconstrained	✓	✓	*Similar*	✓	✓
Q	✓	✓	*Different*	✓	✓
V	✓	✓	*Different*	✓	✓

Matpower and QOPF produced different generation profiles for the three dispatches. The difference in the generation profiles was most noticeable in the voltage constrained dispatch. Hence, the marginal price profiles from Matpower and QOPF are different for each dispatch.

The marginal price profile from Matpower matched the price profile from the PvG pricing model when the Matpower generation profile (and other bus data) for each of the three dispatches was passed through the PvG pricing model. Without modifying the equations of the PvG pricing model, the QOPF dispatches were also passed through the pricing model and the price profiles from QOPF and the PvG pricing model agreed to 4 d.p. When the pq pricing model is used instead, the marginal prices from this model are identical to the Matpower and QOPF marginal prices. Hence, the pq pricing model prices are identical to the prices of the PvG pricing model. Therefore, the PvG and pq pricing models are equivalent when the unit generation cost of reactive power is zero. That is:

$$M_{Q_i} = 0 \qquad \forall i \in Gen \quad (8.32)$$

Matpower (a PvG–type OPF) and QOPF (a pq–type OPF) however, are not equivalent. This is because Matpower and QOPF have each produced a different set of marginal prices for each dispatch. The Matpower price sets are valid because the pq pricing model and the PvG pricing model have produced matching price sets. Likewise, the pq and PvG pricing models have produced price sets matching those from QOPF, thus validating the QOPF prices.

The example in Appendix F further demonstrates that PvG–type OPFs and pq–type OPFs are not equivalent when $M_{Q_i} \neq 0$. In Section 10.7 this example is used to demonstrate that an optimal dispatch produced by a pq–type OPF is sub–optimal with respect to a PvG–type OPF and vice versa. This implies the optimal dispatch from one OPF type is different to the optimal dispatch from the other OPF type: different dispatches only occur if the pq–OPF and the PvG–OPF are not equivalent. Note that this reasoning process assumes the formulations of the pq–OPF and PvG–OPF have been correctly validated.

8.2.3.3 Conclusions

The software implementation of the PvG pricing model (i.e. NODAL2) has been validated as far as possible, without the use of a software implementation of PvG–OPF. This suggests that the Dispatch Based Pricing equations used in the PvG pricing model are being correctly applied for the purpose of calculating marginal prices for a PvG–type spot market. Moreover, it has been completely demonstrated that the PvG pricing model and pq pricing model (and their corresponding OPFs) are not equivalent when the unit generation costs of reactive power are non–zero.

8.3 APPLICATION: AN UNCONSTRAINED DISPATCH

Examples of how to use the PvG pricing model to calculate *ex post* prices for observed dispatches are presented in this section. The 3–node power system in Figure 7.1 is used for these examples, and it is assumed that it was optimally dispatched by a PvG–type OPF (for example, PvG–OPF). The observed dispatch was unconstrained and had the following characteristics:

- Generator 1 was marginal for real and reactive power;
- Generator 3 was marginal for Q_{G3}, but not dispatched (non–marginal) for real power, $P_{G3} = P_{G3}^{min} = 0$;
- Node 1 is the Dispatch Based Pricing reference node (i.e. $c_{P1} = \lambda_P$ and $c_{Q1} = \lambda_Q$);
- Nodes 1 and 3 are PvG nodes;
- Node 2 is a pqD node.

The resultant PvG pricing equations for this observed dispatch are:

Real power marginal price equations

$$\beta_{P1} = \lambda_P \left(1 - \frac{\partial L_P}{\partial P_1}\right) - \lambda_Q \frac{\partial L_Q}{\partial P_1} - \mu_{Q1} \frac{\partial Q_1}{\partial P_1} - \mu_{Q3} \frac{\partial Q_3}{\partial P_1} \tag{8.33}$$

$$\beta_{P2} = \lambda_P \left(1 - \frac{\partial L_P}{\partial P_2}\right) - \lambda_Q \frac{\partial L_Q}{\partial P_2} - \mu_{Q1} \frac{\partial Q_1}{\partial P_2} - \mu_{Q3} \frac{\partial Q_3}{\partial P_2} \tag{8.34}$$

$$\beta_{P3} = \lambda_P \left(1 - \frac{\partial L_P}{\partial P_3}\right) - \lambda_Q \frac{\partial L_Q}{\partial P_3} - \mu_{Q1} \frac{\partial Q_1}{\partial P_3} - \mu_{Q3} \frac{\partial Q_3}{\partial P_3} \tag{8.35}$$

Reactive power marginal price equations

$$\mu_{Q1} = \beta_{Q1} - \lambda_Q \tag{8.36}$$

$$\beta_{Q2} = -\lambda_P \frac{\partial L_P}{\partial Q_2} + \lambda_Q \left(1 - \frac{\partial L_Q}{\partial Q_2}\right) - \mu_{Q1} \frac{\partial Q_1}{\partial Q_2} - \mu_{Q3} \frac{\partial Q_3}{\partial Q_2} \tag{8.37}$$

$$\mu_{Q3} = \beta_{Q3} - \lambda_Q \tag{8.38}$$

Voltage marginal price equations

$$\beta_{V1} = -\lambda_P \frac{\partial L_P}{\partial V_1} - \lambda_Q \frac{\partial L_Q}{\partial V_1} - \mu_{Q1} \frac{\partial Q_1}{\partial V_1} - \mu_{Q3} \frac{\partial Q_3}{\partial V_1} \tag{8.39}$$

$$\mu_{V2} = \beta_{V2} \tag{8.40}$$

$$\beta_{V3} = -\lambda_P \frac{\partial L_P}{\partial V_3} - \lambda_Q \frac{\partial L_Q}{\partial V_3} - \mu_{Q1} \frac{\partial Q_1}{\partial V_3} - \mu_{Q3} \frac{\partial Q_3}{\partial V_3} \tag{8.41}$$

Price bound equations (which determine the range of values for each marginal price):

$$c^-_{P1} \leq \beta_{P1} \leq c^+_{P1} \tag{8.42}$$

$$c^-_{P3} - v^-_{P3} \leq \beta_{P3} \leq c^+_{P3} \tag{8.43}$$

$$c^-_{Q1} \leq \beta_{Q1} \leq c^+_{Q1} \tag{8.44}$$

$$c^-_{Q3} \leq \beta_{Q3} \leq c^+_{Q3} \tag{8.45}$$

$$\beta_{V1} = \left\langle v^-_{V1} \right\rangle - \left\langle v^+_{V1} \right\rangle \tag{8.46}$$

$$\beta_{V2} = 0 \tag{8.47}$$

$$\beta_{V3} = \left\langle v^-_{V3} \right\rangle - \left\langle v^+_{V3} \right\rangle \tag{8.48}$$

8.3.1 Voltage Prices

V_2 was unrestricted at Node 2. So, the corresponding marginal voltage price in Equation 8.40 (β_{V2}) is fixed through Equation 8.47. In a PvG–type spot market all generator voltages are fixed. Hence, V_1 and V_3 were fixed because Nodes 1 and 3 are PvG nodes. The fixed voltages are demonstrated by Equations 5.16 to 5.18. Hence, the slack variables in Equations 8.46 and 8.48 (υ^+_{V1}, υ^-_{V1}, υ^+_{V3} and υ^-_{V3}) allow the marginal voltage prices in Equations 8.39 and 8.41 (β_{V1} and β_{V3}) to be unrestricted in value.

8.3.2 Marginal Generators

Generator 1 was marginal for real power because P_{G1} was still free to vary after the dispatch had been optimised. That is, Equation 8.8 was not binding. This requires that both slack variables υ^+_{P1} and υ^-_{P1} equal zero. Consequently, the marginal price in Equation 8.33 must be forced to equal its unit generation cost by using Equation 8.42. That is:

$$\beta_{P1} = c_{P1}$$

In Section G.4 it is observed that constraining all generator voltages in a pq–type spot market generally results in all generators being marginal for reactive power. This is also true for generators in a PvG–type spot market where generator voltages are always fixed (power system losses have a small influence on multiple marginal generators). Multiple generators were observed to be marginal for reactive power in the (zero reactive power unit generation cost) Matpower experiments of Table 8.1. Therefore, the results of these experiments can be extrapolated to say that multiple marginal generators for reactive power occur in PvG–type dispatches where reactive power generation has a non-zero unit generation cost.

Fixed generator voltages are assumed to be the reason for both Generator 1 and Generator 3 having been marginal for reactive power in this dispatch of the 3–node system. Hence, DBP 5.6 is satisfied because the marginal prices for reactive power at Nodes 1 and 3 (in Equations 8.36 and 8.38) are fixed by Equations 8.44 and 8.45:

$$\beta_{Q1} = c_{Q1} \qquad \text{and} \qquad \beta_{Q3} = c_{Q3}$$

Note that:

$$\upsilon^+_{Q1} = \upsilon^-_{Q1} = \upsilon^+_{Q3} = \upsilon^-_{Q3} = 0$$

8.3.3 Non–Marginal Nodes

Generator 3 was not dispatched for real power, such that Equation 8.8 was binding:

$$P_{G3} = P^{min}_{G3} = 0$$

This enables v^-_{P3} to be used in Equation 8.43 to allow β_{P3} (in Equation 8.35) to vary. It follows that the marginal (market) price of real power at Node 3 will be less than the unit generation cost. That is:

$$\beta_{P3} \leq c_{P3}$$

This equation indicates that the market is not willing to pay the unit cost of real power generation from the generator at Node 3. As a consequence, Generator 3 was not dispatched because its owner was not willing to sell real power for anything less than c_{P3}.

Node 2 is a pqD node, indicating that the real and reactive power injections (P_2 and Q_2) were fixed. Section 8.2.1 showed that price bound equations are not required for pqD nodes. For Node 2 therefore, β_{P2} and β_{Q2} in Equations 8.34 and 8.37 are unrestricted.

8.3.4 The Reference Node

Node 1 is the reference node for this example. Hence, Equations 8.33 and 8.36 can be simplified, as in Section 7.3.6. That is:

$$\beta_{P1} = \lambda_P \left(= c_{P1}\right) \qquad \text{and} \qquad \beta_{Q1} = \lambda_Q \left(= c_{Q1}\right)$$

The marginal voltage price equation (8.39) cannot be simplified, as all of the partial derivatives in this equation are not equal to zero.

8.4 APPLICATION: A CONSTRAINED DISPATCH

This section illustrates how the PvG pricing model is to be used when calculating *ex post* marginal prices for constrained observed dispatches from a PvG–type spot market. Only voltage constraints, reactive power generation constraints and implicit loss constraints are considered herein. However, other constraint types are implemented in a similar manner within the Dispatch Based Pricing framework.

8.4.1 Voltage Constraints

In a PvG–type spot market, voltage constraints can only occur at pqD nodes, because all generator voltages are already fixed. For every voltage constraint, the term in Equations 8.15 to 8.17 will be non–zero:

$$\mu_{Vn} \frac{\partial V_n}{\partial z} \neq 0 \qquad \forall n \in pqD$$

For example, if the observed dispatch had a voltage constraint at Node 2, Equations 8.33 to 8.35 and 8.37 will include $\mu_{V2}\frac{\partial V_2}{\partial Q_2}$. For example, Equation 8.37 becomes:

$$\beta_{Q2} = -\lambda_P \frac{\partial L_P}{\partial Q_2} + \lambda_Q \left(1 - \frac{\partial L_Q}{\partial Q_2}\right) - \mu_{Q1}\frac{\partial Q_1}{\partial Q_2} - \mu_{Q3}\frac{\partial Q_3}{\partial Q_2} - \mu_{V2}\frac{\partial V_2}{\partial Q_2}$$

To satisfy DBP 5.6, another marginal price must be fixed. β_{P3} is the only available marginal price, as β_{P1}, β_{Q1} and β_{Q3} are already fixed. Fortunately, a constraint usually forces another generator to be marginal. Therefore, it can be assumed that Generator 3 had also become marginal for real power. Hence, the required fixed marginal prices for this voltage constrained dispatch of the 3–node power system are:

$$\beta_{P1} = c_{P1}, \quad \beta_{Q1} = c_{Q1}, \quad \beta_{P3} = c_{P3} \quad \text{and} \quad \beta_{Q3} = c_{Q3}$$

8.4.2 Non–Marginal Generators for Reactive Power

A generator that is non–marginal for reactive power has been forced against an upper or lower reactive power generation limit (ref. Equation 8.9). The 'Q' dispatch in Table 8.1 is a dispatch for the 14–bus power system where Generator 8 has reached its upper reactive power generation limit.

For this 14–bus dispatch, the marginal prices from the PvG pricing model only matched the marginal prices from Matpower when Node 8 was modelled using Equations 8.16 and 8.19 instead of Equations 8.17 and 8.18. Furthermore, the marginal prices from the PvG pricing model only matched the marginal prices from QOPF when Equations 8.16 and 8.19 were used instead of Equations 8.17 and 8.18. Therefore, a generator node is transformed from a PvG node to a PqG node with a voltage constraint when Equation 8.9 becomes binding. This is because Q_G has been transformed from a dependent variable to an independent fixed parameter.

For the 3–node example, assume that Generator 3 has become non–marginal for reactive power, such that:

$$Q_{G3} = Q_{G3}^{max} \tag{8.49}$$

The required changes to Equations 8.33 to 8.48 for the transformation from PvG node to PqG node are as follows. Equation 8.38 is replaced with:

$$\beta_{Q3} = -\lambda_P \frac{\partial L_P}{\partial Q_3} + \lambda_Q \left(1 - \frac{\partial L_Q}{\partial Q_3}\right) - \mu_{Q1}\frac{\partial Q_1}{\partial Q_3} - \mu_{V3}\frac{\partial V_3}{\partial Q_3} \tag{8.50}$$

and Equation 8.41 is replaced with $\mu_{V3} = \beta_{V2}$

Equation 8.50 illustrates that all $-\mu_{Q3}\frac{\partial Q_3}{\partial z}$ terms must be removed, and that terms describing the new voltage constraint at this PqG node must be added to Equations 8.33

to 8.35, and 8.37 to 8.39:

$$-\mu_{V3}\frac{\partial V_3}{\partial z}$$

Experience has shown that often another generator will become marginal if an OPF constraint becomes binding, even if that generator had not previously been dispatched. Therefore, it can be assumed that Generator 3 was dispatched and became marginal for real power when it became non–marginal for reactive power. That is:

$$P_{G3} > P_{G3}^{min} \quad \text{where} \quad P_{G3}^{min} = 0\,\text{MW}$$

The marginal price of reactive power at Node 3 must be allowed to vary because Generator 3 is non–marginal for reactive power. Therefore, υ_{Q3}^{+} is added to Equation 8.45 because Generator 3 is non–marginal due to the binding upper generation limit (Equation 8.49):

$$c_{Q3}^{-} \leq \beta_{Q3} \leq c_{Q3}^{+} + \upsilon_{Q3}^{+}$$

Making β_{Q3} unrestricted means that a different fourth marginal generator price must be fixed to satisfy DBP 5.6. Therefore, β_{P3} can be fixed because Generator 3 is now marginal for real power. Thus, Equation 8.43 becomes:

$$c_{P3}^{-} \leq \beta_{P3} \leq c_{P3}^{+}$$

All other equations remain unchanged.

8.4.3 Implicit Loss Constraints

In Section 7.4.2, implicit loss constraints were shown to be a cause of multiple marginal generators in an optimal dispatch. When using the PvG pricing model however, implicit loss constraints are the primary economic explanation of multiple marginal generators of real power only. This is because the generator voltage constraints are already used to account for the fixed reactive power marginal prices of the multiple marginal generators for reactive power (see Section 8.3.2).

The PvG pricing model allows for implicit loss constraints in exactly the same way as the pq pricing model. Consider the original unconstrained PvG dispatch in Section 8.3 as an illustration, but assume that Generator 3 was marginal for real power, as well as for reactive power.

Generator 3 however, must remain formulated as non–marginal for real power to satisfy DBP 5.6. This means that β_{P3} must be totally unrestricted. That is, Equation 8.43 must become:

$$c_{P3}^{-} - \left\langle v_{P3}^{-} \right\rangle \leq \beta_{P3} \leq c_{P3}^{+} + \left\langle v_{P3}^{+} \right\rangle$$

When marginal losses act as constraints however, this equation will become:

$$\beta_{P3} = c_{P3}$$

This is because the slack variables become equal to zero:

$$v_{P3}^{+} = v_{P3}^{-} = 0$$

The slack variables equal zero because the cost of the marginal losses makes the marginal price (β_{P3}) equal to the real power unit generation cost of Generator 3 (c_{P3}). This demonstrates that Generator 3 was indeed marginal for real power.

8.5 CONCLUSIONS

A PvG pricing model has been proposed. Validation experiments have been used to demonstrate that this PvG pricing model and the pq pricing model of Chapter 7 are different.

The PvG pricing model has been validated as far as possible without the use of a software implementation of PvG–OPF. However, the model has not been used to investigate the behaviour of marginal prices for reactive power in a PvG–type spot market, for two reasons:

1. a software implementation of PvG–OPF does not exist. Therefore, observed optimal dispatches from a PvG–type spot market cannot be generated.

2. PvG–OPF is required to produce benchmark marginal prices to validate the 'PvG' part of the NODAL2 source code. It is also required to verify that the PvG pricing equations calculate marginal prices that correctly describe the cost of dispatching real and reactive power generation in a PvG–type spot market. Therefore, an element of uncertainty remains in the validity of marginal prices generated by the PvG pricing model, because PvG–OPF has not been implemented in software.

The behaviour of marginal prices of reactive power in a pq–type spot market is investigated in Chapters 9 and 10.

Chapter 9

OPTIMAL DISPATCH PRICE BEHAVIOUR

9.1 INTRODUCTION

This chapter investigates the behaviour that can be expected of reactive power prices in a pq–type spot market (introduced in Chapter 7). The pq–OPF (Equations 7.1 to 7.12) models the optimisation process of this spot market. Both pq–OPF and the pq pricing model (Equations 7.13 to 7.19) are capable of calculating marginal prices for this spot market.

For all cases in this chapter, it is assumed that:

- an optimal (economic) dispatch has been achieved, and
- the Clearing Manager has correctly identified all constraints that have become binding during the optimisation process (as discussed in Section 3.2).

When these assumptions are valid, duality theory states that pq–OPF (i.e. QOPF, the software implementation of pq–OPF) and the pq pricing model (i.e. NODAL2, its software implementation) will produce identical marginal prices. This is depicted in Figure 6.2. For convenience therefore, QOPF has been used to generate both the optimal dispatches and the corresponding *ex post* marginal prices for all case studies presented within this chapter. However, NODAL2 was used to verify the *ex post* prices calculated by QOPF. QOPF and NODAL2 produce identical prices for each and every case study.

Dandachi *et al* [1996] recognised the impracticalities of re–optimising the reactive power dispatch too frequently, citing altering transformer taps and altering shunt devices during re–optimisation. In New Zealand's spot market the real power dispatch is usually only re–optimised at the start of each trading period[1]. Therefore, the dispatches of both real power and reactive power in this pq–type spot market are assumed to be optimised only at the beginning of each trading period.

[1] If the demand profile changes dramatically during a trading period, the real power dispatch may be re–optimised once or twice during that trading period, to ensure the generation profile follows the pre–dispatch generation schedule. This schedule is just the list of real power unit generation costs of all generators. Following this schedule ensures a merit order dispatch.

Marginal prices for real and reactive power are only valid for a single instant in time. Therefore, the behaviour of reactive power marginal prices for the optimal dispatches described in this chapter only apply to the instant at the beginning of the trading period when the dispatch is optimised. Only optimal dispatches are considered in this chapter.

Reactive power is much less expensive that real power. For illustration purposes therefore, the reactive power cost function for each generator is formulated to give a unit generation cost that is 10% of the real power unit generation cost for the same generator. This applies to all cases. It simplifies reality, where this percentage varies due to many factors, such as the use of hydro generation instead of thermal generation.

The 9–bus and 14–bus power systems are used for the case studies in this chapter.

9.2 BEHAVIOURAL CASE STUDIES

The behaviour of reactive power marginal prices is investigated for the following types of dispatches[2]:

- unconstrained;
- voltage constrained;
- reactive power generation constrained;
- sub–optimal.

The sub–optimal dispatch investigation is presented in Chapter 10.

The focus of the investigations in this chapter is on the behaviour of prices in a pq–type spot market where reactive power has a unit generation cost (that is, cost functions are used to economically dispatch reactive power, as well as real power)[3].

Power–flow algorithms assume that a power system operates its generators with constant voltage set–points. In reality however, it is physically acceptable to let these voltage set–points vary within the rated voltage limits of the generation equipment. Hence, all case studies investigate the behaviour of prices for optimal dispatches from both perspectives:

- dispatches with fixed generator voltages (i.e. every generator node is voltage constrained), and

[2]See Ring [1995] or Read and Ring [1995a] for a full description of the behaviour of real power marginal prices.

[3]The paper, Ward *et al* [1999] has been written as background thesis work. It presents a spot market in which reactive power is costless. That is, only real power is economically dispatched. The focus is on how reactive power marginal prices change when this market is extended to include reactive power cost functions, so that both real and reactive power are economically dispatched. This paper is presented in Appendix H.

- dispatches with unconstrained generator voltages (except the reference voltage).

The effects of constraints on prices is difficult to observe in a variable generator voltage dispatch. Fixed generator voltages however, magnify any price behaviour resulting from binding constraints. This is because they restrict the ability of a power system to respond to any demand for reactive (or real) power. Therefore, fixed generator voltages are used to illustrate the influence of voltage and reactive power generation constraints, on the behaviour of reactive power marginal prices.

Since the focus is on price behaviour in a pq–type market, all generator nodes are modelled as a PQG node in every case study (as proposed in Chapter 7). For the fixed generator voltage dispatches, each generator node is modelled as a PQG node with a voltage constraint.

9.3 INFLUENCES ON REACTIVE POWER MARGINAL PRICES

9.3.1 Non–marginal PQG and pqD Nodes

The marginal prices of real and reactive power at each PQG_{nm} and pqD node (i.e. β_{Pi} and β_{Qi}) are composed of a number of marginal cost components (as described by Equation 2.1 and in Section 3.3). In the Dispatch Based Pricing framework, these marginal cost components can be identified as:

1. the unit costs of real and reactive power (λ_P and λ_Q);
2. the costs of marginal real power losses ($\frac{\partial L_P}{\partial P_i}$ and $\frac{\partial L_P}{\partial Q_i}$);
3. the costs of marginal reactive power losses ($\frac{\partial L_Q}{\partial P_i}$ and $\frac{\partial L_Q}{\partial Q_i}$);
4. the dual marginal costs of all binding primal constraints (For example, the costs of the fixed reference voltage: $\mu_{V_{ref}} \frac{\partial V_{ref}}{\partial P_i}$ and $\mu_{V_{ref}} \frac{\partial V_{ref}}{\partial Q_i}$)

The real and reactive power prices at the reference node are the only exceptions to this composition (see Section 7.3.6).

The behaviour of β_{Pi} and β_{Qi} for a particular power system are dictated by the responses of the above cost components to changes in the demand profile. β_{Pi} and β_{Qi} are a function of the first three components when calculating prices for any observed dispatch (either unconstrained or constrained).

The cost of each constrained power system resource is reflected in β_{Pi} and β_{Qi}, through the fourth marginal cost component. Each extra binding primal constraint results in an extra fourth dual marginal cost component. These constraint cost components also influence the behaviour of the marginal prices.

Consider an example. Assume the 14–bus power system has been dispatched using QOPF, with fixed (i.e. constrained) generator voltages. Furthermore, a thermal constraint became binding on Branch 5–6 during the dispatch process. This is a total of six binding constraints. With respect to Equation 7.15, the equation used to calculate the marginal price of reactive power at Node 14 would then be:

$$\beta_{Q14} = -\lambda_P \frac{\partial L_P}{\partial Q_{14}} + \lambda_P \left(1 - \frac{\partial L_Q}{\partial Q_{14}}\right) - \mu_{V1}\frac{\partial V_1}{\partial Q_{14}} - \mu_{V2}\frac{\partial V_2}{\partial Q_{14}} - \mu_{V3}\frac{\partial V_3}{\partial Q_{14}} - \mu_{V6}\frac{\partial V_6}{\partial Q_{14}}$$
$$- \mu_{V8}\frac{\partial V_8}{\partial Q_{14}} - 2\chi_{5-6}\left(\left\langle \bar{P}^*_{5-6}\frac{\partial \bar{P}_{5-6}}{\partial Q_{14}}\right\rangle + \left\langle \bar{Q}^*_{5-6}\frac{\partial \bar{Q}_{5-6}}{\partial Q_{14}}\right\rangle\right) \quad (9.1)$$

Thus, it can be seen that the marginal loss components and the six constraint components influence the behaviour of the reactive power marginal price at Node 14 (i.e. β_{Q14}).

The marginal price equations are a set of simultaneous equations. Thus, the values of the shadow price (marginal cost) variables within this equation also influence β_{Pi} and β_{Qi} at all other nodes within the power system (as discussed in Section 5.9.4).

9.3.2 Marginal PQG Nodes

The form of price Equation 9.1 also applies to any node (n) where the generator is marginal for reactive (or real) power. However, it has been demonstrated in Sections 5.5, 7.3.3 and 8.3.2 that β_{Qn} (or β_{Pn}) is forced to equal the unit generation cost of the generator, c_{Qn} (or c_{Pn}). That is: $\beta_{Qn} = c_{Qn}$.

Equation 6.9 implies that c_{Qn} is constant when the generator cost function is linear. (note, $c_{Qn} = \frac{\partial M_Q}{\partial Q_{Gi}}$). A constant unit generation cost (c_{Qn}) means that β_{Qn} is fixed at marginal generator nodes. Only linear cost functions are considered in this chapter. This allows the mechanisms of the behaviour of reactive power marginal prices to be clearly identified. Ward *et al* [1999] describes the effect of non–linear cost functions on the behaviour of the reactive power marginal prices (β_{Qn}).

9.3.3 Predicted Reactive Power Price Behaviour

Reactive power prices are defined by equations very similar to those used for real power prices, in a pq–type spot market (see Chapter 7). Naturally, Read and Ring [1995a, Section 7.5] extrapolated their discussion on the behaviour of real power prices, to conclude that reactive power prices at pq nodes behave in a similar but not identical fashion to real power prices.

They note that, in the absence of "marginal production costs" (i.e. generation cost functions), reactive power marginal prices will only be non-zero when reactive power is constrained. These non–zero prices represent the marginal costs of the binding constraints. This statement is applicable to non-marginal PQG nodes and pqD nodes.

Reactive power marginal prices at these nodes will be non-zero because the reactive power injection at these nodes is constrained.

Read and Ring argued that these reactive power marginal prices will be high at nodes in the vicinity of a binding upper reactive power generation constraint where there is a shortage of reactive power. Furthermore, they predicted that reactive power marginal prices will be low at nodes in the vicinity of a binding lower reactive power generation constraint where there is a surplus of reactive power. This was also concluded in Chapter 2 of this thesis. Read and Ring also predicted that β_Q would increase in the presence of upper voltage constraints. This is disproved in Section 9.5.

Many points of reactive power price behaviour have been discussed in Read and Ring [1995a, Section 7.5]. However, no experimental work confirming these points has been published. In this chapter, case studies are presented that verify the predictions of Read and Ring. But, the studies go a step further by demonstrating the behaviour of reactive power prices when using non–zero marginal production costs (that is, reactive power generation cost functions).

9.4 UNCONSTRAINED DISPATCHES

The magnitudes of the costs of real and reactive power marginal losses at each node (i) are:

$$\lambda_P \frac{\partial L_P}{\partial P_i} \qquad \text{and} \qquad \lambda_Q \frac{\partial L_Q}{\partial P_i},$$

These costs generally increase with increased electrical distance between Node i and the reference node. This is because the marginal real and reactive power losses increase with electrical distance. The marginal price of real power (β_{P_i} in Equation 5.2) depends on these marginal loss cost components. Accordingly, Read and Ring [1995a] showed that β_{P_i} generally increases from node to node in the direction of the flow of real power when the cost of reactive power is zero (i.e. $\lambda_Q = 0$). They showed this occurs due to the increasing electrical distance between each consecutive node and the source of the real power generation. The phenomenon of "Real Power Price Inversion" is the rare exception to this rule. Here, the marginal price of real power decreases in the direction of real power flow, because the total real power marginal losses of the power system have decreased for an increase in real power demand [Ward *et al* 1998][4].

This section investigates the behaviour of reactive power marginal prices (β_{Q_i}),

[4] The investigation into the phenomena of price inversion was performed as background work for this thesis and is discussed in Ward *et al* [1998]. The unit generation costs of reactive power were assumed to be zero for this paper. This paper is included in Appendix H

resulting from the influence of the marginal loss cost components:

$$\lambda_P \frac{\partial L_P}{\partial Q_i} \quad \text{and} \quad \lambda_Q \frac{\partial L_Q}{\partial Q_i} \tag{9.2}$$

The effects of these loss components on the behaviour of β_{Qi} can only be observed for unconstrained dispatches. The effects of the marginal costs of binding constraints mask the effect of the loss components. Hence, only unconstrained dispatches are considered in this section.

9.4.1 Variable Generator Voltages

Reactive power is optimally dispatched in the same way that real power is optimally dispatched, according to a merit order (ref. Section 7.3.2). However, there is an extra dimension in optimising the reactive power dispatch because capacitive power system components can generate reactive power, in addition to generators. This necessitates the use of two cases to investigate the behaviour of marginal prices in an unconstrained dispatch.

In Case 1 the power system is composed totally of inductive components. This ensures that generators are the only source of reactive power. It enables the effect of the marginal loss components on β_{Qi} to be easily identified (ref. Equation 9.2). In Case 2, the power system is composed of a mixture of inductive and capacitive branches. This represents reality more closely where, for example, transmission lines generate reactive power when highly capacitive or lightly loaded.

The Cornell University 9–bus power system in Appendix E is used for both cases. Its simple loop–structure enables the relationship between average reactive power flow and reactive power marginal price behaviour to be easily identified.

<u>Case 1</u>

The shunt susceptances (B) of all the loop branches in the 9–bus power system (see Figure E.1) were set to a very small value to ensure that all branches were inductive. That is:

$$B_{4-5} = B_{5-6} = B_{6-7} = B_{7-8} = B_{8-9} = B_{9-4} = 0.0001 \text{ pu}$$

The objective function used by QOPF to optimise the dispatch of this inductive power system was:

$$f\,(cost)_{P,Q} = 10.7\,P_{G1} + 10.9\,P_{G2} + 11.0\,P_{G3} + 1.07\,Q_{G1} + 1.09\,Q_{G2} + 1.10\,Q_{G3} \tag{9.3}$$

The voltages of Generators 2 and 3 were allowed to vary. V_1 was fixed as the reference voltage at 1.0 pu. To prevent any other binding voltage constraints, the

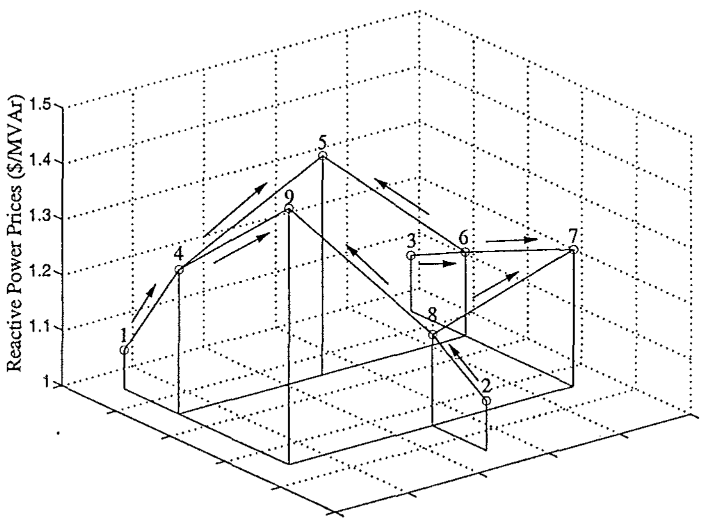


Figure 9.1 The schematic price–profile of reactive power marginal prices (β_{Qi}) around the Cornell 9–bus power system. All branches are inductive.

upper voltage limits on PQG Nodes 2 and 3, and all pqD nodes were set to:

$$V_{2\ldots9}^{max} = 2.1 \text{ pu}$$

The reactive power marginal prices corresponding to this unconstrained, variable generator voltage dispatch are depicted by the 'schematic price–profile' in Figure 9.1.

In this 'schematic price–profile' the layout of the points in the plane defined by the two horizontal axes corresponds to the schematic of the 9–bus power system presented in Figure E.1. The vertical lines represent the nodes of the power system. The height of each vertical line represents the value of the marginal price (in $/MVAr) at that node. The vertical axis has a suppressed zero. The arrows indicate the direction of average reactive power flow (which is defined by Equation 3.3).

Case 2

The power system data presented in Appendix E was used without change in this case. In this unmodified 9–bus power system, the loop branches are capacitive and Branches 1–4, 3–6 and 8–2 are inductive.

The power system was dispatched with respect to the objective function described by Equation 9.3. Again, the generator voltages were allowed to vary, except the reference voltage ($V_1 = 1.0$ pu). The resultant reactive power marginal prices are depicted by the schematic price–profile in Figure 9.2, with arrows indicating the direction of average reactive power flow.

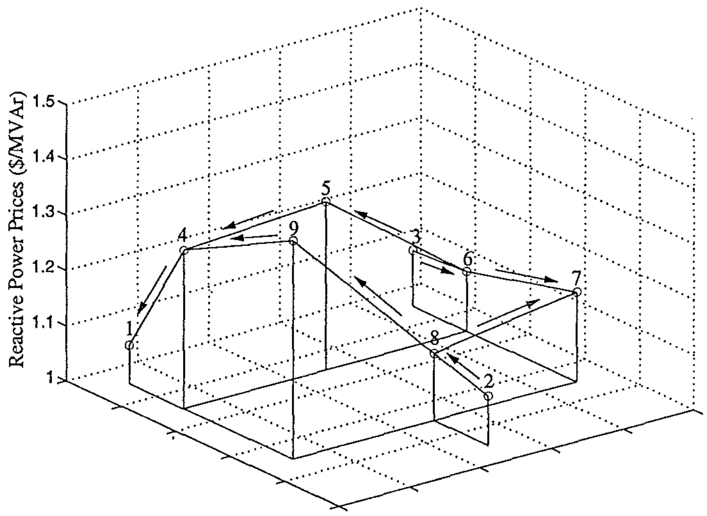


Figure 9.2 The schematic price–profile of reactive power marginal prices (β_{Q_i}) around the Cornell 9–bus power system. Branches are either inductive or capacitive.

Observations

In both cases, the marginal price of reactive power at every node is given by:

$$\beta_{Qi} = \lambda_P \frac{\partial L_P}{\partial Q_{Di}} + \lambda_Q + \lambda_Q \frac{\partial L_Q}{\partial Q_{Di}} + \mu_{V1} \frac{\partial V_1}{\partial Q_{Di}} \tag{9.4}$$

This equation has been formulated with respect to reactive power demand (ref. Equation 5.10).

An inspection of the schematic price–profile in Figure 9.1 reveals that the direction of average reactive power flow is from the generator nodes (1, 2, 3) to the demand nodes (5, 7, 9), without exception. Furthermore, marginal prices increase in the direction of average reactive power flow, from generator to load.

NODAL2 was used to determine how the individual cost components of Equation 9.4 contributed to the behaviour of β_{Qi} in this inductive network. From one node to the next, the marginal loss components for real power and reactive power were both found to increase in the direction of average reactive power flow. This was true of every branch. However, the magnitude of this change in the cost of real power marginal losses ranged between 88% and 270% of the magnitude change in the cost of reactive power marginal losses. For example, the change in the cost of real power marginal losses from Node 5 to Node 6 was 270% of the change in the cost of the reactive power marginal losses:

$$\frac{\left|\lambda_P\left(\frac{\partial L_P}{\partial Q_{D5}} - \frac{\partial L_P}{\partial Q_{D6}}\right)\right|}{\left|\lambda_Q\left(\frac{\partial L_Q}{\partial Q_{D5}} - \frac{\partial L_Q}{\partial Q_{D6}}\right)\right|} = \frac{\left|0.1263 - (-0.0516)\right| \ \$/\mathrm{MVAr}}{\left|1.2684 - 1.2025\right| \ \$/\mathrm{MVAr}} = 270\% \tag{9.5}$$

It must be remembered that $\frac{\partial L_Q}{\partial Q_{Di}}$ is the change in the total reactive power losses for a change in reactive power demand at Node i (see Equation 3.2). It is not the change in the losses across a single branch.

The only other changeable cost component in Equation 9.4 is the cost of the reference voltage constraint. However, NODAL2 only reports the value of μ_{V1}, not the value of the whole $\mu_{V1}\frac{\partial V_1}{\partial Q_{Di}}$ component (note that $\mu_{V1} = \$2.2079/\mathrm{pu}$). In the fixed generator voltage experiment (Case 3 in the next section), the change in the $\mu_{V2}\frac{\partial V_2}{\partial Q_{Di}}$ and $\mu_{V3}\frac{\partial V_3}{\partial Q_{Di}}$ components from one node (i) to the next, are no more than 20% of the change in the sum of the marginal loss cost components:

$$\lambda_P\frac{\partial L_P}{\partial Q_{Di}} + \lambda_Q\frac{\partial L_Q}{\partial Q_{Di}} \tag{9.6}$$

between the same two nodes. Extrapolating this result back to Case 1, it can be assumed that the change in $\mu_{V1}\frac{\partial V_1}{\partial Q_{Di}}$ is negligible compared to the changes in the marginal loss components. Therefore, it can be concluded that the marginal loss cost components are causing β_Q to increase in the direction of average reactive power flow (as depicted in Figure 9.1), and not the voltage constraint cost component.

The marginal reactive power prices in Case 2 however, do not always increase in the direction of average reactive power flow. The loop branches are sources of reactive power generation because they are capacitive. Furthermore, the schematic price–profile in Figure 9.2 indicates that Generator 1 is absorbing reactive power. These factors cause β_Q to decrease in the direction of average reactive power flow, along Branches 1–4, 4–5 and 9–4.

Consider Branch 9–4 as an example. The price decrease across this branch demonstrates that the sum of the total marginal real and total marginal reactive power losses (Equation 9.6) at Node 4 is less than at Node 9. An analysis of the individual components revealed that the cost of marginal real power losses increases from Node 9 to Node 4, in the direction of average reactive power flow:

$$\Delta Cost(L_P) = \lambda_P\frac{\partial L_P}{\partial Q_{D4}} - \lambda_P\frac{\partial L_P}{\partial Q_{D9}} > 0 \tag{9.7}$$

However, the cost of marginal reactive power losses was found to decrease from Node 9 to Node 4:

$$\Delta Cost(L_Q) = \lambda_Q \frac{\partial L_Q}{\partial Q_{D4}} - \lambda_Q \frac{\partial L_Q}{\partial Q_{D9}} < 0 \tag{9.8}$$

Since $\Delta Cost(L_Q)$ has decreased, β_Q decreases in the direction of average reactive power flow because:

$$\left|\Delta Cost(L_P)\right| < \left|\Delta Cost(L_Q)\right| \tag{9.9}$$

To explain the decrease in the cost of marginal reactive power losses, let the cost of marginal real and reactive power losses be reformulated in terms of finite differences. For example:

$$\lambda_Q \frac{\Delta L_Q}{\Delta Q_{Di}}$$

Assume that any change in reactive power demand at Node 9 (ΔQ_D) is supplied totally by Generators 2 and 3. This implies that the cost at Node 4 (i.e. $\lambda_P \frac{\Delta L_P}{\Delta Q_{D4}} + \lambda_Q \frac{\Delta L_Q}{\Delta Q_{D4}}$) should be greater that the cost at Node 9 (i.e. $\lambda_P \frac{\Delta L_P}{\Delta Q_{D9}} + \lambda_Q \frac{\Delta L_Q}{\Delta Q_{D9}}$). This is because Node 4 is further from Generators 2 and 3 than Node 9. However, the sum of the cost of marginal reactive power losses decreases from Node 9 to Node 4 (as Equation 9.8 illustrates). That is, the total losses ($\lambda_P \Delta L_P + \lambda_Q \Delta L_Q$) have decreased.

The total losses can only decrease from Node 9 to Node 4 if part of ΔQ_D is generated by the capacitive branches, instead of by Generators 2 and 3. Therefore, it is possible for β_Q to decrease in the direction of average reactive power flow, if there are reactive power sources in addition to the generators. It must be noted that any decrease in β_Q must be attributed to a decrease in the sum of the marginal loss components (Equation 9.6). This is because it is possible for β_Q to decrease when the real power cost decreases and the reactive power cost increases, and when the change in the real power cost is greater than the change in the reactive power cost. That is: $\Delta Cost(L_P) < 0$, $\Delta Cost(L_Q) > 0$ and $|\Delta Cost(L_P)| > |\Delta Cost(L_Q)|$.

The capacitive branches raised the voltage profile. This tightened the reference voltage constraint and is evident by an increase in the marginal voltage constraint shadow price from Case 1 to Case 2 (μ_{V1}=\$5.0213/pu). This increased shadow price may have also helped nullify the rule–of–thumb that "prices increase in the direction of average power flow", if the whole term ($\mu_{V1} \frac{\partial V_1}{\partial Q_{Di}}$) was comparable to the loss terms.

Summary

In summary, the average flow of real power is always from generator(s) to the demand node because generators are the only source of real power. Likewise, the average flow of reactive power is also from the generator(s) to the demand node if generators are the only source of reactive power (that is, all network branches are inductive). In an inductive network therefore, β_Q increases in the direction of the average flow of reactive power due to the increasing electrical distance from the generators. It is possible that rare exceptions may occur, such as a reactive power equivalent to "Real Power Price Inversion" [Ward *et al* 1998].

In practise, this rule–of–thumb cannot be applied to actual power systems because transmission lines are usually capacitive. In the presence of capacitive network components the sum of the total real and reactive power marginal losses (Equation 9.6) do not always increase in the direction of average reactive power flow. This occurs because some capacitive branches increase their reactive power injection to meet an increase in reactive power demand, and as a consequence cause the total real and/or reactive power losses (L_P and/or L_Q) to decrease. In an unconstrained, capacitive/inductive power system therefore, the trend in β_Q is dependent of the positions of all real and reactive power sources, the X_k/R_k ratio of the network branches and the position of the reference voltage.

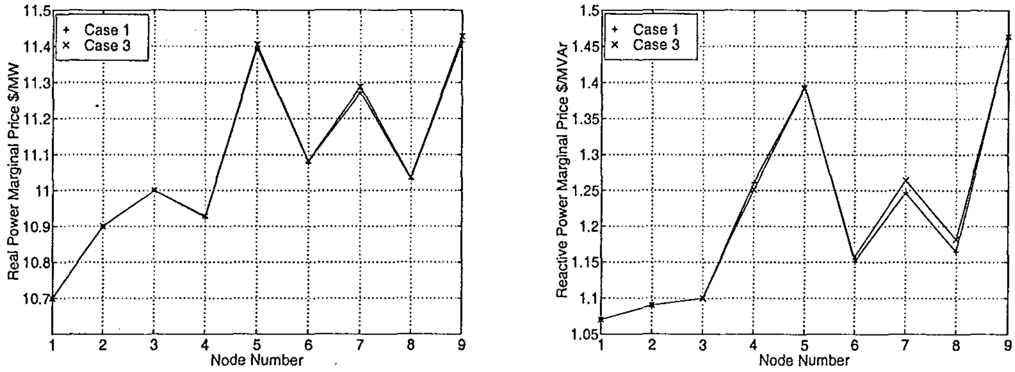
9.4.2 Reactive Power Generation Surplus

The reactive power marginal prices at the pqD nodes in Figure 9.2 are appreciably less than the corresponding marginal prices in Figure 9.1. At every pqD node except Node 4, the marginal real power losses $\left(\frac{\partial L_P}{\partial Q_i}\right)$ have decreased from Case 1 to Case 2, and the marginal reactive power losses $\left(\frac{\partial L_Q}{\partial Q_i}\right)$ have increased because the loop branches are generating reactive power in Case 2. However, $\frac{\partial L_P}{\partial Q_i}$ has decrease more than $\frac{\partial L_Q}{\partial Q_i}$ has increased, thus explaining the drop in reactive power marginal prices at pqD Nodes 5 to 9.

The lower prices indicate a surplus of reactive power in the network, because most generators and branches are capable of generating reactive power. This surplus has decreased the real power losses associated with transmitting any extra reactive power demand. It can therefore be concluded that any unconstrained dispatch of a highly capacitive network will generally have a lower reactive power price profile than its inductive counter-part.

9.4.3 Fixed Generator Voltages

This section investigates the behaviour of reactive power marginal prices for an unconstrained dispatch, when all generator voltages are fixed. That is, Case 1 and Case 2 are re–dispatched with the voltages of all three generators fixed to 1.0 pu. The result



(a) Real power marginal prices

(b) Reactive power marginal prices

Figure 9.3 Marginal price profiles for variable and fixed generator voltage dispatches of the inductive version of the Cornell 9–bus power system.

is Cases 3 and 4 below. For both cases, Equation 9.4 is modified thus:

$$\beta_{Qi} = \lambda_P \frac{\partial L_P}{\partial Q_{Di}} + \lambda_Q + \lambda_Q \frac{\partial L_Q}{\partial Q_{Di}} + \mu_{V1} \frac{\partial V_1}{\partial Q_{Di}} + \mu_{V2} \frac{\partial V_2}{\partial Q_{Di}} + \mu_{V3} \frac{\partial V_3}{\partial Q_{Di}} \tag{9.10}$$

The extra components are the marginal costs of the fixed generator voltage constraints.

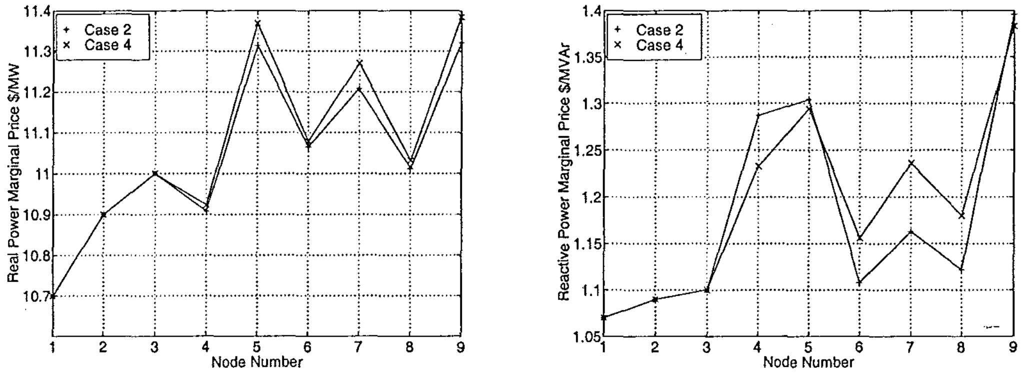
The real and reactive power price profiles of Case 3 are compared with those of Case 1, in Figure 9.3. The price profiles of Cases 2 and 4 are compared in Figure 9.4.

Case 3

As in Case 1, β_Q increased in the direction of average reactive power flow, even with the two extra fixed generator marginal costs. This is because the change in the sum of the three voltage costs between any to nodes, only ranged between 13% and 22% of the change in the sum of the loss costs. For example, the percentage change between Nodes 5 and 6 is:

$$\frac{\left|\mu_{V1}\left(\frac{\partial V_1}{\partial Q_6} - \frac{\partial V_1}{\partial Q_5}\right) + \mu_{V2}\left(\frac{\partial V_2}{\partial Q_6} - \frac{\partial V_2}{\partial Q_5}\right) + \mu_{V3}\left(\frac{\partial V_3}{\partial Q_6} - \frac{\partial V_3}{\partial Q_5}\right)\right|}{\left|\lambda_P\left(\frac{\partial L_P}{\partial Q_6} - \frac{\partial L_P}{\partial Q_5}\right) + \lambda_Q\left(\frac{\partial L_Q}{\partial Q_6} - \frac{\partial L_Q}{\partial Q_5}\right)\right|} = 16\% \tag{9.11}$$

At all nodes the actual magnitudes of the voltage cost components were small in comparison to the magnitudes of the loss components, ranging between 3% and 11%.



(a) Real power marginal prices (b) Reactive power marginal prices

Figure 9.4 Marginal price profiles for variable and fixed generator voltage dispatches of the Cornell 9–bus power system; capacitive and inductive branches exist.

At Node 6 for example:

$$\frac{\left|\mu_{V1}\frac{\partial V_1}{\partial Q_6}+\mu_{V2}\frac{\partial V_2}{\partial Q_6}+\mu_{V3}\frac{\partial V_3}{\partial Q_6}\right|}{\left|\lambda_P\frac{\partial L_P}{\partial Q_6}+\lambda_Q\frac{\partial L_Q}{\partial Q_6}\right|}=7\% \tag{9.12}$$

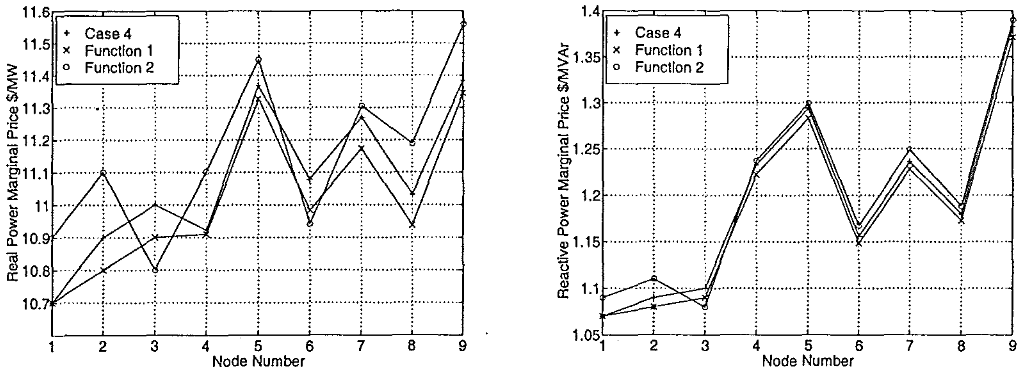
Therefore, given the low value of μ_{V1} in Case 1 and the low percentages (as demonstrated by Equation 9.12), the increase in the real power and reactive power price profiles from Case 1 to Case 3 is primarily due to an increase in the sum of the marginal loss components (Equation 9.6).

Case 4

Fixing all generator voltages when dispatching the original (capacitive/inductive) 9–bus power system resulted in a completely different dispatch to that of Case 2. This is reflected in the different marginal price profiles of Figure 9.4. However, it was still found that β_Q does not always increase in the direction of average reactive power flow, as in Case 2.

With reference to Equation 9.11, the changes in the voltage costs were much more extreme in this case than in Case 3. At each node, the percentage change in β_Q from Case 2 to Case 4 was in the range of 33% to 341%. For example, the percentage change from Node 5 to Node 6 was 197% (compared with 16% in Case 3).

The magnitudes of the voltage cost components were between 17% and 43% of the marginal loss components (ref. Equation 9.12).



(a) Real power marginal prices

(b) Reactive power marginal prices

Figure 9.5 A comparison of the price profiles for three different half–hourly trading periods in a pq–type spot market, for the Cornell 9–bus system.

9.4.4 Conclusions

The percentages associated with Equation 9.12 demonstrate that the marginal loss components set the height of the reactive power price profile in an unconstrained power system. This is independent of whether branches are inductive or capacitive, and independent of whether the power system was dispatched with fixed or variable generator voltages. The loss/voltage percentages (ref. Equation 9.11) identify the marginal cost components that determine the shape of the reactive power marginal price profile. In Case 3, these percentages show that only the marginal loss components determine the shape. In Case 4, these percentages show that the marginal loss cost components and the marginal cost components of the fixed generator voltages determine the shape of the profile.

9.4.5 Different Real and Reactive Power Cost Functions

In New Zealand's Wholesale Electricity market, the price of real power can vary widely over time. Generating companies may, for a variety of reasons, use different real power generation cost functions for different trading periods during a day, when submitting offers to the 'Real-time Physical Market'. In an unconstrained power system, changes in cost functions between two consecutive trading periods result in real power price variations other than the price variations that occur slowly over time as a function of the demand–profile. This is also true of a spot market that economically dispatches reactive power.

Consider for example, a spot market set up on the Cornell University 9–bus power system. Here, three generating companies are bidding against each other to supply real and reactive power. If one company made its generator much more expensive than the other two, the market would ignore this generator in favour of the two cheaper generators. That is, if these two could meet the total demand by themselves. Therefore, it can be assumed that the real power and reactive power generation cost functions for the generators are comparable, since the generating companies are being competitive with each other. It is also assumed that the changes in all the cost functions between different trading periods are not dramatic.

Three different optimal dispatches, representing three different half–hourly trading periods of the spot market, are presented. The generator voltages are fixed for all dispatches. The only differences between the dispatches are the generation cost functions of real and reactive power. The objective function for each dispatch is composed of three pairs of real and reactive power generation cost functions (one pair for each generator):

$$\text{Case 4} = 10.7\,P_{G1} + 10.9\,P_{G2} + 11.0\,P_{G3} + 1.07\,Q_{G1} + 1.09\,Q_{G2} + 1.10\,Q_{G3}$$

$$\text{Function 1} = 10.7\,P_{G1} + 10.8\,P_{G2} + 10.9\,P_{G3} + 1.07\,Q_{G1} + 1.08\,Q_{G2} + 1.09\,Q_{G3}$$

and

$$\text{Function 2} = 10.9\,P_{G1} + 11.1\,P_{G2} + 10.8\,P_{G3} + 1.09\,Q_{G1} + 1.11\,Q_{G2} + 1.08\,Q_{G3}$$

These objective functions represent the total generation cost to the power system for the three trading periods.

In all three dispatches, all generators were marginal for real and reactive power. Hence, the unit generation costs of each generator (c_P and c_Q) are equal to the coefficients of the cost functions. For example, $c_{P1} = \$10.7/\text{MW}$ and $c_{Q1} = \$1.07/\text{MVAr}$ for Case 4.

Table 9.1 summarises the generation from Generators 1, 2 and 3, and the total system losses for each of the three dispatches.

Table 9.1 Optimal power system dispatches for the three objective functions.

	$P_1 + \mathrm{j}\,Q_1$	$P_2 + \mathrm{j}\,Q_2$	$P_3 + \mathrm{j}\,Q_3$	Total System Losses
Case 4	$165.94 + \mathrm{j}\,24.11$	$92.33 + \mathrm{j}\,6.83$	$59.69 - \mathrm{j}\,4.18$	$2.96 - \mathrm{j}\,88.24$
Function 1	$156.95 + \mathrm{j}\,23.14$	$97.17 + \mathrm{j}\,6.79$	$63.81 - \mathrm{j}\,4.55$	$2.93 - \mathrm{j}\,89.62$
Function 2	$148.89 + \mathrm{j}\,23.25$	$61.38 + \mathrm{j}\,6.21$	$107.85 - \mathrm{j}\,3.01$	$3.11 - \mathrm{j}\,88.55$

The distribution of real power across the generators is dependent on the unit cost of real power at each generator. A higher unit generation cost results in less real power from that generator. Consequently, the distribution of real power changes with objective function. The reactive power generation distribution changes very little between objective functions.

For each optimal dispatch, the cost of losses is implicitly minimised when minimising the total cost of real and reactive power generation. Therefore, the costs of the total marginal losses for the three dispatches are similar because the objective functions are similar (i.e. they calculate similar total generation costs). This occurs even though the real power generation profile changes significantly. Consequently, the reactive power price profiles depicted in Figure 9.5 are similar, since reactive power marginal prices are dependent on the costs of the total marginal losses in an unconstrained system ($\lambda_P \frac{\partial L_P}{\partial z}$ and $\lambda_Q \frac{\partial L_Q}{\partial z}$). The similarities in the reactive power price profiles also indicate that the pressure on the voltage constraints have not changed significantly (ref. Equation 9.10).

The real power price profiles have been included for completeness (but without comment).

Reactive power marginal prices therefore, will not change dramatically from one half–hourly period to the next due to changing generation cost functions in a pq–type spot market. Large price changes will only occur if system constraints become binding, or generators become non–marginal.

9.5 VOLTAGE CONSTRAINED DISPATCHES

9.5.1 Introduction

Fixed generator voltages have been introduced in Section 7.4.1 as binding primal voltage constraints. Such constraints are 'acceptable' if it is decided that this is the best way for the power system to operate, and if the market is willing to accept the associated behaviour of real power prices and reactive power prices. However, the price behaviour resulting from binding voltage constraints at non-generator (pqD) nodes is generally unacceptable. This is because these voltage constraints can often be relieved (by installing fixed capacitors for example), thus removing the price behaviour associated with the voltage constraint.

In New Zealand's distribution system, the limits on the acceptable voltage range are 230 V $\pm$ 6% [NZ Regs, 1997]. When the needs of a power system force the voltage at a node against the limit of a range such as this, the voltage is constrained. This is modelled with a binding voltage constraint.

A voltage (V_{13}^{min}) can only be considered as constrained if it is constrained at a voltage different to the voltage that would occur if the voltage constraint was relaxed (i.e. the unconstrained voltage). This unconstrained voltage (V_{13}^{uncnstr}) is illustrated in Figure 9.6. Note that $V_{13}^{\text{uncnstr}} < V_{13}^{min}$.

This section investigates the effects of lower and upper primal voltage constraints on reactive power marginal prices. Figure 9.6 illustrates a binding primal lower voltage constraint. In particular, the behaviour of marginal prices is investigated when the voltage at pqD Node 13 of the IEEE 14–bus power system is constrained. The IEEE

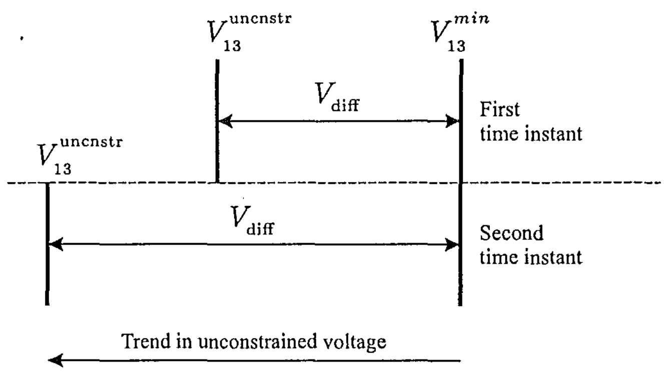


Figure 9.6 A primal voltage constraint is tightened when the underlying unconstrained voltage (V_{13}^{uncnstr}) moves further away from the actual constrained voltage (V_{13}^{min}), from one instant in time to the next.

14–bus power system is used because its mesh structure clearly illustrates the effect of a nodal–type constraint (e.g. a voltage constraint) on the marginal prices of the surrounding nodes.

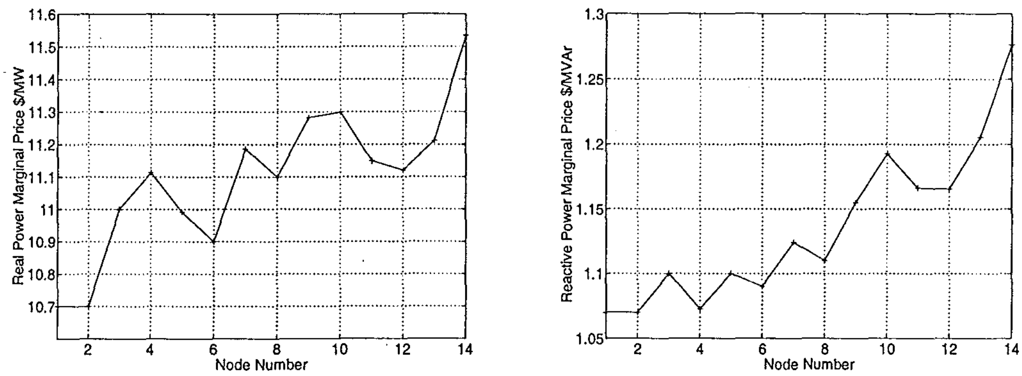
The effect of tightening primal voltage constraints is also presented. Voltage operating limits are often legally unchangeable (for example, the voltage limits regulated in NZ Regs. [1997]). For example, it may not be possible to change V_{13}^{min} in Figure 9.6. Therefore, a binding voltage constraint (i.e. limit) is tightened when the underlying unconstrained voltage (V_{13}^{uncnstr}) moves further away from the actual constrained voltage (V_{13}^{min}) due to changing power system conditions.

In the following cases, the voltage constraint is tightened by changing the reactive power demand at Node 13 (Q_{D13}), which in turn shifts the unconstrained voltage. This demand–change might occur in reality when a wholesale customer alters their reactive power demand from one trading period to the next, for example when a power company makes the transition from a day tariff to night tariff.

For all cases in this section (9.5), the voltage constrained 14–bus power system was dispatched using QOPF with respect to the objective function:

$$\begin{aligned} f\,(cost)_{P,Q} &= 10.7\,P_{G1} + 10.7\,P_{G2} + 11.0\,P_{G3} + 10.9\,P_{G6} + 11.1\,P_{G8} \\ &\quad + 1.07\,Q_{G1} + 1.07\,Q_{G2} + 1.10\,Q_{G3} + 1.09\,Q_{G6} + 1.11\,Q_{G8} \end{aligned} \tag{9.13}$$

The real and reactive power price profiles for an unconstrained dispatch of the 14–bus power system are presented in Figure 9.7 for the purpose of comparison with the voltage constrained price profiles. This unconstrained dispatch was obtained by running the 14–bus power system data of Appendix E, unmodified, through QOPF. The coefficients of the objective function (9.13) are described by the 'Generator Cost' data. The voltage at Node 13 for this unconstrained dispatch is $V_{13}^{\text{uncnstr}} = 1.0502$ pu.



(a) Real power marginal prices (b) Reactive power marginal prices

Figure 9.7 Real and reactive power marginal price profiles for the unconstrained dispatch of the IEEE 14–bus power system.

9.5.2 Lower Voltage Constraints

In Cases 5 and 6, the voltage at Node 13 has been forced against a lower voltage limit. A binding lower voltage constraint was obtained by setting the value of V_{13}^{min} in both cases, above the original unconstrained voltage value of 1.0502 pu, to:

$$V_{13}^{min} = 1.0510\,\text{pu}$$

The only difference between these two cases is the value of Q_{D13}.

<u>Case 5</u>

This case illustrates the change in real and reactive power marginal prices when a lower voltage constraint becomes binding in a power system. In this case, the reactive power load at Node 13 is the same as the original IEEE 14–bus power system data (i.e. $Q_{D13} = 5.80\,\text{MVAr}$).

Figure 9.8 depicts the trends in reactive power prices around this voltage constrained 14–bus power system. These reactive power marginal prices are also depicted by the 'Case 5' price profile in Figure 9.9(b). The unconstrained profiles of Figure 9.7 are included in Figure 9.9 for comparison.

The marginal price of reactive power is high at the constrained, Node 13. But, it decreases quickly with increased electrical distance from Node 13. The quick decrease of prices indicates that, only reactive power demand close to the constrained Node 13 strongly influences the constrained voltage. This highlights the localised nature of reactive power.

An increase in demand at Node 13 would draw more reactive power out of the

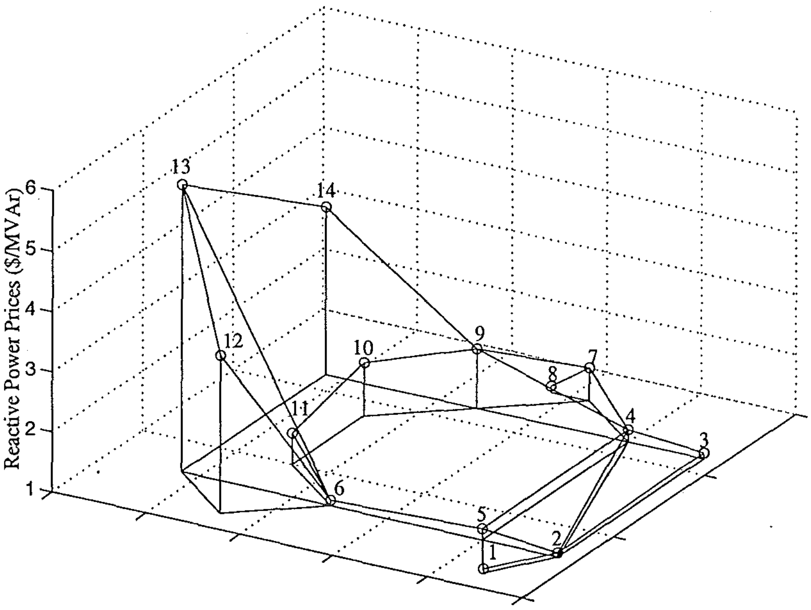


Figure 9.8 The Case 5 schematic price–profile depicting reactive power marginal prices for the voltage constrained dispatch of the IEEE 14–bus power system. Node 13 has a binding lower voltage constraint.

system and cause the unconstrained voltage to drop. This increases the difference between the constrained and the unconstrained voltages, and thus puts more stress on the lower voltage constraint. Accordingly, the high price at Node 13 discourages reactive power consumers from putting stress on the constraint by using reactive power. It also reflects the amount the market is willing to pay in order to relax the constraint on the voltage at that node.

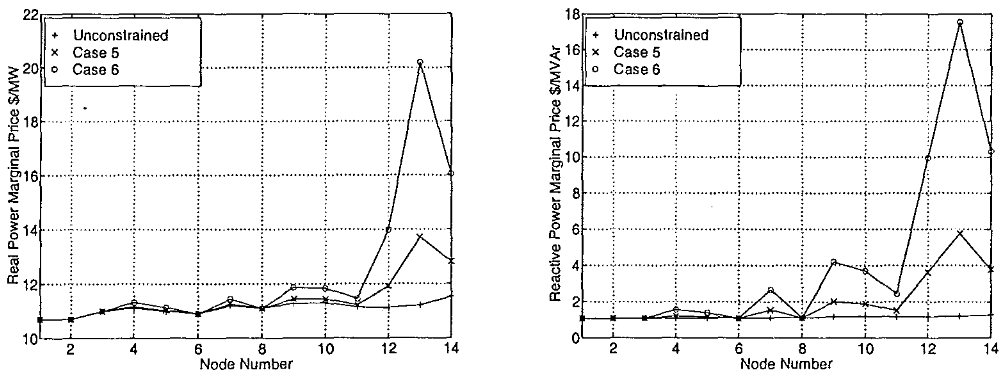
Case 6

Case 6 demonstrates the response of real and reactive power prices when the binding, lower voltage constraint at Node 13 is tightened. The constraint was tightened by increasing the reactive power demand at Node 13 to $Q_{D13} = 6.80$ MVAr.

Figure 9.9 shows that tightening the voltage constraint causes both real and reactive power prices to increase. β_{Q13} (at \$17.54/MVAr) is now almost the same magnitude as β_{P13} (at \$20.19/MVAr). This is a significant proportional increase from the unconstrained case in Figure 9.7, where the unconstrained reactive power prices at all nodes were between 9.5% and 11.1% of the unconstrained real power prices (these percentages were obtained by comparing the prices depicted in Figures 9.7(a) and 9.7(b)).

9.5.3 Upper Voltage Constraints

In this section, Cases 7 and 8 are used to demonstrate the behaviour of real and reactive power marginal prices, in the presence of a binding upper voltage constraint. This is



(a) Real power marginal prices

(b) Reactive power marginal prices

Figure 9.9 Marginal price profiles of Cases 5 and 6 for the IEEE 14–bus power system, with a binding lower voltage constraint on Node 13. The unconstrained Figure 9.7 profiles are included for reference.

achieved by setting the value of V_{13}^{max} in both cases, below the original unconstrained voltage value of 1.0502 pu, to:

$$V_{13}^{max} = 1.0500 \text{ pu}$$

As with Cases 5 and 6, the only difference between these two cases is the value of the reactive power load, Q_{D13}. In both cases, all generators are marginal for both real power and reactive power generation.

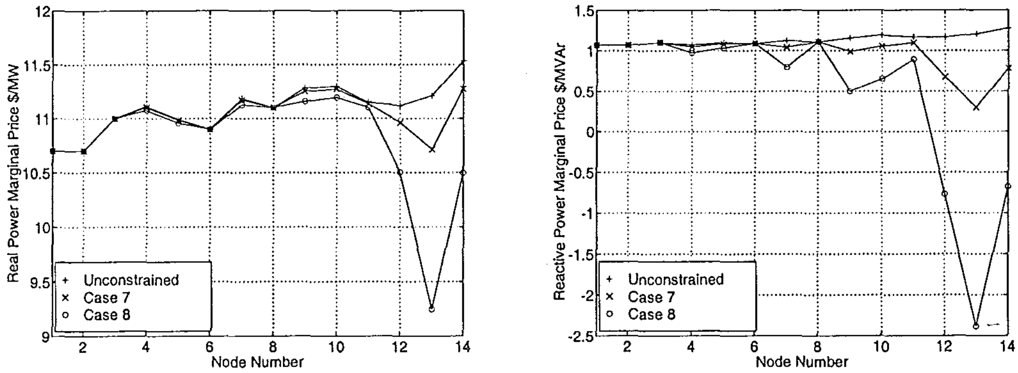
Case 7

The reactive power load at Node 13 has been set to the same original power system value, as in Case 5. That is, $Q_{D13} = 5.80\,\text{MVAr}$.

Figure 9.10(b) shows that reactive power marginal prices drop rapidly with decreasing electrical distance from the constrained node. The low reactive power prices encourage an increase in reactive power consumption (Q_{Di}) in order to relieve the pressure on the voltage constraint; increased consumption reduces the voltage (V_{13}). The price increase when moving away from Node 13, shows that increasing reactive power consumption becomes less effective at relieving the constraint with increased electrical distance from the constraint.

Case 8

The binding upper voltage constraint at Node 13 is tightened in this case. This was achieved by reducing the reactive power load at Node 13 in order to increase the unconstrained voltage at the same node. Hence, the reactive power demand for this



(a) Real power marginal prices

(b) Reactive power marginal prices

Figure 9.10 Marginal price profiles of Cases 7 and 8 for the IEEE 14–bus power system, constrained by an upper voltage limit on Node 13. The unconstrained Figure 9.7 profiles are included for reference.

case is:

$$Q_{D13} = 4.80 \text{ MVAr}$$

Tightening the upper voltage constraint resulted in lower real and reactive power prices than in Case 7 (shown in Figure 9.10). Moreover, Figure 9.10(b) demonstrates that reactive power prices can even go negative.

From the consumers' perspective, a negative price at a node indicates that the power system is willing to pay consumers to increase their reactive power load as a means of reducing the voltage at that node, in turn relieving the pressure on the constraint. From the perspective of the power system, the negative price acts as an incentive for the power system to relax the voltage constraint so that it does not have to pay consumers to use reactive power.

9.5.4 Discussion on Voltage Constraints

With respect to the pq pricing model (Equations 7.13 to 7.19) the marginal cost of reactive power at Node 13 for Cases 5 to 7, is described by:

$$\beta_{Q13} = -\lambda_P \frac{\partial L_P}{\partial Q_{13}} + \lambda_Q \left(1 - \frac{\partial L_Q}{\partial Q_{13}}\right) - \mu_{V1}\frac{\partial V_1}{\partial Q_{13}} - \mu_{V2}\frac{\partial V_2}{\partial Q_{13}} - \mu_{V3}\frac{\partial V_3}{\partial Q_{13}} - \mu_{V6}\frac{\partial V_6}{\partial Q_{13}} - \mu_{V8}\frac{\partial V_8}{\partial Q_{13}} - \mu_{V13}\frac{\partial V_{13}}{\partial Q_{13}} \quad (9.14)$$

The presence of non–zero μ_{Vi} dual variables for Nodes 1, 2, 3, 6 and 8 indicate that the voltages at all generator nodes are fixed. The presence of the (non–zero) voltage shadow price μ_{V13}, indicates that the voltage at Node 13 has been constrained. All μ_{Vi} shadow prices are the non–zero LaGrange multipliers of binding primal voltage constraints. By Equations 7.16 and 7.19, these voltage shadow prices are:

$$\begin{aligned}
\mu_{V1} &= \beta_{V1} = \left\langle v^{+}_{V1} \right\rangle - \left\langle v^{-}_{V1} \right\rangle \\
\mu_{V2} &= \beta_{V2} = \left\langle v^{+}_{V2} \right\rangle - \left\langle v^{-}_{V2} \right\rangle \\
\mu_{V3} &= \beta_{V3} = \left\langle v^{+}_{V3} \right\rangle - \left\langle v^{-}_{V3} \right\rangle \\
\mu_{V6} &= \beta_{V6} = \left\langle v^{+}_{V6} \right\rangle - \left\langle v^{-}_{V6} \right\rangle \\
\mu_{V8} &= \beta_{V8} = \left\langle v^{+}_{V8} \right\rangle - \left\langle v^{-}_{V8} \right\rangle \\
\mu_{V13} &= \beta_{V13} = \left\langle v^{+}_{V13} \right\rangle - \left\langle v^{-}_{V13} \right\rangle
\end{aligned}$$

By complimentary slackness (i.e. Duality 5.5), if a lower primal voltage constraint is binding at Node 13 (as in Cases 5 and 6), v^{-}_{V13} is non–zero, v^{+}_{V13} is zero, and μ_{V13} becomes:

$$\mu_{V13} = -v^{-}_{V13}$$

v^{+}_{V13} becomes non–zero for binding upper primal voltage constraints, as in Cases 7 and 8.

μ_{V13} and its corresponding partial derivative (i.e. sensitivity coefficient) describe the marginal cost to the power system of the voltage constraint. Formulating Equation 9.14 in terms of reactive power demand, rather than reactive power injection results in:

$$\beta_{Q13} = \cdots + \mu_{V13} \frac{\partial V_{13}}{\partial Q_{D13}}. \tag{9.15}$$

If the voltage V_{13} was unconstrained, an incremental increase in reactive power demand Q_{D13} would result in a drop in V_{13}. Hence, the partial derivative is negative. Assuming a lower voltage constraint therefore, the marginal price of reactive power is:

$$\beta_{Q13} = \cdots - \left(-v^{-}_{V13}\right) \left| \frac{\partial V_{13}}{\partial Q_{D13}} \right|$$

This suggests that reactive power prices will be higher than in the unconstrained case (where $\mu_{V13} = 0$). This price increase is evident when comparing the unconstrained profiles with the Case 5 and 6 profiles (see Figure 9.9(b)). The partial derivative becomes larger when the constraint is tightened, indicating an increase in the sensitivity of V_{13} to increases in reactive power demand. This increase in sensitivity is also reflected by an increase in μ_{V13}, and consequently, an increase in reactive power prices from Case 5 to Case 6.

When an upper constraint is binding, the marginal price of reactive power at Node 13 is:

$$\beta_{Q13} = \cdots - \left(+v^{+}_{V13}\right)\left|\frac{\partial V_{13}}{\partial Q_{D13}}\right|$$

This results in a decrease in marginal prices from the unconstrained case, since the partial derivative is negative. This constrained price behaviour was observed in Cases 7 and 8.

The decline in β_Q with increased electrical distance is best illustrated through a brief example. Consider the marginal price of reactive power at Node 5:

$$\beta_{Q5} = \cdots + \mu_{V13}\frac{\partial V_{13}}{\partial Q_{D5}} \tag{9.16}$$

The value of μ_{V13} in Equation 9.16 is equal to the value of μ_{V13} in Equation 9.15, because these two equations are part of a set of simultaneous equations. But, the voltage V_{13} is not as sensitive to an increase in reactive power demand at Node 5 (Q_{D5}) as it is to an increase in reactive power demand at Node 13 (Q_{D13}). This is because the sensitivity of V_{13} (and β_{Q13}) decreases with increased electrical distance. Consequently, the partial derivative in Equation 9.16 will be smaller than the one in Equation 9.15. This discussion also applies to the cost components representing the fixed generator voltages, since fixed voltages are just voltage constraints.

The discussion above only apples when the shadow price and partial derivative of one primal voltage constraint (e.g. $\mu_{V13}\frac{\partial V_{13}}{\partial Q_{13}}$) is greater than the summation of all the other marginal cost components in Equation 9.14. If all cost components are comparable, the behaviour of β_{Q13} is dictated by the behaviour of all the components.

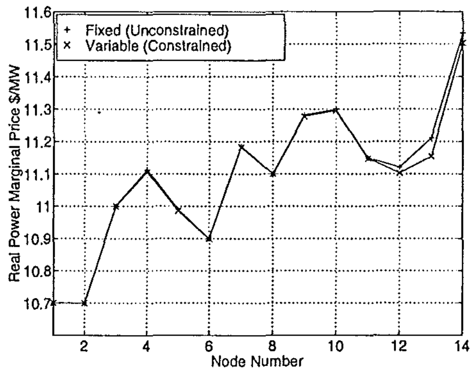
9.5.5 Variable Generator Voltage Magnitudes

Fixed generator voltages magnify real and reactive power prices when a primal voltage constraint is binding at a pqD node.. This price magnification was observed by re-dispatching the voltage constrained power system of Case 8, with variable generator voltages. Although, the voltage at Node 1 was constrained to provide a reference voltage for the power system:

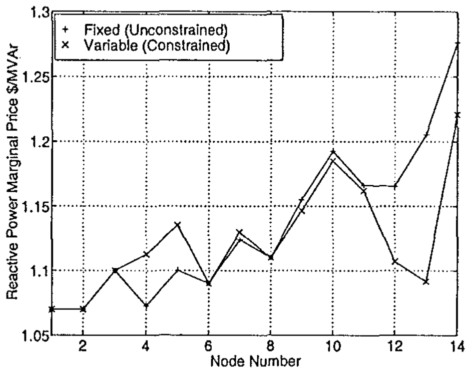
$$V_1^{max} = V_1^{min} = 1.06\,\text{pu}$$

No other changes were made to the original IEEE 14–bus power system, aside from those implemented for Case 8.

The resultant dispatch was only restricted by the binding voltage constraints at Nodes 1 and 13. All generators were marginal for real and reactive power generation. Figure 9.11 presents price profiles for the real and reactive power marginal prices. The



(a) Real power marginal prices



(b) Reactive power marginal prices

Figure 9.11 Marginal price profiles for the IEEE 14–bus power system, with variable generator voltages and an upper voltage constraint on Node 13. The Figure 9.7 unconstrained, fixed generator voltage profiles are included for reference.

unconstrained, fixed generator voltage, price profiles of Figure 9.7 have been included for comparison.

The magnitudes of the reactive power prices for this dispatch are now very small in comparison to Case 8 prices in Figure 9.10(b). Rather, they are comparable to the unconstrained price profiles of Figure 9.7. There are two reasons for the small reactive power prices. Firstly, the shadow prices, μ_V, corresponding to the generator voltages are now zero, since the generator voltages are no longer fixed (refer to Table 9.2). The only exception to this is μ_{V1}, which is non–zero because the reference voltage is fixed. From Equation 9.14, it can be seen that the marginal price of reactive power at every node is reduced, since these shadow prices are now zero.

Fixed Generator Voltages ($/pu)	Variable Generator Voltages ($/pu)
μ_{V1}=-2.7405	μ_{V1}=0.3497
μ_{V2}=0.5510	μ_{V2}=0.0000
μ_{V3}=1.285	μ_{V3}=0.0000
μ_{V6}=-37.8170	μ_{V6}=0.0000
μ_{V8}=-2.2340	μ_{V8}=0.0000
μ_{V13}=41.4660	μ_{V13}=1.1051

Table 9.2 The shadow prices of voltage constraints in the fixed and variable generator voltage dispatches of the IEEE 14–bus power system in Case 8, as calculated by NODAL2.

The second reason comes from the drop in the cost of the voltage constraint, μ_{V13}, as a result of allowing generator voltages to vary. The shadow price μ_{V13}, is defined to be the cost (in $/pu) of relaxing the voltage constraint at Node 13 by 1 pu; the partial derivative, functions as a conversion factor between $/pu and $/MVAr. An extreme value of μ_{V13} indicates the high importance that the spot market assigns to relieving the stress on the power system incurred by this constraint. The extent to which a change in the reactive power demand Q_{Di} will relieve the stress, is indicated in the magnitude of the reactive power price (β_{Qi}), through the values of μ_{V13} and the partial derivative (i.e. sensitivity coefficient). Extreme values of β_Q were experienced in Cases 5 to 8.

Fixed generator voltages constrain all other voltages in the power system by restricting real and reactive power flows. These flow restrictions may require the use of, normally, uneconomical transmission routes and/or reactive power sources when obtaining the power necessary for maintaining the constrained voltage level at Node 13. Such abnormal practises place stress on the power system.

Allowing generator voltages to vary within their rated operating ranges therefore, relieves the stress on the power system. This means that the nett levels of real and reactive power at Node 13 required to maintain the constrained voltage (V_{13}) are more easily obtained since there are fewer power flow restrictions. This lack of stress is reflected by the low value of μ_{V13} in Table 9.2, and by the low reactive power prices depicted in Figure 9.11(b).

There was very little change in any of the reactive power prices, and no observable change in real power prices when the voltage constraint at Node 13 was tightened for this variable generator voltage dispatch. This result is the opposite of the results from Cases 7 and 8. Consequently, any tightening of the voltage constraint places very little extra stress on the power system, and the reactive power prices remain the same because the generator voltages are free to vary. However, there is threshold above (or below) which the power system has difficulty in maintaining the constrained voltage. This results in high stress and thus high prices. The high prices indicate the importance to the system, of relieving this constraint.

9.6 REACTIVE POWER GENERATION CONSTRAINTS

9.6.1 Introduction

A generator that is non–marginal for reactive power has been forced against a primal reactive power generation constraint. Constraints that limit the reactive power output of a generator can be imposed for either physical or economic reasons. This section investigates the effects of binding lower and upper primal reactive power generation constraints on the behaviour of reactive power marginal prices (β_{Qi}).

Specifically, the cases presented describe the behaviour of reactive power marginal prices for dispatches of the IEEE 14–bus power system with a primal reactive power generation constraint imposed on the generator at Node 8. In all cases, this reactive power generation–constrained power system is optimally dispatched using objective function 9.13, repeated here for convenience:

$$\begin{aligned} f\,(cost)_{P,Q} = {} & 10.7\,P_{G1} + 10.7\,P_{G2} + 11.0\,P_{G3} + 10.9\,P_{G6} + 11.1\,P_{G8} \\ & + 1.07\,Q_{G1} + 1.07\,Q_{G2} + 1.10\,Q_{G3} + 1.09\,Q_{G6} + 1.11\,Q_{G8} \end{aligned} \tag{9.17}$$

For all cases, the maximum generation limit on Generator 1 in the original IEEE 14–bus power system data has been increased to $Q_{G1}^{max} = 50\,\text{MVAr}$, to ensure that that all generators (except Generator 8) are marginal for reactive power. All generators are marginal for real power. Unless indicated, all generator voltages have been fixed. There are no other constraints on the power system.

The unconstrained price profiles in Figure 9.7 are used to contrast the constrained price profiles presented in the impending cases. The reactive power generation injected by Generator 8, in the unconstrained dispatch used to generate the price profiles of Figure 9.7, is $Q_{G8} = 16.0797\,\text{MVAr}$.

9.6.2 Lower Reactive Power Generation Constraints

Binding lower reactive power generation constraints are considered in Cases 9 and 10. Generator 8 has been forced against a lower generation limit. The value of Q_{G8}^{min} is the only difference between Case 9 and Case 10. It has been increased in the latter case to tighten the lower reactive power generation constraint.

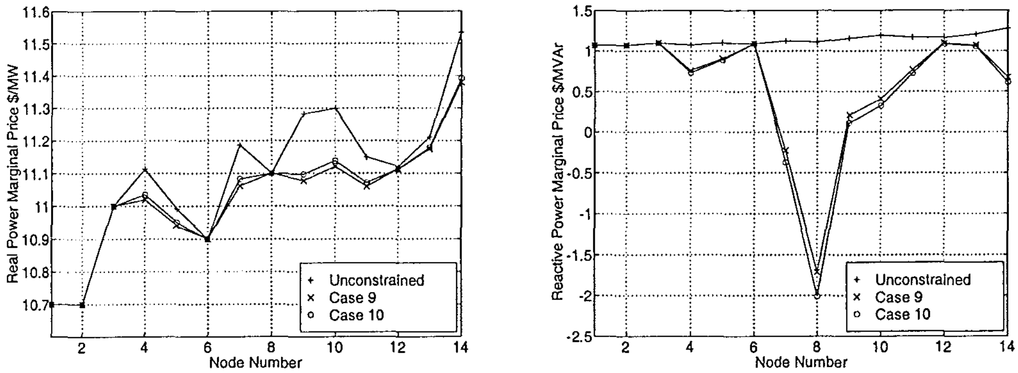
<u>Case 9</u>

To obtain a binding lower reactive power generation constraint, the minimum reactive power limit was set above the unconstrained generation level of 16.0797 MVAr to:

$$Q_{G8}^{min} = 16.5 \text{ MVAr}$$

The resultant real and reactive power marginal price profiles are depicted in Figure 9.12. The unconstrained profiles of Figure 9.7 are included for comparison. A schematic price–profile has not been presented, as the negative β_Q values make the schematic price–profile confusing rather than informative.

β_{Q8} at PQG Node 8 has decreased with respect to the unconstrained price and become negative. The negative price requires that the owner of Generator 8 pay the market the value of β_{Q8} if the owner wants Generator 8 to continue generating this level of reactive power. Customers at Nodes 7 and 8 will be paid by the market for their reactive power usage. The negative prices encourages either, the customer to use more reactive power allowing Generator 8 to operate above its present constrained level, or



(a) Real power marginal prices (b) Reactive power marginal prices

Figure 9.12 Marginal price profiles of Cases 9 and 10, for the IEEE 14–bus power system, where Generator 8 is constrained by a lower reactive power generation limit. The unconstrained Figure 9.7 profiles are included for reference.

Generator 8 to slacken its lower generation limit to meet the unconstrained demand for reactive power. Either way, the desired effect is to relieve the constraint.

Case 10

In this case, the lower generation constraint is tightened by increasing the minimum reactive power generation limit to:

$$Q_{G8}^{min} = 17.0 \text{ MVAr}$$

Tightening this constraint made the reactive power prices in the region of the constraint more negative. Unlike the upper constraint in the next section however, the changes in the reactive power marginal prices are small. Tightening the constraint further causes the prices to become more negative in a non-linear fashion, indicating the increasing importance to the market of relieving the constraint on the power system.

9.6.3 Upper Reactive Power Generation Constraints

Generator 8 is forced against an upper generation limit in Cases 11 and 12. Case 12 differs from Case 11, only in that the upper generation limit on the generator (Q_{G8}^{max}) is reduced, to tighten the upper reactive power constraint.

Case 11

To force a binding upper reactive power generation constraint on the power system, the maximum reactive power generation limit was set below the unconstrained

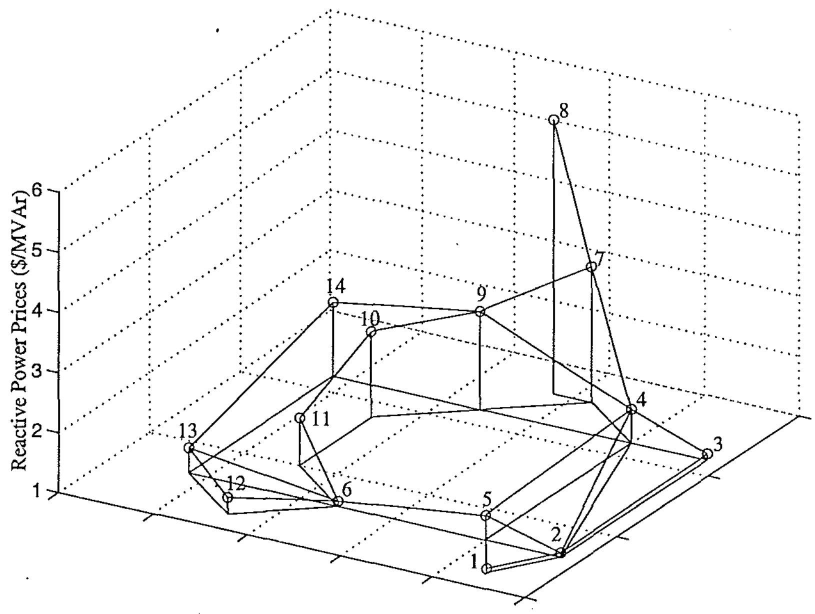


Figure 9.13 The schematic price–profile of reactive power prices (β_{Q_i}) from Case 11, for the IEEE 14–bus power system. Generator 8 has been forced against an upper reactive power generation limit.

generation level of 16.0797 MVAr to:

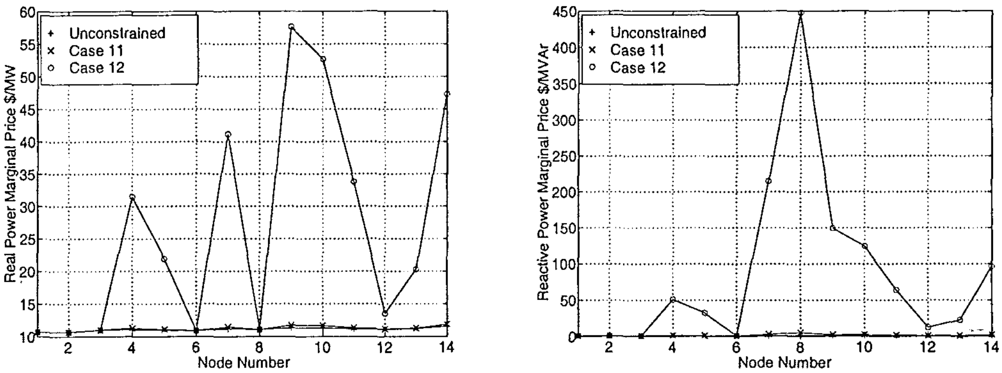
$$Q_{G8}^{max} = 16 \text{ MVAr}$$

This reactive power generation constraint causes the reactive power marginal prices in the vicinity of Generator 8 to increase dramatically (see the schematic price–profile in Figure 9.13). The increase in prices shows that the market is willing to pay in order to increase the reactive power generating capacity of Generator 8. The high price also prevents more stress being applied to the constraint by discouraging customers from using more reactive power. That is, the high price prevents the constraint from being tightened. The low prices at a distance from the constraint means those distant customers can use more reactive power without significantly increasing the amount of stress on the reactive power constraint.

<u>Case 12</u>

The change in reactive power marginal prices as the binding upper reactive power generation limit is tightened, is described in this case. The constraint was tightened by decreasing the maximum reactive power generation limit to:

$$Q_{G8}^{max} = 15.765 \text{ MVAr}$$



(a) Real power marginal prices (b) Reactive power marginal prices

Figure 9.14 Marginal price profiles of Cases 11 and 12, for the IEEE 14-bus power system, constrained by an upper reactive power generation limit on Generator 8. The unconstrained Figure 9.7 profiles are included for reference.

Tightening the constraint by a small amount causes the reactive power prices to increase dramatically, especially with respect to the unconstrained case from Figure 9.7(b) (see Figure 9.14(b)). This increase in marginal prices impresses the importance of relieving the generation constraint on the market. At Nodes 1, 2, 3 and 6, β_{Qi} are small, as they are held at the unit reactive power generation costs of the generators marginal for reactive power (as described in Section 7.3.3).

The reactive power marginal prices are more volatile when tightening this upper reactive power generation constraint than when the lower generation constraint is tightened in Case 10.

9.6.4 Discussion on Reactive Power Generation Constraints

The marginal price equation for reactive power in a reactive power constrained power system is very similar to Equation 9.14 in the section on binding voltage constraints. For example, the equation for the marginal price of reactive power at Node 8 is:

$$\beta_{Q8} = -\lambda_P \frac{\partial L_P}{\partial Q_8} + \lambda_Q \left(1 - \frac{\partial L_Q}{\partial Q_8}\right) - \mu_{V1}\frac{\partial V_1}{\partial Q_8} - \mu_{V2}\frac{\partial V_2}{\partial Q_8} - \mu_{V3}\frac{\partial V_3}{\partial Q_8} - \mu_{V6}\frac{\partial V_6}{\partial Q_8} - \mu_{V8}\frac{\partial V_8}{\partial Q_8} \quad (9.18)$$

In this equation however, the only constraint shadow prices (μ_{Vi}) are for the fixed generator voltages.

Fixed generator voltages are binding primal constraints. By complimentary slackness therefore, the μ_{Vi} shadow prices are non–zero:

$$\mu_{V1} = \beta_{V1} = \left\langle \upsilon^{+}_{V1} \right\rangle - \left\langle \upsilon^{-}_{V1} \right\rangle$$
$$\mu_{V2} = \beta_{V2} = \left\langle \upsilon^{+}_{V2} \right\rangle - \left\langle \upsilon^{-}_{V2} \right\rangle$$
$$\mu_{V3} = \beta_{V3} = \left\langle \upsilon^{+}_{V3} \right\rangle - \left\langle \upsilon^{-}_{V3} \right\rangle$$
$$\mu_{V6} = \beta_{V6} = \left\langle \upsilon^{+}_{V6} \right\rangle - \left\langle \upsilon^{-}_{V6} \right\rangle$$
$$\mu_{V8} = \beta_{V8} = \left\langle \upsilon^{+}_{V8} \right\rangle - \left\langle \upsilon^{-}_{V8} \right\rangle$$

In Section 7.3.3, it was shown that the dual reactive power marginal price at that node is constrained between the unit generation costs of the last and next units of reactive power when a generator is marginal for reactive power. Therefore, when Generator 8 is marginal for reactive power, the reactive power marginal price is thus:

$$\$1.11/\text{MVAr} \leq \beta_{Q8} \leq \$1.11/\text{MVAr}$$

However, when a primal reactive power generation limit is reached Generator 8 becomes non–marginal and this dual price constraint must be relaxed. This is achieved by including a slack variable into the equation. In Case 11, Generator 8 is non–marginal for reactive power because $Q_{G8} = Q^{max}_{G8}$. Consequently, the dual price constraint equation becomes:

$$\$1.11/\text{MVAr} \leq \beta_{Q8} \leq \$1.11/\text{MVAr} + \upsilon^{+}_{Q2}$$

for reasons described in Section 7.3.4.

In Cases 9 to 12, QOPF fixes all generator voltages at pre–specified voltage set–points and adjusts the real and reactive power generation profiles to maintain these set–points. At Node 8 for example, $V_8 = V^{set}_8 \left(= V^{min}_8 = V^{max}_8\right)$. When Generator 8 is marginal, reactive power from this generator is used to maintain V_8 at V^{set}_8. Whereas, reactive power must be provided by a different reactive power source when Generator 8 becomes non-marginal for reactive power.

Generator 8 is forced against an upper reactive power generation limit in Case 11 (i.e. $Q_{G8} = Q^{max}_{G8}$). This reactive power constraint prevents Generator 8 from supplying all the reactive power required to boost V_8 to the voltage set–point. Reactive power must therefore, be generated by a remote, and possibly more expensive, reactive power source to ensure that V_8 is equal to V^{set}_8.

The tendency of the power system is therefore to attempt to reduce V_8 so that a remote reactive power source does not have to be used to maintain V^{set}_8. This is because using a remote source puts stress on the power system. Consequently, the lower generator voltage constraint is binding $\left(\text{i.e. } V_8 = V^{set}_8 = V^{min}_8\right)$ and the corresponding slack variable (υ^-_{V8}) is non–zero. Thus:

$$\mu_{V8} = \beta_{V8} = \upsilon^-_{V8}$$

Reducing the value of Q^{max}_{G8}, as in Case 12, tightens the upper reactive power constraint and moreover, the lower generator–voltage constraint. The tighter the upper generation constraint, the harder it becomes to maintain the voltage set–point and the greater the value of υ^-_{V8}. An increase in υ^-_{V8} results in an increase in the marginal price of reactive power because:

$$\beta_{Q8} = \cdots - \left(-\upsilon^-_{V8}\right) \frac{\partial V_8}{\partial Q_8} \cdots$$

An increase in reactive power demand tightens the lower voltage constraint. Therefore, a high reactive power marginal price occurs to discourage the use of more reactive power. High prices were observed in Cases 11 and 12. The partial derivative $\left(\frac{\partial V_8}{\partial Q_8}\right)$ also increases in value as the reactive power constraint is tightened. This reflects the increased sensitivity of V_8 to an incremental increase in Q_{G8} (i.e. $V_8^{uncnstr}$ has moved further from V_8^{set}).

This price behaviour is the opposite of reactive power price behaviour in the presence of a voltage constraint at a pqD node (compare Figures 9.10(b) and 9.14(b)). In the vicinity of a binding upper reactive power generation constraint, reactive power prices increase because an increase in reactive power demand tightens the upper generation constraint and consequentially tightens the lower generator voltage constraint. In the vicinity of a binding upper voltage constraint however, reactive power prices decrease because an increase in reactive power demand relieves the voltage constraint (see Figure 9.10(b)).

Reactive power prices drop quickly with electrical distance from an upper reactive power constrained (i.e. non–marginal) generator node. This is because increasing remote reactive power demand has less effect on tightening the lower generator voltage constraint than local reactive power demand. A similar argument is also applicable to binding lower reactive power generation constraints.

Table 9.3 presents the values of the voltage shadow prices for Cases 10 and 12. The negative sign means that a lower (rather than an upper) primal voltage constraint was binding. These shadow prices show that a lower reactive power generation constraint at Node 8 results in an upper voltage constraint at the same node (Case 12). In like manner, an upper generation constraint results in a lower voltage constraint (Case 10). This confirms the discussion above.

Reactive power marginal prices are more volatile for binding lower voltage constraints than for binding upper voltage constraints as a comparison of the marginal price profiles in Figure 9.9(b) and Figure 9.10(b) demonstrates. Consequently, when

Lower Q_G constraint Case 10 (\$/pu)	Upper Q_G constraint Case 12 (\$/pu)
μ_{V1}=-3.4379	μ_{V1}=145.3015
μ_{V2}=-1.7202	μ_{V2}=483.5036
μ_{V3}=-0.1329	μ_{V3}=301.7433
μ_{V6}=-3.8137	μ_{V6}=661.9529
μ_{V8}=10.5128	μ_{V8}=-1528.0841

Table 9.3 The shadow prices of fixed generator voltages, in the presence of a reactive power generation constraint at Node 8.

a generator becomes non–marginal for reactive power generation, reactive power marginal prices are much more volatile when the upper generation constraint is binding, rather than when the lower generation constraint is binding.

The above discussion applies only when the marginal cost component of the fixed generator voltage at the node with the reactive power generation constraint (e.g. $\mu_{V8}\frac{\partial V_8}{\partial Q_8}$) is larger than the other components. If this marginal cost component is comparable to the other component, the price behaviour may contradict the above discussion. Such a contradiction may occur if the power system has sufficient, cheap, carefully located reactive power sources to maintain fixed generator voltages without placing undue stress on the power system when generators become non–marginal for reactive power.

9.6.5 Variable Generator Voltage Magnitudes

In this section, the influence of variable generator voltages on reactive power marginal prices is investigated for dispatches bound by a reactive power generation constraint. QOPF was used to optimally dispatch the IEEE 14–bus power system with variable generator voltages. However, the Node 1 voltage was used as a reference at:

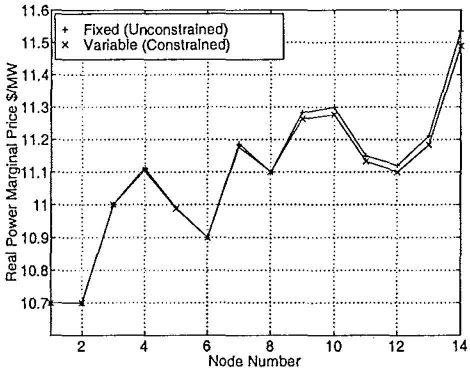
$$V_1^{min} = V_1^{max} = 1.06 \text{ pu}$$

The objective function was Equation 9.17.

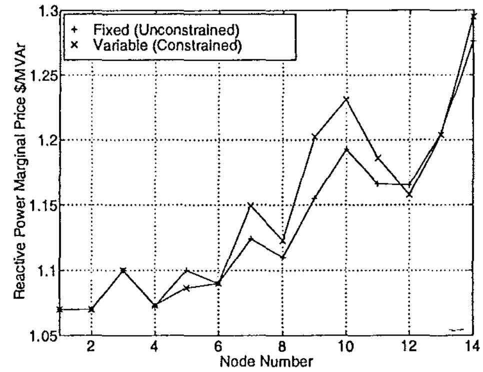
The following reactive power generation limits were modified to ensure that Generator 8 is the only non–marginal generator for reactive power:

$$\begin{aligned} Q_{G1}^{min} &= -200 \text{ MVAr} \\ Q_{G8}^{min} (= Q_{G8}^{max}) &= 9 \text{ MVAr} \end{aligned}$$

Q_{G8}^{min} is lower here than in the fixed generator voltage section because in this new dispatch, the unconstrained reactive power generation is now $Q_{G8}^{uncnstr} = 10.32$ MVAr instead of 16.0767 MVAr.



(a) Real power marginal prices



(b) Reactive power marginal prices

Figure 9.15 Marginal price profiles for the IEEE 14–bus power system, with variable generator voltages and an upper reactive power constraint on Node 8. The Figure 9.7 unconstrained, fixed generator voltage profiles are included for reference.

Equation 9.19 indicates that V_1 and $Q_{G8}^{uncnstr}$ are the only constrained resources in this dispatch:

$$\beta_{Q8} = c_{Q8} + \left\langle \overset{+}{\upsilon}_{Q8} \right\rangle - \left\langle \overset{-}{\upsilon}_{Q8} \right\rangle = -\lambda_P \frac{\partial L_P}{\partial Q_8} + \lambda_Q \left(1 - \frac{\partial L_Q}{\partial Q_8} \right) - \mu_{V1} \frac{\partial V_1}{\partial Q_8} \tag{9.19}$$

The resultant marginal price profiles are illustrated in Figure 9.15. The unconstrained, fixed generator voltage price profiles of Figure 9.7 are included for comparison.

The reactive power marginal prices for this dispatch are now very small in comparison to the prices in Figure 9.14(b), because only the generator voltage at Node 1 is still fixed resulting in a non–zero shadow price (μ_{V1} = \$1.6350/pu). The μ_V terms in Equation 9.18 corresponding to the other generator voltages equal zero. Consequently, the reactive power constraint has almost no effect on reactive power prices. The reactive power constraint only influences β_{Qi} by increasing the cost of marginal losses. This constraint forces any reactive power demand to be supplied by alternate sources, thus altering the marginal losses.

μ_{V1} is no longer large enough to significantly affect the behaviour of β_Q. This is because the other generator voltages are no longer restricting the flow of real and reactive power. The restriction occurs when reactive power is required to maintain any fixed generator voltages. With no restrictions, the use of alternative reactive power sources place almost no stress on the power system. No restrictions means a small value of μ_{V1}. Therefore, a major cause of high reactive power marginal prices when reactive power generation is constrained is the costs of fixed generator voltages (μ_V), not the cost of the generation constraint itself (υ_{Qi}).

The upper reactive power generation limit was tightened using the same procedure as Cases 11 and 12. However, tightening the constraint made almost no change to the reactive power prices.

A PQG node that is non–marginal for reactive power is equivalent to a pqD node (Section 7.3.4). Consequently, β_Q at a reactive power constrained generator node behaves in the same manner as β_Q at an unconstrained pqD node. This statement is verified by the similarities between the 'fixed' and 'variable' price profiles of Figure 9.15(b).

In conclusion, reactive power prices for this variable generator voltage dispatch will generally behave in the same way as the prices in Cases 9 to 12. But, only if the magnitude of the $\mu_{V1}\frac{\partial V_1}{\partial Q_i}$ term is larger than the combined magnitudes of the real and reactive power marginal loss cost components. If this term is smaller, the price behaviour may differ to that in Cases 9 to 12.

9.6.6 Summary

To summarise, reactive power marginal prices decrease when a lower reactive power generation constraint is binding. They increase when an upper reactive power generation constraint is binding. In the presence of a reactive power generation constraint, the main contributors to the volatile and subdued behaviour of reactive power marginal prices are the marginal cost components of any binding constraints, such as fixed generator voltages. The exception to this is when the stress caused by binding constraints is low. This can cause low constraint costs, allowing the marginal loss components to dominate. When the marginal loss components dominate the unconstrained price behaviour described in Section 9.4 applies.

9.7 CONCLUSIONS

In this chapter the behaviour of reactive power marginal prices for optimal dispatches from a pq–type spot market has been investigated. This price behaviour is valid only at the instant in time in the trading period when the dispatch of real and reactive power is optimised.

It has been demonstrated that two types of marginal cost components influence the behaviour of reactive power marginal prices. These are:

- the costs of marginal losses, and
- the marginal costs of binding resource constraints.

In the absence of binding resource constraints, reactive power marginal prices behave in exactly the same way as real power marginal prices in a pq–type spot market. This is because real and reactive power marginal losses are formulated in exactly the same way (as set out in Section 3.3). Any difference between the behaviour of real and reactive power marginal prices (that can be attributed to the cost of marginal losses) is due to the presence of reactive power generation sources other than generators, such as capacitive transmission lines.

For constrained dispatches, reactive power marginal prices were shown to be stable and comparable to the reactive power prices of unconstrained dispatches, when all generator voltages are allowed to vary. When generator voltages are fixed however, marginal prices for reactive power can become equal to, or even exceed, the real power marginal prices in the vicinity of another binding constraint. This is in spite of the fact that the reactive power unit generation costs are set at 10% of the corresponding real power unit generation costs. The fixed generator voltages tighten any binding constraints by restricting reactive power flows and the power system's ability to work round the constraint. This results in a high marginal cost–component on the binding constraint and hence high reactive power marginal prices. The high marginal costs on binding constraints obscure the reactive power price behaviour resulting from the low costs of real and reactive power marginal losses.

Read and Ring [1995a, Section 7.5] predicted that, in the vicinity of a reactive power generation constraint, reactive power marginal prices will be high (or extreme) where there is a shortage of reactive power and low (or negligible) when there is a surplus of reactive power. These results verified this prediction and demonstrated that prices can even go negative. Read and Ring also predicted high reactive power marginal prices in the vicinity of a binding upper voltage constraint, implying that using reactive power will tighten this constraint. However, reactive power marginal prices have been demonstrated to go low and even negative, thus indicating that the use of reactive power actually relieves upper voltage constraints.

The influence of voltage and reactive power generation constraints on reactive power price behaviour, can be used to predict the behaviour of reactive power marginal prices in the presence of any other type of constraint. Reactive power prices will behave according to whether an incremental increase in reactive power demand at a node tightens or relaxes the binding constraint. A respective increase or decrease in the partial derivative of the constraint reflects this. The changes in the reactive power marginal prices are then dependent on whether that partial derivative and its corresponding shadow price are added to or subtracted from the reactive power price equation (Equation 7.15).

Chapter 10

SUB–OPTIMAL DISPATCH PRICE BEHAVIOUR

10.1 INTRODUCTION

In an ideal pq–type spot market, competition between market participants results in an optimal dispatch of real and reactive power. That is, the social welfare function is maximised (see Section 2.3) subject to a set of operating constraints on the power system. As in Chapter 9, this optimisation process is assumed to occur only at the start of each trading period. Note that the optimal dispatches of real power and reactive power are assumed to be coincident. In reality, many factors prevent the optimal dispatches of real power and reactive power. Factors may include poor market operation, uncompetitive market–participant behaviour, and unknown operating constraints [Ring 1995]. A dispatch influenced by factors such as these is defined to be sub–optimal in the Dispatch Based Pricing framework.

Read and Ring [1995a, Section 7.5] predicted that obtaining a feasible solution for a sub–optimal dispatch will be difficult as real and reactive power marginal prices at different nodes will not be consistent with each other. Price inconsistencies arise because there are no specific binding primal constraints to economically explain why the total cost of generation was not be minimised further to obtain a more optimal dispatch.

Ring [1995] proposed a "'Best Compromise' pricing approach" to remove these price inconsistencies, so that *ex post* marginal prices can be calculated for the sub–optimal dispatch. This approach exploits the methodology of Dispatch Based Pricing (presented in Section 3.2) by using arbitrary primal constraints to explain why a more optimal dispatch was not obtained. The sub–optimal dispatch can then be considered as optimal with respect to these constraints and *ex post* marginal prices can then be calculated. Arbitrary constraints add extra dual marginal cost components to the β_{Qi} equation (7.15). Each cost component can be "viewed as a penalty on those responsible for the sub–optimal dispatch". In the previous chapter it was shown that the behaviour of reactive power marginal prices for a constrained optimal dispatch is dependent on the behaviour of the marginal cost components of the binding constraints. Therefore, any reactive power price behaviour resulting from the behaviour of the arbitrary constraint cost components is described by the conclusions of Chapter 9.

Read and Ring [1995a, Section 7.5] proposed that the problem of price inconsistencies can also be eliminated by treating the sub-optimal dispatch of reactive power (i.e. the reactive power generation profile) as a constraint. This proposal is validated in this chapter. Examples are used to show that the pq pricing model (and Dispatch Based Pricing in general) is formulated in such a way that it automatically considers the real and reactive power generation profiles to be constrained when a dispatch is sub-optimal. The behaviour of the *ex post* reactive power marginal prices from these examples is then described. The implications of this price behaviour in a pq-type spot market are also highlighted.

Discussions in this chapter are concerned only with reactive power for the purpose of illustration. However, these discussions and the conclusions are equally applicable to the behaviour of real power marginal prices, since the form of the real and reactive power price equations are identical. Although, the underlying power system operation causing this price behaviour, may be different.

10.2 OBTAINING A SUB-OPTIMAL DISPATCH

The IEEE 30-bus power system (in Appendix E) is used for all cases in this chapter. Figure 10.1 depicts the process used to obtain sub-optimal dispatches for this power system.

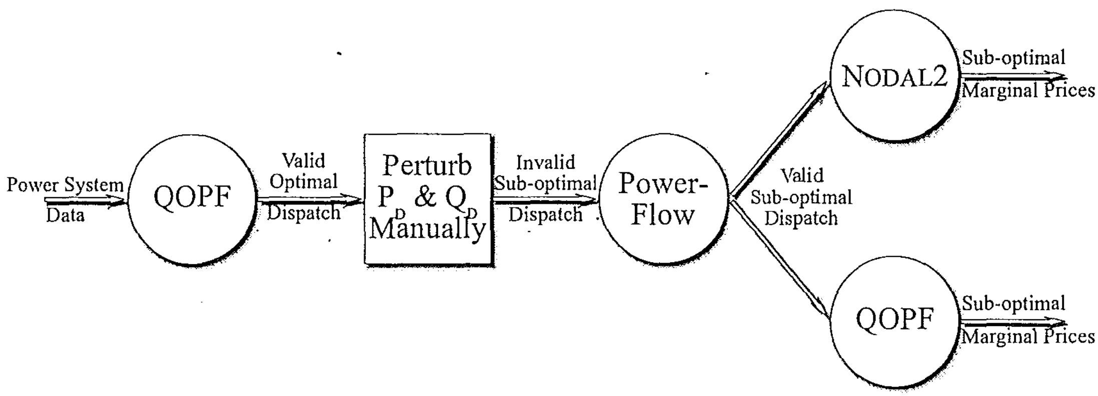


Figure 10.1 The perturbation process, used to obtain marginal price profiles for sub-optimal dispatches.

Optimal dispatches were used as the starting point for deriving sub-optimal dispatches. This is based on the assumption that dispatches are generally close to optimal if not optimal. All optimal dispatches in this chapter were generated using QOPF, with respect to the following objective function:

$$\begin{aligned} f\,(cost)_{P,Q} =& c_{P1} P_{G1} + c_{P2} P_{G2} + c_{P5} P_{G5} + c_{P8} P_{G8} + c_{P11} P_{G11} + c_{P13} P_{G13} \\ &+ c_{Q1} Q_{G1} + c_{Q2} Q_{G2} + c_{Q5} Q_{G5} + c_{Q8} Q_{G8} + c_{Q11} Q_{G11} + c_{Q13} Q_{G13} \end{aligned} \tag{10.1}$$

$$\begin{aligned} =& 10.8\, P_{G1} + 10.6\, P_{G2} + 11.0\, P_{G5} + 10.9\, P_{G8} + 11.1\, P_{G11} + 10.7\, P_{G13} \\ &+ 1.08\, Q_{G1} + 1.06\, Q_{G2} + 1.10\, Q_{G5} + 1.09\, Q_{G8} + 1.11\, Q_{G11} + 1.07\, Q_{G13} \end{aligned} \tag{10.2}$$

The real and reactive power demand profiles for these optimal dispatches were perturbed to obtain sub–optimal dispatches. This was achieved by manually perturbing the real and reactive power loads at the following nodes:

$$P_{Di}^{opt} \quad \text{and} \quad Q_{Di}^{opt} \qquad\qquad \forall i \in (3, 8, 15, 17, 24, 26)$$

Note that '*opt*' identifies the demand values for the optimal dispatch. As an example, the following demand perturbations were implemented for most cases:

$$\begin{aligned} Q_{D3} &= Q_{D3}^{opt} + 4 \text{ MW} & Q_{D8} &= Q_{D8}^{opt} - 4 \text{ MW} \\ Q_{D15} &= Q_{D15}^{opt} + 4 \text{ MW} & Q_{D17} &= Q_{D17}^{opt} - 4 \text{ MW} \\ Q_{D24} &= Q_{D24}^{opt} - 4 \text{ MW} & Q_{D26} &= Q_{D26}^{opt} + 4 \text{ MW} \end{aligned}$$

For continuity, real power and reactive power are always added to the demands at Nodes 3, 15 and 26, and always subtracted from the demands at Nodes 8, 17 and 24. The demand profiles are perturbed rather than the generation profiles. This reflects the fact that the demand profiles change from one trading period to the next (or within a trading period) and the generation profiles follow.

The perturbation process causes the real and reactive power generation profiles to be sub–optimal with respect to the perturbed demand profile. This process of obtaining a sub–optimal dispatch models the scenario where the generators do not respond optimally to changes in the real and reactive power demand profiles.

The resultant data does not describe a valid dispatch after the demand profile is perturbed. For this reason, the invalid dispatch data is passed through the PSERC power–flow to obtain a valid, sub–optimal, dispatch. NODAL2 is then used to generate *ex post* marginal prices for all valid sub–optimal dispatches in this chapter. All sub–optimal dispatches presented in this chapter have been derived using the process depicted in Figure 10.1.

10.3 MULTIPLE MARGINAL GENERATORS

In Section 7.4.2, implicit loss constraints are identified as one reason for the occurrence of multiple marginal generators (for either real power or reactive power). For example, it is possible for all generators to be marginal for both real and reactive power with no binding explicit constraints, because the marginal losses are acting as constraints. For this example, the Marginal Price Criterion (i.e. DBP 5.6) is satisfied when only one generator is formulated as marginal because there are no binding explicit constraints. All other generators must be formulated as non-marginal (as demonstrated in Section 7.4.2). Formulating a generator as marginal follows the process described in Section 7.3.3. Formulating a generator as non-marginal follows the process described in Section 7.4.2.

A distinction is made between a formulated-non-marginal generator for reactive power and a truly non-marginal generator for reactive power. They are mathematically equivalent because Equation 7.18 is used for both of these generator types, to let β_{Qi} vary freely. However, the former generator type is still physically capable of changing its generation output because the output is within the generation limits defined by Equation 7.8 (that is, formulated-non-marginal generators are in fact physically marginal because Equation 7.8 is not binding). Whereas, the latter type cannot change its output because the output has reached the physical generation limit defined by Equation 7.8 (that is, this non-marginal generator is truly non-marginal because Equation 7.8 is binding).

In the cases presented in this chapter, Generators 1, 2, 5, 8, and 13 are marginal for both real and reactive power for every 'valid optimal dispatches' generated by QOPF. That is, these generators are physically marginal because the real and reactive power outputs of each are within the physical generation limits specified by the data of the 30-bus power system. Generator 11 is only marginal for reactive power. QOPF has not dispatched Generator 11 for real power because it is too expensive in comparison with the other generators.

10.4 FORMULATED-NON-MARGINAL GENERATORS

The case studies and examples presented in Sections 7.4.2 and Chapter 9 apply only to optimal dispatches. In those dispatches, several generators were marginal for real and reactive power due to losses acting as implicit constraints. When calculating *ex post* marginal prices however, these generators were formulated as non-marginal to satisfy the marginal price criterion (DBP 5.6). In the price equations for these formulated-non-marginal generators, the slack variables were equal to zero because the dispatches were optimal (i.e. $\upsilon_{Pi} = 0$ and $\upsilon_{Qi} = 0$ as described in Section 7.4.2).

This section investigates the role of formulated–non–marginal generators in sub–optimal dispatches, and the implications of the behaviour of the real and reactive power marginal prices associated with these generators.

10.4.1 Obtaining the Price Profiles

Four increasingly sub–optimal dispatches are considered herein. With respect to the perturbation process presented in Section 10.2, the following perturbations were used to obtain these increasingly sub–optimal dispatches: 1 MW, 1 MVAr; 2 MW, 2 MVAr; 3 MW, 3 MVAr; 4 MW, 4 MVAr.

The 'valid optimal dispatch' of Figure 10.1 is an optimal unconstrained, variable generator voltage dispatch of the 30–bus power system. For this dispatch the following changes were made to the original 30–bus power system data, to ensure that all generators were marginal for both real and reactive power, and that the reference voltage is the only binding constraint:

- $V_1^{max} = V_1^{min} = 1.06\,\text{pu}$. This is the reference voltage.
- $V^{max} = 2.06\,\text{pu}$ for all other nodes (2...30).
- $P_{G5}^{max} = 400\,\text{MW}$
- $Q_G^{min} = 10\,Q_{G_{original\_data}}^{min} \quad \forall i \in \text{PQG}$
- $Q_G^{max} = 10\,Q_{G_{original\_data}}^{max} \quad \forall i \in \text{PQG}$

The *ex post* marginal price profiles of these sub–optimal dispatches and of the 'valid optimal dispatch' are compared in Figure 10.2. Table 10.1 contains the information required by the pq pricing model (i.e. NODAL2) to generate the marginal price profiles of Figure 10.2.

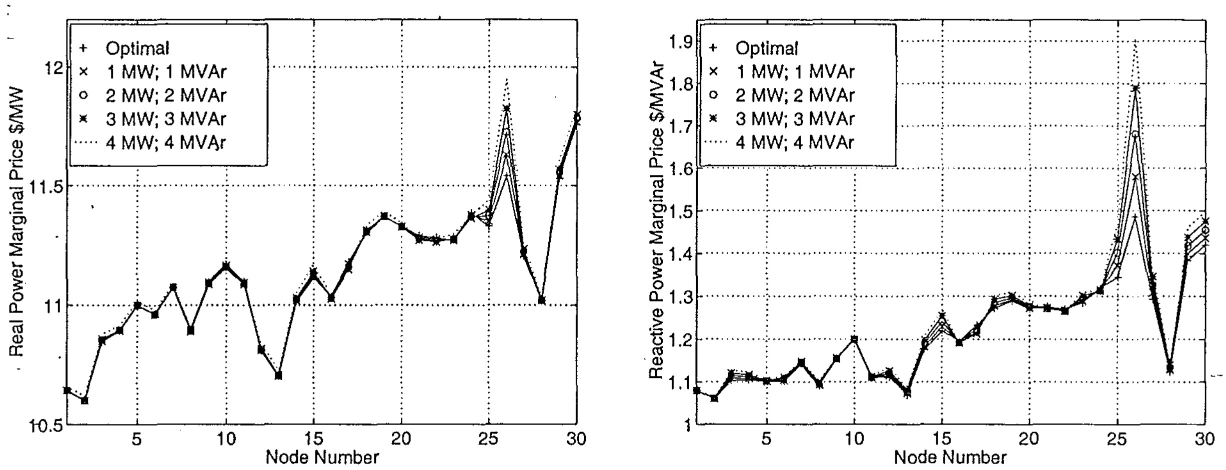
Table 10.1 Information required by NODAL2 to calculate *ex post* marginal prices for the observed unconstrained, variable generator voltage, sub–optimal dispatch of the IEEE 30–bus system.

Node	*Description*	*Unit Generation Cost*
1	Ref	$\beta_{P1} = \lambda_P,\ \beta_{Q1} = \lambda_Q$
5	P_{fm}	\$ 11.0/MW
1	Q_{fm}	\$1.08/MVAr
1	V_b	n/a

The contents of the 'Description' column in Table 10.1 are defined as:

Ref indicates the reference node;

$\mathbf{P}_{fm}$ indicates that the generator at Node 5 is to be formulated as marginal for real power in the pricing model;



(a) Real power marginal prices

(b) Reactive power marginal prices

Figure 10.2 Price profiles generated by Nodal2 for optimal and sub-optimal, variable generator voltage dispatches of the IEEE 30-bus system. Generator 5 is formulated as marginal for real power, and Generator 1 for reactive power.

Q_{fm} indicates that the generator at Node 1 is to be formulated as marginal for reactive power in the pricing model;

V_b indicates that there is a binding primal voltage constraint at Node 1.

Hence, the contents of Table 10.1 show that DBP 5.6 is satisfied because there is one constraint and two formulated-marginal generators.

In addition to Generator 5, Generators 1, 2, 8 and 13 are identified in Section 10.3 as being marginal for real power in the 'valid optimal dispatch'. Likewise, Generators 2, 5, 8, 11 and 13 are identified as being marginal for reactive power in addition to Generator 1 (as stated in Section 10.3). To satisfy DBP 5.6 however, the generators in these two groups are formulated as non-marginal for real power and reactive power respectively, when creating the pq pricing model equations to calculate marginal prices for these optimal and sub-optimal dispatches.

The equation for the reactive power marginal price at every node of all five dispatches represented in Figure 10.2, is:

$$\beta_{Qi} = -\lambda_P \frac{\partial L_P}{\partial Q_i} + \lambda_Q \left(1 - \frac{\partial L_Q}{\partial Q_i}\right) - \mu_{V1} \frac{\partial V_1}{\partial Q_i} \qquad \forall i \in \mathrm{PX} \quad (10.3)$$

10.4.2 Binding Reactive Power Generation Constraints

Definition DBP 5.2 states that a generator (at a node n) is marginal for reactive power only if the marginal price β_{Q_n} is equal to the unit generation cost c_{Q_n}. This definition applies to both formulated–marginal generators and formulated–non–marginal generators. An inspection of the price profiles in Figure 10.2(b) shows that only β_{Q1} is equal to the unit generation cost, c_{Q1} (see Equation 10.2 for the unit generation costs). For all other generators (all of which are formulated as non–marginal for reactive power) β_{Qm} was close to c_{Qm} but not equal, even though they are operating within their physical generation limits. That is, $\upsilon_{Qm} \neq 0$ in the following price bound equation:

$$\beta_{Qm} = c_{Qm} + \upsilon_{Qm} \qquad \forall m \in (2, 5, 8, 11, 13)$$

Equations 7.18 and 7.40 have been applied here.

The reactive power price profiles in Figure 10.2(b) demonstrate that β_{Qm} is not equal to c_{Qm} for the generators formulated as non–marginal for reactive power, even though these generators are physically marginal for reactive power. By calculating β_{Qm} as not equal to c_{Qm}, the pq pricing model (and Dispatch Based Pricing in general) has indicated that these generators are non–marginal for reactive power and unable to respond to any demand for extra reactive power. Therefore, the reactive power output of the formulated–non–marginal generators must actually be constrained (even though the outputs are within the physical reactive power generation limits), otherwise a more optimal dispatch would have been found. Note that this 'more optimal dispatch' is the original 'valid optimal dispatch' depicted in Figure 10.2.

If the reactive power output of the formulated–non–marginal generators were not constrained then their shadow prices (υ_{Qm}) will be non–zero and β_{Qm} will be equal to c_{Qm}. This will result in marginal prices that are inconsistent with each other (definition DBP 5.1) and an infeasible problem because DBP 5.6 is not satisfied. Hence, these generation constraints satisfy DBP 5.6 so that all prices are consistent with each other, and so that the pq pricing model can find a feasible solution. The marginal cost of each of these binding generation constraints is the value of the non–zero shadow price, υ_{Qm}. These prices can be thought of as the penalty for a sub–optimal dispatch, thus performing the same function as the marginal costs of the arbitrary constraints in Ring's 'Best Compromise' approach. However, further research is required to ensure that these non–zero constraint prices produce signals appropriate to penalty costs.

To conclude therefore, when a power system is sub–optimally dispatched the reactive power marginal prices generated by any Dispatch Based Pricing model imply that:

1. all formulated–marginal generators are marginal for reactive power,

2. all formulated–non–marginal generators are non–marginal for reactive power (even though they may be operating well within their physical generation limits), and that

3. the phenomenon of 'multiple marginal generators', resulting from implicit loss constraints, cannot occur in sub-optimal dispatches.

10.4.3 Verification using QOPF

10.4.3.1 Sub-Optimal Dispatch Description

The conclusions of Section 10.4.2 can be verified using QOPF. Consider another sub-optimal dispatch of the 30-bus power system. The objective function, power system data and perturbation data are identical to those used to obtain the sub-optimal dispatch from which the '4 MW; 4MVAr' profiles in Figure 10.2 are generated. In this new dispatch however, all generator voltages are fixed while obtaining the 'valid optimal dispatch'. These generator voltage constraints are used to increase the number of marginal generators, making the verification experiments easier to interpret.

QOPF was used instead of the power-flow software after this optimal fixed-generator-voltage dispatch had been perturbed, (ref. Figure 10.1). QOPF was used simultaneously to obtain the 'valid sub-optimal dispatch' and to calculate *ex post* marginal prices for this sub-optimal dispatch. The upper and lower generation limits of all generators were set equal to the actual generation power outputs (to 12 d.p.) to prevent QOPF from finding a more optimal solution. This is illustrated by the following excerpt from the QOPF print-out, where 'Pmin'='Pmax':

```
============  Generation Constraints  ============
Bus                Active Power Limits
 #    Pmin mu    Pmin      P         Pmax      Pmax mu
---   -------  --------  --------  --------   -------
  1   0.1630     0.23      0.23      0.23        -
  2   0.0073   109.92    109.92    109.92        -
  5   0.0105    70.81     70.81     70.81        -
  8   0.0257    58.53     58.53     58.53        -
 11   0.0294     0.00       -        0.00        -
 13     -       46.80     46.80     46.80        -
```

This real (active) power generation print-out is analogous to the reactive power generation print-out from QOPF. Also, the voltage magnitude and angle values were stored back into the QOPF input data fields 'Vm' and 'Va' (see Appendix E) to help with convergence.

'Pmax mu' and 'Pmin mu' are the shadow prices or Lagrange multipliers of the upper and lower real power generation limits. They correspond to υ^+_{Pi} and υ^-_{Pi} of Equations D.9 and D.10, where i is any of the generator nodes: 1, 2, 5, 8, 11 and 13. υ^+_{Qi}

and υ^-_{Qi} of Equations D.11 and D.12 correspond to '`Qmax mu`' and '`Qmin mu`' (see Appendix F for an example). Each dash (i.e. '-') represents a shadow price of less than 1×10^{-8}. Hence, that shadow price equals zero and the corresponding constraint is not binding[1].

A generator at Node i is defined to be marginal for real power (reactive power) only if both υ^+_{Pi} and υ^-_{Pi} (υ^+_{Qi} and υ^-_{Qi}) equal zero, when using QOPF to calculate real power (or reactive power) *ex post* marginal prices for a sub-optimal dispatch. Since the real power (reactive power) output of each generator, '`P`' ('`Q`') is constrained through '`Pmax`'='`Pmin`' ('`Qmax`'='`Qmin`'), each generator can only be marginal if QOPF has not attempted to change '`P`' ('`Q`'). Attempting to change '`P`' causes either the upper or lower generation limit ('`Pmax`' or '`Pmin`') to become binding, resulting in a non-zero shadow price. Hence, the QOPF print-out shows that only Generator 13 is marginal for real power.

Generator 13 is representative of the formulated-marginal generators for real power that occur when using the pq pricing model. Therefore, $\beta_{P13} = c_{P13} = \$10.7/\mathrm{MW}$ for this marginal generator.

'`Pmin`' and '`Pmin mu`' indicate that Generators 1, 2, 5, and 8 are non-marginal for real power, even though an inspection of the original power system data in Appendix E reveals that these generators are operating within their physical generation limits. Hence, these are the formulated-non-marginal, but physically marginal, generators described in Section 10.4.2. Generator 11 is truly non-marginal because it has not been dispatched for real power (i.e. $\text{P'}_{11}=0$), just as with Generator 11 in the variable generator voltage case described in Section 10.4.1. Hence, Generator 11 represents all formulated-non-marginal and truly non-marginal generators.

Table 10.2 Checkmarks represent the marginal (i.e. formulated-non-marginal) generators for the observed sub-optimal, fixed generator voltage dispatch of the IEEE 30-bus power system. Extra real power generation limits are relaxed with each case.

	Marginal Generators for Real Power						Marginal Generators for Reactive Power					
	1	2	5	8	11	13	1	2	5	8	11	13
Case 13						√	√	√	√	√	√	√
Case 14		√				√		√	√	√	√	√
Case 15		√	√			√		√		√	√	√

In summary, the full QOPF print-out revealed that seven generators were marginal for this 'valid sub-optimal dispatch'[2]. These generators are identifiable as marginal because either '`Pmin mu`'=0 and '`Pmax mu`'=0, or '`Qmin mu`'=0 and '`Qmax mu`'=0. Thus,

[1]Dashes are not used by QOPF to indicate generators with reactive power (or real power) outputs of 0 MVAr. QOPF reports such generators as having an output of 0.0000 MVAr.

[2]There are 12 potential generators because each generator is capable of being marginal for both real and reactive power.

Generator 13 was marginal for real power (see the print-out above) and all six generators were marginal for reactive power, as indicated by the checkmarks for Case 13 in Table 10.2. These seven generators must be formulated as marginal when using the pq pricing model to calculate *ex post* prices for this sub-optimal dispatch. If different generators are formulated as marginal, the marginal prices calculated by the pq pricing model will be different to the prices calculated by QOPF.

The seven formulated-marginal generators satisfy DBP 5.6 because there are six voltage constraints, one constraint representing each of the fixed generator voltages. Generators 1, 2, 5, 8 and 11 are formulated as non-marginal for real power when using the pq pricing model.

The real power generation limits on Generators 2 and 13 were relaxed to ensure that DBP 5.6 was indeed being satisfied, thus producing Case 14. That is:

$$\text{`Pmin'}_{2,13} = \text{`P'}_{2,13} - 1\ \text{MW} \quad \text{and} \quad \text{`Pmax'}_{2,13} = \text{`P'}_{2,13} + 1\ \text{MW}$$

The limits of Generator 2 were relaxed to make Generator 2 marginal for real power, thus giving the power system an extra degree of freedom. The limits of Generator 13 were relaxed to ensure that Generator 13 remained marginal. The degree of freedom for Generator 13 already existed in Case 13. To remove the extra degree of freedom introduced by relaxing the limits of Generator 2 so as to satisfy DBP 5.6, QOPF forced `Q'}_1$ against $\text{`Qmin'_1, thus making Generator 1 non-marginal for reactive power. This is illustrated by the checkmarks in Case 14 of Table 10.2. Relaxing a third pair of constraints (`Pmin'}_5$ and $\text{`Pmax'_5) resulted in Generator 5 becoming marginal for real power and becoming non-marginal for reactive power (i.e. Case 15).

The marginal price profiles for each of these three cases are different from the price profiles of the other cases. This is because the generators that are marginal for each case are different from the marginal generators of the other two cases. This is discussed further in Section 10.6. For each case, NODAL2 is able to produce real and reactive power price profiles that are identical to the QOPF profiles, by formulating as marginal only the checkmarked (i.e. marginal) generators. The uncheckmarked (i.e. non-marginal) generators are formulated as non-marginal. Thus, conclusions 1 and 2 of Section 10.4.2 are verified.

10.4.3.2 Generation Constraints

The process by which QOPF calculates marginal prices for a sub-optimal dispatch is as follows. QOPF attempts to dispatch generators according to a merit order dispatch, subject to the generation constraints used to prevent QOPF from finding a more optimal dispatch. Initially, sufficient generators are allocated as marginal by QOPF to satisfy DBP 5.6. After allocation, QOPF tries to minimise the objective function by attempting to change the outputs of the remaining generators, resulting in binding generation limits. Consequently, these remaining unallocated generators are non–marginal, as illustrated by the excerpt and the checkmarks in Table 10.2. After attempting to minimise the objective function, QOPF then recalculates the marginal prices. All real and reactive power marginal prices are dependent on the costs of these generation constraints as well as the costs of any other binding constraints.

The QOPF process of price calculation is the primal equivalent of the process used by NODAL2. NODAL2 assumes all generators to be marginal with zero slack variables υ^-_{Pi}, υ^+_{Pi}, υ^-_{Qi}, υ^+_{Qi} (the shadow prices of generation constraint Equations D.9 to D.12). NODAL2 then adjusts the values of these slack variables to explain why a more optimal dispatch could not be found. The slack variables that become non–zero during this process identify the non–marginal generators for real and/or reactive power.

NODAL2 calculates marginal prices for real and reactive power, based on the non–zero slack variables of the generation constraints for these non–marginal generators, and on the non–zero slack variables of any other binding constraints. The final price profiles from NODAL2 are identical to those from QOPF. This is because the sub–optimal dispatch is actually optimal with respect to the generation constraints. Figure 6.2 illustrates that QOPF and NODAL2 prices are identical for optimal dispatches.

10.4.3.3 Multiple Marginal Generators

In an optimal dispatch, the total generation cost (i.e. the objective function) has been minimised. When this optimal dispatch is run through QOPF, the generation profile does not change. This is because QOPF cannot further minimise the objective function by changing the (already optimal) generation profile. Accordingly, the losses do not change either.

A constraint implies a boundary that cannot be moved. In Section 7.4.2, marginal losses of an optimal dispatch are classed as implicit constraints if the marginal cost of those makes the unit generation costs of any two marginal generators consistent. The losses are classed as constraints because they will not change when the optimal dispatch is run through QOPF.

In a sub–optimal dispatch, the total generation cost (i.e. the objective function) has not been minimised. Therefore, when this dispatch is run through QOPF, QOPF will always attempt to further minimise the total cost of generation. However, generation constraints are used to prevent QOPF from further optimising the sub–optimal dispatch. Therefore, when QOPF attempts to minimise the objective function, generation constraints will always become binding before losses begin acting as implicit constraints. Consequently, the multiple marginal generator phenomenon described in Section 7.4.2 cannot occur in sub–optimal dispatches.

10.4.3.4 Summary

In summary, Dispatch Based Pricing automatically treats the real and reactive power generation output from all of the formulated–non–marginal generators as constraints when calculating *ex post* marginal prices for any observed sub–optimal dispatch. That is, NODAL2 considers the generation profiles to be constrained and invokes generation constraints to reflect this. Generation constraints are only invoked in the absence of the arbitrary constraints from the 'Best Compromise' pricing approach.

10.4.4 Load–Following Generators

Definition DBP 5.2 implies that only generators marginal for reactive power can supply any demand for more reactive power. Therefore, the conclusions of Section 10.4.2 imply that only formulated–marginal generators respond to any change in reactive power demand when the dispatch is sub–optimal. The formulated–non–marginal generators do not respond, even though they are physically capable of doing so. This is equivalent to the formulated–marginal generators behaving as load–following generators.

Generator 1 is the only formulated–marginal generator for reactive power, in the dispatches used to generate the price profiles depicted in Figure 10.2(b). Therefore, only this generator will respond to any changes in reactive power demand. Furthermore, at each node, the marginal price of using this load–following, Generator 1 to supply any change in reactive power demand at that node, is depicted by the price profiles in Figure 10.2(b).

If there are any explicitly–stated constraints, extra generators must be formulated as marginal to satisfy DBP 5.6 (e.g. Cases 13 to 15 in Table 10.2). These multiple marginal generators follow the load. However, they are not true load–following generators. This is because the contribution from each generator is optimally determined by the OPF, rather than by the impedances of the network as in a power–flow problem.

The load–following scenario for reactive power (and real power) therefore, only applies to sub–optimal dispatches where there is only one marginal generator for reactive power. That is, dispatches where any extra marginal generators are formulated as non–marginal, because there are insufficient explicit and implicit constraints (with respect to DBP 5.6) to explain why these generators are operating within their physical generation limits.

10.5 INCREASINGLY SUB–OPTIMAL DISPATCHES

The discussions and conclusions presented thus far have been based on the assumption that this sub–optimal dispatch was the result of a poor optimisation process at the start of a trading period. However, there is another scenario. Any dispatch that is optimal at the start of a trading period will become increasingly sub–optimal with time due to the continually changing demand profile, until the start of the next trading period when the dispatch is re–optimised.

The price profiles in Figure 10.2(b) were obtained by successively moving the real and reactive power demands at the six pqD nodes (3, 8, 15, 17, 24 and 26) away from their original levels at the point when the dispatch was last re–optimised. Hence, these four price profiles demonstrate how reactive power marginal prices change as an initially optimal dispatch becomes more sub–optimal with time.

In Section 10.4.3.2, these four sub–optimal dispatches were concluded to be optimal with respect to the binding reactive power generation constraints on the formulated–non–marginal generators at nodes 2, 5, 8, 11 and 13. It has been demonstrated in Section 9.6 that the behaviour of reactive power prices in a reactive power generation constrained dispatch is dependent on any binding voltage constraints. It follows therefore, that the behaviour of the reactive power marginal prices shown in Figure 10.2(b) is dependent on the marginal cost component of the reference voltage in Equation 10.3:

$$\mu_{V1} \frac{\partial V_1}{\partial Q_i}$$

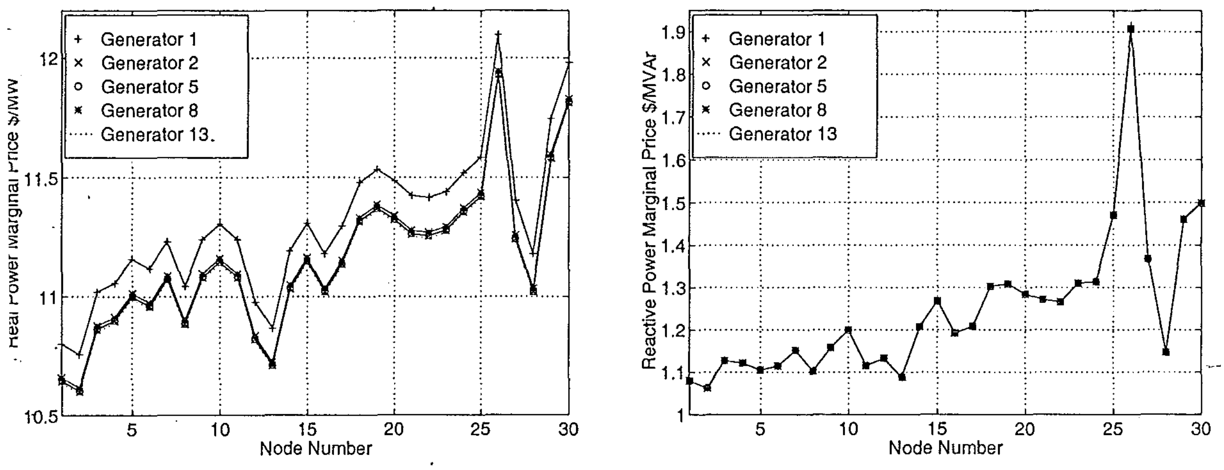
The price profiles demonstrate that reactive power prices become more extreme as the trading period progresses. This is because the changes in the demand profile over time are causing this voltage constraint to be tightened. The price at each node increases or decreases as the dispatch becomes more sub–optimal, depending on whether a marginal change in reactive power demand at that node tightens or relaxes the voltage constraint.

Any price changes due to the changing demand profile are magnified further because reactive power must come from the reactive power load–following, Generator 1. Since all other generators are non–marginal, the power system's ability to respond to any changes in reactive power is severely constrained. This inability is reflected in the high marginal prices.

10.6 DIFFERENT FORMULATED–MARGINAL GENERATORS

10.6.1 Obtaining the Price Profiles

The contents of Table 10.2 demonstrate that the Clearing Manager has a choice as to the marginal generators he or she formulates are marginal and as to the marginal generators he or she formulates as non–marginal when calculating *ex post* marginal prices for a sub–optimal dispatch. In other words, there are several different but equally valid sets of generation constraints that can be used to explain any observed sub–optimal dispatch.



(a) Real power marginal prices

(b) Reactive power marginal prices

Figure 10.3 The price profiles obtained from NODAL2 by formulating each marginal generator as marginal for real power in turn, except Generator 11. The 'valid sub-optimal dispatch' used for all of these profiles is the one used to generate the '4 MW; 4 MVAr' profiles in Figure 10.2.

These different sets provide the Clearing Manager with a choice as to the two generators that will behave as load–following generators to follow any changes in the real and reactive power demand profiles during a trading period. At the start of the trading period, this choice is only available if the dispatch is sub–optimal, since the concept of load–following generators applies only to sub–optimal dispatches.

The same 'valid sub–optimal dispatch' is used to generate all the real and reactive power price profiles in Figure 10.3. It is the same 'valid sub–optimal dispatch' used to obtain the '4 MW; 4 MVAr' price profiles in Figure 10.2. Hence, the '4 MW; 4 MVAr' and 'Generator 5' price profiles are the same. The price profiles of Figure 10.3 were obtained by formulating each generator as marginal for real power in turn, and then resolving the reformulated pq pricing model of the 'valid sub–optimal dispatch'.

The price profiles in Figure 10.3(a) illustrate the cost at each node of obtaining another unit of real power from different load–following generators, during a trading period. These profiles also demonstrate the different marginal price profiles that can be obtained for a sub–optimal dispatch as the start of the trading period, just by formulating different generators as marginal.

It has not been possible to investigate the effect of changing the generator formulated as marginal for reactive power. The NODAL2 source code has been written in such a way, that an error results when the reference node is not formulated as marginal for reactive power. Node 1 is identified in Table 10.1 as the Dispatch Based Pricing reference node. For this reason, Generator 1 is the formulated–marginal generator of reactive power for all of the price profiles in Figure 10.2 and Figure 10.3.

10.6.2 Discussion

The following discussion only applies to sub–optimal dispatches. That is, dispatches where there are insufficient explicit and implicit constraints to explain why multiple generators are physically marginal.

For the valid sub–optimal dispatch associated with Figure 10.3, only one marginal generator can be formulated as marginal for real power because the reference voltage forms the only explicit constraint: DBP 5.6 must be satisfied. The marginal price at this formulated–marginal generator node (n) is:

$$\beta_{Pn} = \lambda_P \left(1 - \frac{\partial L_P}{\partial P_n}\right) - \lambda_Q \frac{\partial L_Q}{\partial P_n} - \mu_{V1} \frac{\partial V_1}{\partial P_n} \tag{10.4}$$

However, β_{Pn} must be fixed as equal to the unit generation cost of the formulated–marginal, Generator n by using Equation 7.17:

$$\beta_{Pn} = c_{Pn} \tag{10.5}$$

Rearranging Equation 10.4 with respect to the reference prices gives:

$$\lambda_P \left(1 - \frac{\partial L_P}{\partial P_n}\right) - \lambda_Q \frac{\partial L_Q}{\partial P_n} = c_{Pn} + \mu_{V1} \frac{\partial V_1}{\partial P_n} \tag{10.6}$$

Table 10.1 shows that the reference prices λ_P and λ_Q are the marginal prices of the generator at Node 1 (i.e. $\lambda_P = \beta_{P1}$ and $\lambda_Q = \beta_{Q1}$). For the 'Generator 1' price profile, Generator 1 is formulated as marginal and Equation 10.5 illustrates that β_{P1} is fixed:

$$\beta_{P1} = c_{P1} \, (= \lambda_P) \tag{10.7}$$

Hence, the reference price is equal to the unit generation cost of the reference generator. For the other price profiles, the reference node is different to the formulated–marginal node (i.e. $n \neq 1$) and therefore formulated–non–marginal, resulting in β_{P1} and λ_P being unrestricted. By Equation 10.6 therefore, λ_P is dependent on the unit generation cost of a different generator for each of these other profiles.

For each profile, the marginal cost at every pqD node and every formulated–non–marginal and truly non–marginal generator node is:

$$\beta_{Pi} = \lambda_P \left(1 - \frac{\partial L_P}{\partial P_i}\right) - \lambda_Q \frac{\partial L_Q}{\partial P_i} - \mu_{V1} \frac{\partial V_1}{\partial P_i} \qquad \forall i \in \text{pqD} \cup \text{PQG}_{nm} \tag{10.8}$$

PQG_{nm} represents the set of all non–marginal and formulated–non–marginal generator nodes. This price equation and Equation 10.6 illustrate that the marginal price at every node is dependent on the unit generation cost of the formulated–marginal generator (i.e. c_P) through the reference prices (i.e. λ_P and λ_Q).

The same valid sub–optimal dispatch is used to generate all the price profiles in Figure 10.3(a) (and Figure 10.3(b)). Therefore, the marginal losses $\left(\frac{\partial L_P}{\partial P_i} \text{ and } \frac{\partial L_Q}{\partial P_i}\right)$ and the voltage constraint partial derivative $\left(\frac{\partial V_1}{\partial P_i}\right)$ do not change from one price profile to the next in Figure 10.3(a). Hence, the only terms that change are λ_P, λ_Q and μ_{V1}. μ_{V1} changes so as to be consistent with the other shadow prices in the equation. λ_P and λ_Q change according to the unit generation cost of the formulated–marginal Generator n (i.e. c_{Pn}).

The real power unit generation costs of the generators are defined to all be different, by Equations 10.1 and 10.2. Accordingly, the real power price profile experiences a translation along the 'Marginal Price' axis each time a different marginal generator is formulated as marginal, rather than formulated as non–marginal. If NODAL2 had allowed generators (other than the reference generator) to be formulated as marginal for reactive power, the reactive power price profile would shift up or down for different reactive power formulated–marginal generators, in the same way that the real power profile does.

It must be noted that if the dispatch had been optimal the price profile would not have moved, regardless of which generator was formulated as marginal. This is because all prices are consistent with each other through the costs of explicit constraints and the costs of marginal losses (as described in Section 7.4.2). That is, fixing the price at the marginal generator node specifies the optimal marginal prices at all other nodes in the power system.

In conclusion, the Clearing Manager must be careful in their choice of formulated–marginal generator when calculating *ex post* marginal prices for the start of a trading period, if the dispatch was sub–optimal. For consistency with Dispatch Based Pricing then, the most expensive marginal generator should be formulated as marginal when using the pq pricing model, unless all market participants agree that another generator should be formulated–marginal. Choosing the most expensive generator reflects a merit order dispatch. During a trading period however, it is possible to exploit this load–following characteristic of Dispatch Based Pricing. The characteristic can be used to determine the cost of supplying real or reactive power using different load–following generators, during a trading period.

10.7 PvG–TYPE OPTIMAL DISPATCHES

Thus far it has been shown that, at the node of a formulated–non–marginal generator for reactive power the reactive power marginal price (β_{Qm}) does not equal the unit generation cost (c_{Qm}) when the dispatch is sub–optimal. This result also applies when the Dispatch Based Pricing model used to calculate *ex post* marginal prices for an observed optimal dispatch, is not the dual equivalent of the OPF used to obtain this observed optimal dispatch.

In the example presented in Appendix F, the IEEE 14–bus power system has been optimally dispatched using the pq–OPF formulation (Equations 7.1 to 7.12). That is, QOPF was used. The resultant pq–type optimal dispatch was unconstrained, apart from the fixed generator voltages. Section F.2 states that all generators were marginal for both real power and reactive power.

Ex post marginal prices were calculated for this dispatch, using both the pq pricing model and the PvG pricing model. These two price sets (both generated by NODAL2) are presented in Sections F.5 and F.6 respectively. As discussed in Chapters 7 and 8, these models calculate prices for pq–type and PvG–type spot markets respectively. Tables F.1 and F.2 show that only Generator 8 was formulated as marginal for real power when calculating both sets of marginal prices. All other generators (m =1, 2, 3, 6) were formulated as non–marginal for real power, even though they were physically marginal for real power (see the QOPF output). All generators were formulated as marginal for reactive power.

In the pq pricing model price set, β_{Pm} is equal to c_{Pm} for all the formulated–non–marginal (i.e. physically marginal) generators. This implies that this dispatch is optimal for a pq–type spot market, and that QOPF is the primal equivalent of the pq pricing model. In the PvG pricing model price set however, β_{Pm} is not equal to c_{Pm} for any of the formulated–non–marginal generators. This implies that Generators 1, 2, 3 and 6 are actually non–marginal (i.e. constrained) for real power, even though the generators are all within their physical generation limits. Therefore, it can be concluded that any pq–type optimal dispatch is actually sub–optimal in the context of a PvG–type spot market (which is modelled with a PvG–type OPF), and vice versa. That is, the PvG–pricing model considers this pq–type optimal dispatch (produced by QOPF) to be sub–optimal. This is because the QOPF formulation is not the primal equivalent of the PvG pricing model.

10.8 CONCLUSIONS

Read and Ring [1995a, Section 7.5] have been cited as proposing that a sub–optimal reactive power dispatch (i.e. the reactive power generation profile) can be treated as a constraint, so as to eliminate the problem of price inconsistencies that normally prevent the calculation of *ex post* marginal prices for sub–optimal dispatches. In this chapter, this proposal has been shown to be inherent within the equations of the Dispatch Based Pricing framework. Specifically, it has been demonstrated that the pq pricing model automatically treats the real and reactive power outputs of the formulated–non–marginal generators as constrained when the dispatches of real and reactive power are sub–optimal.

Read and Ring expected that reactive power marginal prices should be "relatively small" (i.e. approaching $0/MVAr) when the dispatch is near optimal, in the absence of reactive power generation cost functions. A similar result has been demonstrated in Section 10.4 for dispatches where reactive power generation cost functions are employed. For these dispatches, the sub–optimal price profile approaches the optimal price profile as the dispatch becomes more optimal. Accordingly, it has been concluded and shown that marginal prices become more extreme throughout each trading period as the dispatch becomes more sub–optimal.

It has also been demonstrated that Dispatch Based Pricing can be used to calculate the cost of supplying another unit of real or reactive power from different load–following generators. The different marginal price profiles that result are dependent on the unit generation costs of the formulated–marginal generators.

All of the above conclusions also apply when the OPF formulation used to obtain the optimal dispatch is not the primal equivalent of the dual pricing model used to calculate the *ex post* marginal prices.

Chapter 11

APPLICATION OF DISPATCH BASED PRICING

11.1 INTRODUCTION

The North Island and South Island of New Zealand geographically divide the country's National Grid into two smaller grids. These two grids are connected only by a 1240 MW high voltage dc link. The geographies of the two islands cause the two power systems to have very different physical and operational characteristics. The National Grid is the high voltage network operated by Transpower New Zealand Ltd. It excludes the networks of the retail power companies.

The physical process currently used to dispatch reactive power in the South Island section of the National Grid is presented in Section 11.2. This section summarises discussions held with Transpower New Zealand Ltd employees at the control centre for the South Island section of the National Grid. The voltage and reactive power profiles presented in this section are generated using SCADA data obtained from the control centre. The data were recorded on Thursday, November 19, 1998.

The reactive power dispatch information along with the conclusions of the previous chapters, are used to propose a spot market for real and reactive power for the South Island National Grid. Some of the implications concerning how this spot market might influence the operation of the South Island National Grid are also discussed.

11.2 THE SOUTH ISLAND NATIONAL GRID

11.2.1 Grid Operation Voltage Limits

In the South Island National Grid reactive power is currently controlled for the purpose of maintaining certain voltage levels within the National Grid transmission network. The following nominal operating voltage ranges apply to the North and South Island Grids [GOSP 1997]:

220 kV	± 10%
110 kV	± 10%
66 kV	± 5%
50 kV	± 5% (North Is. only)

These voltage ranges indicate extreme limits. Generally, operating voltage ranges are much smaller. For example, the usual operating voltage range for the Waitaki power scheme is 220 kV to 236 kV. Such operating limits depend on:

- upper and lower tap limits of transformers;
- the maximum voltage ratings of generators and power stations: these help define the upper voltage limits;
- supply–point voltage limits.

11.2.2 Supply–Point Voltage Limits

The supply–point is defined to be the low voltage (LV) side of the transformer banks connecting the power company networks to the South Island National Grid. At the supply–points, the following nominal voltage ranges exist [GOSP 1997]:

33 kV	$\pm$ 5%
22 kV	$\pm$ 2.5%
11 kV	$\pm$ 2.5%

However, these percentage limits are often redefined within supply contracts between Transpower New Zealand Ltd and the individual power companies.

11.2.3 On–Load Tap Changing Transformers

In the South Island National Grid, On–Load Tap Changing (OLTC) transformers are used primarily in the transformer banks that connect the power company networks to the grid. These OLTC's maintain the supply–point voltages within their contracted limits, as stated in the previous section. When OLTC's approach their tap limits, reactive power must be redispatched. This redispatch changes the grid voltage profile, allowing OLTC's to move away from their tap limits.

11.2.4 Reactive Power Dispatch

The South Island control centre dispatches reactive power from the South Island power stations using two methods: by 'voltage set–point' and by 'reactive power output'. These methods are described below. There are also some stations that are not dispatched by the South Island control centre.

11.2.4.1 By Voltage Set–Point

The South Island control centre provides certain power stations with a voltage set–point. For each power station, the associated automatic voltage regulators (AVR's) hold the required voltage set–point. The reactive power output of the station varies with changing load conditions in order to maintain this set–point. In this way reactive power is dependently dispatched (see Section 5.7). The Cobb, Coleridge and Manapouri power stations are dispatched in this manner.

There are also six capacitor banks. These are also provided with a voltage set–point. The control centre switches sufficient capacitors into the network to obtain the required voltage set–point, thus nominally dispatching reactive power. After this 'dispatch' has occurred, both voltage and reactive power change at the point of connection according to the relationship: $Q = \frac{V^2}{X}$. The Islington site actually consists of switched capacitors, a static VAr compensator and synchronous condensers.

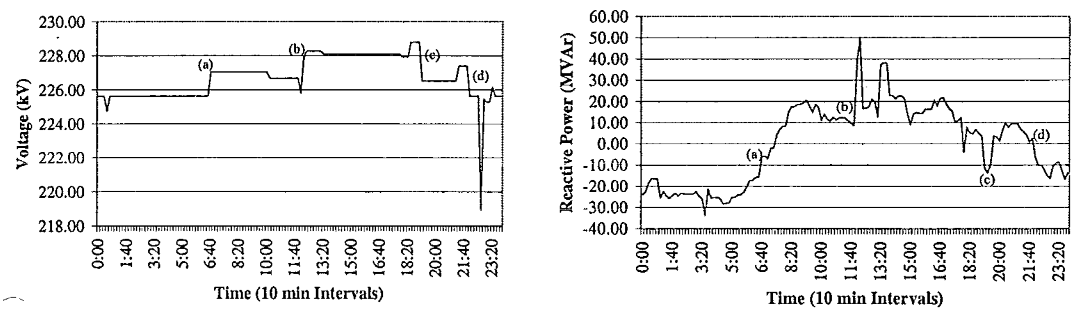


Figure 11.1 Voltage and reactive power profiles at the Manapouri power station 220 kV busbar, for Thursday, November 19, 1998.

Figure 11.1 depicts the voltage and reactive power output at the Manapouri power station 220 kV busbar over a 24 hr period. The voltage and reactive power are heavily dictated by the loads of the nearby aluminium smelter and city of Invercargill. It can be seen that the reactive power profile changes slowly over the 24 hr period, following the demand of the city and smelter. Consequently, the voltage set–point was only changed four times during this period (points a–d). In general, the voltage set–points of the three power stations are changed between 2 and 4 times a day.

11.2.4.2 By Reactive Power Output

The South Island control centre issues these power stations (or blocks of) with a certain reactive power output. When a reactive power output is issued to a station, the station's control system varies the machine excitation voltages until the required reactive power station output is obtained. By varying the excitation voltages the terminal voltage of the power station also varies. The station terminal voltage corresponding to the new reactive power output is used as the new voltage set–points for the generator AVR's. In this way, reactive power is independently dispatched (see Section 5.7). Once the AVR's begin maintaining this voltage set–point, the reactive power output of the power station is allowed to vary with load conditions, in order to maintain this set–point. Holding the set–point indicates that, after being dispatched for reactive power, these generators operate in exactly the same way as the 'voltage set–point' generators. The following station blocks are dispatched in this manner:

Aviemore	Roxburgh, at 110 kV and 220 kV busbars
Benmore	Ohau (A, B and C)
Clyde	Tekapo B
Waitaki	

The voltage and reactive power profiles at the Clyde power station No. 1 220 kV busbar are depicted in Figure 11.2. Reactive power was redispatched to 0 MVAr at 6:19 am (point a), and to -20 MVAr at 7:34 pm (point b). The corresponding new voltage set–points required to obtain the 0 MVAr and -20 MVAr are visible by the significant steps in the voltage profile. Once dispatched, the Clyde power station AVR's maintain the new voltage set–point and the reactive power output is allowed to drift.

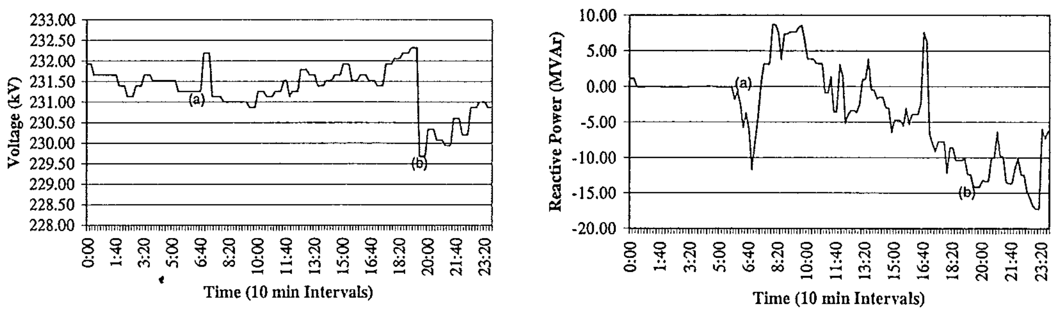


Figure 11.2 Voltage and reactive power profiles at the Clyde power station No. 1, 220 kV busbar, for Thursday, November 19, 1998.

The AVR's actually monitor the LV (11 kV) busbar but the SCADA does not record data at the LV busbar. Thus, the fluctuations in the voltage profile are likely to be the result of changes in the current flowing through the connection transformer. Since Clyde uses thyristor control, it is expected that its LV profile remains as constant as the Manapouri 220 kV voltage depicted in Figure 11.1 (between the points of redispatch that is).

11.2.4.3 Undispatched Power Stations

The control centre has no influence over reactive power generation from the Branch River, High Bank, Tekapo A or Waipori power stations. The operators at these minor stations control their own reactive power output or voltage set–point to manage local voltage conditions. Waipori uses a power–factor set–point rather than a voltage set–point.

11.2.5 Reactive Power Generation Constraints

Generally, the control centre attempts to minimise the reactive power output of all power stations, if possible. The primary reason for this is to minimise rotor winding heating and enables generators to respond to voltage contingencies. Minimisation of the reactive power output is a form of reactive power generation constraint.

11.3 A SOUTH ISLAND NATIONAL GRID SPOT MARKET

In this section, a spot market for real and reactive power is proposed for the South Island National Grid. This market proposal is based on the current methods used to dispatch reactive power in the South Island National Grid. It is assumed to have 48 half–hourly trading periods in a day, to match the existing New Zealand Electricity Market (NZEM) described in Section 2.5.2. At minimum, real power generation is assumed to be economically dispatched (i.e. the output of each generator is re–optimised) once every trading period, as it is in the NZEM.

Regarding the economic dispatch of reactive power, Dandachi *et al* [1996] noted the impracticalities of frequent "system–wide dispatches of quantities like transformer taps and shunt capacitors". Their suggestion was to fully re–optimise reactive power only at certain points on the load cycle, and every half–hour optimise the reactive power dispatch while minimising the number of controls shifted. For simplicity of discussion however, reactive power is assumed to experience a full economic (i.e. optimal) dispatch every time real power is economically dispatched in this new spot market, that is at least once every trading period. It is also assumed this dispatch occurs at the start of the trading period.

11.3.1 The Optimal Reactive Power Dispatch

Real and reactive power generation are economically dispatched in this notional spot market using a pq–type optimisation process. In Section 11.2.4.2, it was stated that reactive power can be dispatched as an independent resource (i.e. by 'reactive power output') from the majority of South Island power stations. This implies that each station terminal voltage is a dependent resource, because it assumes a value consistent with the real and reactive power output of that station. Therefore, the terminal busbars of the stations listed in Section 11.2.4.2 should be classified as PQG nodes when optimising the reactive power dispatch at the start of the trading period.

PQG nodes are only present in the formulations of the pq–OPF (Equations 7.1 to 7.12) and the pq pricing model (Equations 7.13 to 7.19). Hence, this South Island spot market is a pq–type spot market. In this pq–type market, a pq–type OPF (such as pq–OPF or QOPF) must be used to optimally dispatch the real and reactive power generation. Also, the Clearing Manager must use an equivalent of the pq pricing model to calculate *ex post* marginal prices for each instant in time when the dispatch was optimised.

Generator voltage set–points are defined to be voltage constraints in the context of a pq–type spot market (see Section 7.4.1). Therefore, the terminal busbars of Cobb, Coleridge and Manapouri can be classed as PQG nodes with a binding voltage constraint, where[1]:

$$V_i^{set} = V_i^{min} = V_i^{max} \tag{11.1}$$

The terminal busbar of each capacitor bank should also be modelled as a PQG node with a binding voltage constraint[2]. For these nodes however, there is no real power generation. That is, $P_{Gi} = P_{Gi}^{min} = P_{Gi}^{max} = 0$.

In this pq–type spot market the behaviour of reactive power marginal prices at the instant of optimisation is described in Chapter 9. If however, the dispatch was sub–optimal (i.e. poorly optimised) then the price behaviour is described in Chapter 10.

11.3.2 Between Optimal Dispatches

The PvG pricing model can be used to calculate the actual cost of obtaining another unit of power from the load–following generators, after each optimisation of real and reactive power generation (see Sections 10.4.4 and 10.6).

Once the power system has been optimised, demand for power is actually supplied in a power–flow type manner. That is, all generators respond to any change in reactive power load, where the electrical distance between each generator and the load–change–node determine the proportion supplied by that generator. Furthermore, any demand for extra real power (greater than minor fluctuations) is usually supplied by a single swing bus generator.

This power–flow dispatch is identical to an unconstrained, sub–optimal, PvG–type dispatch thus making it possible to use the PvG pricing model for calculating marginal

[1]This PQG classification actually requires the control centre to dispatch these stations by 'reactive power output' instead of by 'voltage set–point' because Q is an independent variable at PQG nodes. The voltage set–point would then be used to define very tight operating voltage limits for this PQG node (e.g. Equation 11.1). The alternative is to dispatch these stations by voltage set–point and class the busbars as PvG nodes. This results in a mixed pq/PvG–type spot market, requiring the pq/PvG Dispatch Based Pricing model (Equations 5.1 to 5.9); the price behaviour arising from this permutation has not been investigated.

[2]These nodes should actually be modelled as PvG nodes, just like the 'voltage set–point' generator nodes.

prices. The reasoning for this is as follows. In Section 11.2.4.2 it was shown that once a power station has attained the required reactive power output, the terminal busbar voltage is fixed and the reactive power output is allowed to vary. Thus, voltage is the independent resource and reactive power is the dependent resource. Hence, all PQG nodes must be reclassified as PvG nodes after the generators have been optimally dispatched for real and reactive power. The nodes of the 'voltage set–point' power stations remain as PvG nodes, likewise the Islington node with the static VAr compensator. The static capacitors however, would only be switched in or out when the dispatch is optimised. Therefore, it is proposed that they should be treated as fixed shunt impedances and their nodes reclassified as pqD nodes (i.e. standard load nodes). Given that all nodes are either PvG of pqD nodes, this is a PvG–type dispatch. Also, this dispatch is sub–optimal because it is a PvG–type dispatch in the context of a pq–type spot market, and because the demand profile is constantly changing (see Section 10.7).

All generators can be formulated as marginal for reactive power when using the PvG pricing model to calculate *ex post* marginal prices. This is because all generator voltages are fixed thus satisfying DBP 5.6 (see the examples in Chapter 8 and Appendix F). Likewise, to satisfy DBP 5.6 only one generator can be formulated as marginal for real power when the dispatch is unconstrained. All other generators must be formulated as non–marginal for real power, even if they are within their generation limits. Applying the conclusions of Section 10.4.2 to this dispatch, all these formulated–non–marginal generators (even those operating within their physical generation limits) must actually be non–marginal for real power because the dispatch is sub optimal. Note that when a dispatch is optimal, formulated–non–marginal generators can actually be marginal (ref. Section 7.4.2).

Table 11.1 Fixed PvG and power–flow variables for a power–flow type dispatch.

Fixed PvG Pricing Variables	Fixed Power–Flow Variables
V and θ at the formulated–marginal node	V and θ at the swing bus (S)
P and V at all other PvG nodes	P and V at all other generator nodes (PV)
P and Q at the pqD nodes	P and Q at all demand nodes (PQ)

The fixed variables of the PvG pricing equations for this unconstrained sub–optimal PvG-type dispatch are presented in Table 11.1 along with the fixed variables in a standard power–flow algorithm. There is only one possible dispatch solution in a power–flow problem because the problem is fully defined by the fixed variables. Thus, when comparing the two sets of fixed variables it is evident that the PvG pricing problem is also fully defined. That is, there is only optimal solution: a set of marginal prices that describes the cost of supplying any demand for power using a power–flow dispatch. For example, the prices will indicate that the optimal proportion of reactive power to be supplied by each generator (as calculated by an OPF in response to a unit change in reactive power demand) is exactly the same as the generation proportions calculated by a standard power–flow algorithm.

QOPF was used to verify that an OPF does dispatch the next unit of power in a power–flow manner when the variables (shown in Table 11.1) are fixed. However, the marginal prices from QOPF (which is a pq–type OPF) did not match the prices from the PvG pricing model (i.e. NODAL2). Until a PvG–type OPF is implemented in software, it is not possible to establish whether this price discrepancy stems from the difference between the pq–type and PvG–type OPF algorithms, or from a source code error in NODAL2.

Regardless of the validity of the final output of the NODAL2 software, the PvG pricing equations model the power–flow (or load–following) behaviour of the South Island National Grid, with the following provisos. The dispatch must be unconstrained and sub–optimal. Also, the reference voltage angle (i.e. $\theta_{ref} = 0$) must be at the node of the generator that is formulated as marginal for real power. If it is not, the next unit of power will not be dispatched in a power–flow manner because there must be an element of optimisation if the OPF is to find a feasible dispatch.

11.4 IMPLEMENTATION ISSUES

In this Section some of the many issues relevant to the implementation of this South Island spot market are identified.

11.4.1 Variable Generator Voltages

The terminal voltages of the power stations identified in Section 11.2.4.2 are allowed to vary within certain limits at the time of optimisation (e.g. 220 kV to 236 kV for the Waitaki power scheme). The "Variable Generator Voltages" sections in Chapter 9 indicated the primary advantage of variable generator voltages, stable and moderate reactive power marginal prices in the vicinity of a single binding constraint. The importance of this becomes evident when looking at the behaviour of a reactive power dispatch.

In Appendix G it is shown that a pq–type OPF will force a merit order dispatch for both real and reactive power in the absence of binding constraints such as fixed generator voltages. This is because the nature of an OPF is to force the dispatch against operating limits (in particular, generation limits when the dispatch is unconstrained). However, this appendix demonstrates that if some other network constraints had been tight enough, these network constraints would have become binding before the reactive power generation limits became binding (i.e. multiple marginal generators for reactive power occur would have occurred).

Assuming that pq–OPF is used to minimise the total cost of real and reactive power generation, the main constraints that are likely to become binding in the South Island National Grid are:

1. supply–point voltage limits;
2. OLTC limits (if included in the pq–OPF formulation);
3. power station terminal voltage limits;
4. reactive power generation limits.

The first three types of constraints are likely to become binding before any reactive power generation limits become binding. This is particularly true of the supply–point voltage limits, which are considered to be very tight by the control centre (e.g. the nominal voltage $\pm 2.5\%$). Depending on the line impedance in the South Island National Grid, it is also possible that reactive power losses will act as implicit constraints, resulting in multiple marginal generators for reactive power.

It can be concluded therefore, that prices should be moderate for a limited number of binding constraints if the power station voltages are allowed to vary within their respective operating limits. However, as the OPF forces more constraints to become binding, prices are likely to become more volatile and extreme.

11.4.2 Fixed Generator Voltages

Marginal prices for an optimal dispatch are much more volatile in the vicinity of a binding constraint when all generator voltages are fixed (as concluded in Chapter 9)[3]. This is because the power system's ability to work around that binding constraint is restricted by the fixed generator voltages. Therefore, if all South Island power stations are operated to voltage set–points, reactive power marginal prices can be expected to be volatile if the pq–OPF forces binding constraints when optimising the dispatch of real and reactive power generation. Fixed generator voltages do however, have the benefit of restricting the amount by which the pq–OPF can optimise the reactive power dispatch, thus reducing the likelihood of binding constraints and volatile prices.

11.4.3 Neglected Constraints

It can be concluded from Chapter 10 that a dispatch is sub–optimal if there are insufficient primal constraints to explain why an observed dispatch could not be made more optimal. Consider therefore, an optimal dispatch of the South Island National

[3] In a pq–type spot market, generator voltages are fixed by applying Equation 11.1 to all generator nodes. Hence, all generator nodes become PQG nodes with voltage constraints, and reactive power is still the independent resource.

Grid. It follows that the marginal prices calculated by the pq pricing model will reflect a sub–optimal dispatch if any binding constraints are neglected or overlooked, even though the dispatch is optimal. The likelihood of the Clearing Manager overlooking binding constraints will be dependent on the final complexity of the model used for this South Island spot market.

These marginal prices include penalties, signalling to the market that the dispatch of real and reactive power was sub–optimal. These penalties can take the form of the 'Best Compromise' pricing proposed by Ring [1995], or the slack variables of generation constraints (Section 10.4.2). Either way, market participants will be penalised inappropriately if constraints are neglected.

11.5 CONCLUSIONS

Certain physical aspects pertaining to the dispatch of reactive power in the South Island National Grid have been presented. Specifically, it has been shown that it is physically possible to dispatch reactive power from most power station blocks as an independent resource. Consequently, any spot market for the South Island National Grid must be a pq–type spot market (or at least a mixed pq/PvG spot market).

In this market, both real and reactive power are independently and economically (i.e. optimally) dispatched. Hence, a pq–type OPF must be used to model this optimisation process. Furthermore, a pq pricing model must be used when calculating *ex post* marginal prices for the instant in time when the dispatch was optimised. At every other point in time, each extra unit of real or reactive power has been shown to be dispatched in an unconstrained, sub–optimal PvG–type manner, under certain conditions. Respecting these conditions, the PvG pricing model can therefore be used to calculate the marginal prices of supplying another unit of power from different load–following generators.

Fixed and variable generator voltages are shown to have advantages and disadvantages by the way they affect the optimal power system dispatch, and ultimately the way in which marginal prices behave.

Chapter 12

CONCLUSIONS AND FUTURE WORK

12.1 INTRODUCTION

Recent literature indicates that spot pricing of reactive power is beginning to receive serious consideration as a method of paying for the ancillary services of system security, power quality, and voltage support. Such a method has particular relevance for countries where real power spot markets are already operating, such as New Zealand. The objective of this thesis has been to increase the understanding of how the reactive power marginal prices in a notional spot market behave when both real and reactive power have non–zero unit generation costs at the generators. This has involved utilising the Dispatch Based Pricing theory derived by Ring [1995]. It has been used to identify that the power system marginal costs are the mechanisms influencing the behaviour of reactive power marginal prices under unconstrained and constrained power system conditions. It has also been used to delineate some of the implications of this marginal price behaviour in this notional spot market.

The research was broadly divided into two stages. Firstly, the Dispatch Based Pricing equations were applied to different types of spot markets. This included the validation of the software used to implement the equations (see Section 12.2). Secondly, the behaviour of reactive power marginal prices was investigated using the validated software (see Section 12.3).

12.2 DISPATCH BASED PRICING VALIDATION

Dispatch Based Pricing, as proposed by Read and Ring [1995d], was used to provide the framework for describing the behaviour of the reactive power marginal prices of this notional spot market. To calculate marginal prices, Dispatch Based Pricing models utilise optimal power flow technology. However, Read and Ring used power–flow node classifications (i.e. PV and PQ) when identifying the node–type to which each Dispatch Based Pricing equation was to be applied. They also used these power–flow classifications in their non–linear optimal power flow (OPF) formulation. Their Dispatch Based Pricing model was derived from this OPF formulation.

The use of power–flow nomenclature makes it unclear as to how the power system resources behave in an OPF context. For example, P varies at every generator node because an OPF optimises the real power injection at these nodes. Therefore, to class an OPF generator node as a PV node is misleading because P is not really fixed or specified, as implied by the 'PV' classification. Therefore, Read and Ring's non–linear OPF formulation and Dispatch Based Pricing model have been redefined (see Chapters 4 and 5) with respect to a new OPF classification system (defined in Chapter 3).

The new OPF nomenclature classifies power system nodes with respect to their fixed and control variables, as defined by the OPF algorithm. The result has been the definition of two types of OPF algorithm, identified by the classifications of the generator nodes. The OPF node classification system has revealed that the formulations of Read and Ring's redefined OPF (called pq/PvG OPF) is the combination of these two types of OPF. They are:

- a pq–type OPF (i.e. pq–OPF of Chapter 7) where reactive power generation is dispatched as an independent resource at generator nodes,
- and a PvG–type OPF (i.e. PvG–OPF of Chapter 8) where reactive power generation is dispatched as a dependent resource at generator nodes.

The pq–OPF and PvG–OPF formulations have been used to derive two new Dispatch Based Pricing models, the 'pq pricing model' in Chapter 7 and the 'PvG pricing model' in Chapter 8. When combined, the equations of these two models form the redefined version of Read and Ring's Dispatch Based Pricing model (called the pq/PvG Dispatch Based Pricing model).

A piece of software called NODAL2 has been introduced in Chapter 6. This software was developed by Transpower New Zealand Ltd to implement Read and Ring's Dispatch Based Pricing model (and hence the pq/PvG Dispatch Based Pricing model). It has been demonstrated that NODAL2 also implements the pq pricing model and the PvG pricing model. This is because all the equations required for these models are contained in the pq/PvG Dispatch Based Pricing model.

OPF software (called QOPF) has been developed in conjunction with Cornell University to implement the pq–OPF formulation. QOPF is distinct in that it is capable of including reactive power generation cost functions in it objective function. The pq–type marginal prices generated by QOPF have been used as a benchmark against which to compare the NODAL2 marginal prices. This comparison has demonstrated that the NODAL2 source code correctly implements the pq pricing model equations (except when the power system is thermally constrained). It has also been established that all terms in the pq pricing equations correctly describe the components of the real and reactive power marginal prices in a pq–type spot market. It must be noted that using QOPF as a benchmark program has provided valuable insight into the optimising behaviour of the OPF algorithm (see Chapter 10). Also, insight has been gained into the use of the OPF as a spot pricing tool for both optimal and sub-optimal dispatches.

Due to time restrictions, PvG–type OPF software that includes reactive power generation cost functions in the objective function has not been developed. This OPF would have been the implementation of the PvG–OPF. However, inductive reasoning has been employed to establish reasonable confidence that the equation–terms of the PvG pricing model correctly describe the marginal prices of real and reactive power in a PvG–type spot market. It has been verified that the NODAL2 source code correctly implements parts of the PvG pricing model equations. However, the source code implementing the remainder of the PvG pricing model can only be validated once a PvG–type OPF has been developed.

The verification experiments for the PvG pricing model indicated that the pq–OPF and the PvG–OPF are different, returning different optimal dispatches for the same set of input data. Furthermore, the pq pricing model and the PvG pricing model have been demonstrated to calculate different real and reactive power marginal price–sets for the same observed dispatch. The conclusion drawn from these results is that the optimisation processes of these two OPF's are distinctly different. That is, these OPF's model two types of spot market with distinctly different optimisation processes, which accordingly produce different real and reactive power marginal prices. The definitions of these two OPF types have been used to define two types of spot market: a pq–type spot market where reactive power is dispatched as an independent resource, and a PvG–type spot market where reactive power is dispatched as a dependent resource. Consequently, the pq pricing model and the PvG pricing model must only be used calculate *ex post* marginal prices for dispatches from the respective spot markets.

12.3 REACTIVE POWER PRICE BEHAVIOUR

To facilitate the investigation into the behaviour of reactive power marginal prices in this thesis, a generic spot market was assumed. In this spot market, the social welfare function was assumed to be the total cost of real and reactive power generation, obtained by summing the real and reactive power generation cost functions of all the generators. The cost functions were assumed to be submitted by generating companies as offers in the 'Real–Time Physical Market' (described in Section 2.5.2.2). Real power and reactive power were assumed to be economically (i.e. optimally) and simultaneously dispatched by minimising this social welfare function. It has been assumed that this optimisation process occurs only at the start of each trading period. This spot market definition was used when discussing pq–type spot markets and PvG–type spot markets.

Section 2.6 indicates the prevalent opinion in literature is, that reactive power marginal prices represent only a fraction of the costs of supplying ancillary services and are thus unusable as signals for encouraging the efficient and economic dispatch of reactive power. Dandachi *et al* [1996] demonstrated that reactive power generation cost functions can be used to cause the magnitudes of reactive power marginal prices to be significant. This thesis has confirmed that work. Further, it has demonstrated that these significant reactive power marginal prices have the potential to be used to fund ancillary services in both pq–type and PvG–type spot markets. It has also been shown that reactive power generation cost functions and significant reactive power marginal prices enable reactive power to be economically dispatched.

Reactive power marginal price behaviour has only been investigated for pq–type spot markets, since it has not been completely established that NODAL2 generates correct marginal prices for PvG–type spot markets. The behaviour of marginal prices was examined under four dispatch conditions: optimal, sub–optimal, unconstrained and constrained (see Chapters 9 and 10). The pq pricing model equations were used to identify the marginal cost components that are the dominant influence on the behaviour of reactive power marginal prices for each condition.

The locations of reactive power sources within the power system (such as capacitive network components) influence the amount QOPF is able to optimise the real and reactive power losses. Accordingly, the magnitudes of the costs of marginal losses are influenced by the locations of the reactive power sources. It has been shown that the costs of real power marginal losses and reactive power marginal losses dominate over any other cost components when an optimal dispatch is unconstrained. Hence, the magnitudes of the reactive power marginal prices are also influenced by the locations of reactive power sources (see Section 9.4).

The loss cost components also determine the trends in prices between connected nodes when the generator voltages are unrestricted. However, it was proved that if generator voltages were fixed, the price trends between nodes were determined instead by the marginal cost components of these fixed generator voltages.

At times, the power system's ability to supply another unit of reactive power may be severely restricted. These restrictions are the result of numerous binding constraints, such as fixed generator voltages. A scenario has been considered, of another constraint becoming binding when this already–constrained power system is optimally dispatched. It has been shown that if this new constraint results in a further significant restriction to the ability of the power system, the cost component of this new constraint will dominate over all other marginal cost components (Section 9.5).

When a dispatch is optimal, every generator whose real power or reactive power output is within its physical generation limits is identified as being marginal for real or reactive power. These generators are defined as being marginal by the fact that the marginal price at the node of each of these generators is equal to the unit generation cost of that generator. It has been demonstrated that a generator will be marginal irrespective of whether it is formulated as marginal or formulated as non–marginal in the Dispatch Based Pricing model. That is, the marginal price at the node of a marginal generator will always equal the unit generation cost of that generator when the dispatch is optimal. This is regardless of whether the generator is formulated as marginal or non–marginal. The implication is there is only one possible set of marginal prices when the dispatch is optimal. Note that the marginal generators respond to all changes in demand.

When a dispatch is sub–optimal, the marginal price of real or reactive power at a generator node is only equal to the unit generation cost if that generator has been formulated as marginal for real or reactive power. This implies that the outputs of all generators formulated as non–marginal must actually be constrained because the marginal prices do not equal the unit generation costs. This is irrespective of whether the formulated non–marginal generators are within their physical generation limits. Hence, only the formulated marginal generators respond to any marginal change in demand. This is because the prices for a sub–optimal dispatch infer that these generators are the only marginal generators.

Marginal prices for sub–optimal dispatch also imply a load–following scenario, where the formulated marginal generators are the load–following generators. It has been demonstrated that formulating different generators as marginal, results in different marginal price–sets. These price–sets communicate the costs of supplying real or reactive power from different load–following generators.

If the PvG pricing model is used to calculate *ex post* marginal prices for an optimal pq–type dispatch, these prices purport that the dispatch was sub–optimal. This is because pq–type and PvG–type OPF algorithms result in different optimal solutions.

The pq–type and PvG–type spot markets have been discussed in the context of the South Island section of the New Zealand national grid (Chapter 11). In this power system, the optimal dispatch of real and reactive power can be modelled using a pq–type OPF. In addition, the pq pricing model can be used to calculate *ex post* marginal prices for these optimal dispatches. Between each optimal dispatch, the dispatch of the next unit of real or reactive power can be described as an unconstrained sub–optimal PvG–type dispatch (this description was subject to certain conditions). The PvG pricing model can then be used to calculate real and reactive power marginal prices for this sub–optimal dispatch. These prices indicate the true cost to the power system of satisfying the demand for another unit of real or reactive power.

12.4 FUTURE WORK

This thesis has set the foundation for future work into the role of reactive power in a deregulated spot market. Initially a PvG–type OPF must be developed to finish the validation process started in Section 8.2.3. This OPF is required to establish total confidence in the equations of the PvG pricing model, and to ensure that the NODAL2 source code correctly implements these equations. To achieve this, the OPF must be able to optimally dispatch reactive power is a dependent resource. This can be achieved by taking the Matpower OPF algorithm (introduced in Chapter 6) and implementing a new gradient function in which Q_G is expressed in terms of the other system variables. This gradient function is represented by Equation 8.31.

This thesis has investigated the behaviour of reactive power marginal prices in a pq–type spot market. Once the PvG pricing model has been validated using a PvG–type OPF, it will also be possible to investigate the behaviour of reactive power marginal prices in a PvG–type spot market. A comparison can then be made of the merits and disadvantages of each type of spot market.

A more flexible spot market would be a mixed pq/PvG–type spot market, the marginal prices of which would be calculated by the pq/PvG Dispatch Based Pricing model of Chapter 5. Such a spot market would accommodate the PQG nodes of power stations where reactive power is dispatched as an independent resource. Also, it would accommodate voltage–constrained nodes of power stations such as Manapouri that should be modelled as PvG nodes where reactive power is dispatched as a dependent resource. A pq/PvG–type OPF must be developed to model the optimisation process of this spot market.

The extension of the spot market to allow reactive power cost functions at non–generator nodes should also be investigated. This will enable market participants owning equipment such as static VAr compensators or capacitor banks to trade solely in the reactive power sub–market.

In this thesis the behaviour of reactive power marginal prices has been described for dispatches where reactive power generation cost functions have been used. However, it has been left as future work to determine whether these cost functions result in price behaviour that generates signals encouraging the efficient and economic use of reactive power.

Reactive power generation cost functions have been used without definition in this thesis. Therefore future work must define the structure of these cost functions, so that generating companies can adequately charge for the reactive power ancillary services they provide.

Certain sectors of any power system experience lower power quality and/or more extreme voltage levels than other sectors. In these sectors, the needs of each customer determine whether such experiences are an issue to that customer. Therefore, real and reactive power marginal prices must be structured so that customers only pay for the level of power quality ancillary services they require (such as their required level of voltage support).

It has been shown that a pq–type OPF will attempt to force generators against reactive power generation limits when optimally dispatching real and reactive power. Work should be performed to determine whether power system security will be compromised by this kind of extreme economic dispatch of reactive power. If so, future work must determine a structure for reactive power generation cost functions and reactive power marginal prices that will incorporate charges reflecting the levels of power system security required by different customers.

Appendix A

SYMBOLS AND SYMBOLIC CONVENTIONS

A.1 INTRODUCTION

This Appendix contains all symbols and symbolic conventions used within this thesis. A number of the definitions are taken from either Ring [1995] or Read and Ring [1995d].

A.2 UNITS AND ACRONYMS

\$/MVAr	Dollars per megavar.
\$/MVArhr	Dollars per megavar–hour.
\$/MW	Dollars per megawatt.
\$/MWhr	Dollars per megawatt–hour.
\$/pu	Dollars per 1 pu voltage.
abbr.	Abbreviation.
AVR	Automatic voltage regulator.
c.f.	Compare with.
d.p.	Decimal places.
DBP	Dispatch Based Pricing.
ECNZ	Electricity Corporation of New Zealand.
EMCO	Electricity Market Company. Now called M–co, The Marketplace Company Ltd.
IEEE	Institute of Electrical and Electronic Engineering.
Matpower	A PvG–type OPF developed by PSERC. Incapable of accepting unit generation costs for reactive power.
Matpower^{pq}	A pq–type OPF, equivalent in ability to QOPF.
MMG	Multiple marginal generators.
n/a	Not applicable.
NGC	National Grid Company.
NODAL2	Dispatch Based Pricing software.
NZEM	New Zealand Electricity Market.
OLTC	On–load tap changing transformer.

OPF	Optimal power flow.
PSERC	Power System Engineering Research Centre.
QOPF	A pq–type OPF developed by PSERC, with the ability to accept unit generation costs for reactive power.
ref.	Reference or refer to.
SC–OPF	Security–constrained optimal power flow.

A.3 PRICING SYMBOLS

Symbols are sorted alphabetically with respect to the Greek alphabet, and then with respect to the English alphabet. Where required, symbols are sorted further with respect to subscripts, and then with respect to superscripts.

A.3.1 Greek Symbols

β_1	Arbitrary dual variable (i.e. shadow price or LaGrange multiplier).
β_{Pi}	Marginal price of real power demand at Node i.
β_{Qi}	Marginal price of reactive power demand at Node i.
β_{Vi}	Marginal price of voltage at Node i.
η_{Pk}	Shadow price for the average real power flow constraint on Branch k.
η_{Qk}	Shadow price for the average reactive power flow constraint on Branch k.
θ	Algorithm variable for voltage angle.
λ	An arbitrary LaGrange multiplier.
λ_P	Marginal price of real power at the reference node.
λ_Q	Marginal price of reactive power at the reference node.
μ_{Qn}	Shadow price for the constraint defining dependent reactive power injections at Node n.
μ_{Vn}	Shadow price for the constraint defining the dependent voltage magnitude at Node n.
υ^+_{Pi}, υ^-_{Pi}	Shadow prices for the upper and lower real power generation limits at Node i.
υ^+_{Qi}, υ^-_{Qi}	Shadow prices for the upper and lower reactive power generation limits at Node i.
υ^+_{Vi}, υ^-_{Vi}	Shadow prices for the upper and lower voltage magnitude limits at Node i.
χ_k	Shadow price for the thermal limit of Branch k.

A.3.2 English Symbols

A

a, b, c	Arbitrary polynomial constant coefficients.

B

B_k	Total shunt susceptance of Branch k.

C

C	Constraint curve.
c_{Pi}	Unit generation cost of real power at Node i when $c^+_{Pi} = c^-_{Pi}$.
c_{Qi}	Unit generation cost of reactive power at Node i when $c^+_{Qi} = c^-_{Qi}$.
c^+_{Pi}, c^-_{Pi}	Costs of next and last units of real power at Node i.
c^+_{Qi}, c^-_{Qi}	Costs of next and last units of reactive power at Node i.
$Cost(L_P)$	Cost of total real power losses.
$Cost(L_Q)$	Cost of total reactive power losses.

D

D	Indicates a demand for the associated resource (subscript).
Dem	Set of all non-generator nodes; includes, but not limited to, pqD nodes.

F

$f\,(cost)_{P,Q}$	An objective function for the total cost of real and reactive power generation.
$f(x,\, y)$	An arbitrary primal objective function, which is a function of x and y.

G

G	Indicates generation of the associated resource (subscript).
Gen	Set of generic generator nodes. Can include both PvG and PQG nodes.
$g(x,\, y)$	An arbitrary primal constraint equation, which is a function of x and y.

I

I	Set of all arbitrary variables (i.e. x_i).
i, j, m, n	Node indices (subscripts).

K

k	Branch k.
K	Set of all branches.

L

L_P	Total real power losses.
L_Q	Total reactive power losses.
L_{Qk}	Reactive power losses in Branch k.
L'_Q	Marginal reactive power loss, equivalent to $\frac{\Delta L_Q}{\Delta Q_i}$.
$\mathcal{L}(z)$	LaGrange equation.

M

$M_P\,(P_G)_i$	A polynomial function describing the real power generation cost at Node i (abbr. M_{Pi}).

$M_Q\left(Q_G\right)_i$ A polynomial function describing the reactive power generation cost at Node i (abbr. M_{Q_i}).

O

opt Indicates the value of the variable when the dispatch is optimal (subscript).

P

P Algorithm variable for real power injection.

$\tilde{p}$ Vector of fixed OPF parameters.

$\mathbf{P_D}$ Vector of real power demand at the nodes indicated by the superscript.

P_{Di} Real power demand at Node i.

P_{Di}^{set} Real power demand set by the customer at Node i.

P_{fm} Indicates the corresponding generator node is formulated as marginal for real power.

P_G Real power generation.

$\mathbf{P_G}$ Vector of real power generation at the nodes indicated by the superscript.

P_{Gi} Real power generation at Node i.

P_{Gi}^{+} Incremental increase in real power generation at Node i (this variable is always positive, i.e. ≥ 0).

P_{Gi}^{-} Incremental decrease in real power generation at Node i (this variable is always positive, i.e. ≥ 0).

P_{Gi}^{max}, P_{Gi}^{min} Upper and lower real power generation limits at Node i.

P_i Real power injection at Node i.

'P'$_i$ Real power generation at Node i, as reported by QOPF.

$\bar{P}_k$ Average flow of real power in Branch k.

P_{ki} Real power, flow in Branch k from Node i to Node j.

$P_{mismatch_i}$ Real power mismatch at Node i.

'Pmax'$_i$, 'Pmin'$_i$ QOPF fields for upper and lower limits on real power generation at Node i.

pq Set of pqD and PQG nodes (OPF).

PQ Set of reactive power controlled nodes (power–flow).

pqD Set of reactive power controlled demand nodes; p and q injections are fixed (OPF).

pQG Set of non–generator nodes where reactive power generation can be optimally dispatched.

PqG Set of generator nodes, non–marginal for reactive power.

PQG Set of reactive power controlled generator nodes; P and Q injections are unrestricted (OPF).

PQG$_{nm}$ Set of all non–marginal and formulated–non–marginal generator nodes; i.e. non–marginal for either real or reactive power.

PV	Set of voltage controlled nodes (power–flow).
PvG	Set of generator nodes with fixed generator voltages (OPF).
PX	Set of all nodes (as defined for this thesis).
PX	Set of all nodes excluding the swing bus (according to Ring [1995]).
PXS	Set of all nodes including the swing bus (according to Ring [1995]).
PY	Set of all nodes in a PvG–type OPF; i.e. (PvG ∪ pqD).

Q

Q	Algorithm variable for reactive power injection.
$\mathbf{Q_D}$	Vector of reactive power demand at the nodes indicated by the superscript.
Q_{Di}	Reactive power demand at Node i.
Q_{Di}^{set}	Reactive power demand set by the customer at Node i.
Q_{fm}	Indicates the corresponding generator node is formulated as marginal for reactive power.
$\mathbf{Q_G}$	Vector of reactive power generation at the nodes indicated by the superscript.
Q_{Gi}	Reactive power generation at Node i.
Q_{Gi}^{max}, Q_{Gi}^{min}	Upper and lower reactive power generation limits at Node i.
Q_{Gi}^{+}	Incremental increase in reactive power generation at Node i (this variable is always positive, i.e. ≥ 0).
Q_{Gi}^{-}	Incremental decrease in reactive power generation at Node i (this variable is always positive, i.e. ≥ 0).
Q_i	Reactive power injection at Node i.
$\bar{Q}_k$	Average flow of reactive power in Branch k.
Q_{ki}	Reactive power, flow in Branch k from Node i to Node j.
$\bar{Q}_{1-2}^{max}$	Maximum average flow of reactive power in Line $1-2$.
`'Qmax'`$_i$, `'Qmin'`$_i$	QOPF fields for upper and lower limits on reactive power generation at Node i.

R

R_k	Series resistance of Branch k.
R_{ki}	Shunt resistance of Branch k at Node i.
Ref	Indicates the reference node.

S

S	The swing bus.

T

T_k^{max}	Square of the thermal limit of Branch k.

U

$\tilde{u}$	Vector of OPF control variables.

V

V Algorithm variable for voltage magnitude.

$\mathbf{V}$ Vector of voltage magnitudes at the nodes indicated by the superscript.

V_b Indicates a node with a binding voltage constraint.

V_{diff} Difference between V_i^{uncnstr} and the constrained voltage for the same Node i.

V_i Voltage magnitude at Node i.

V_i^{uncnstr} Voltage magnitude at Node i when the power system is unconstrained.

V_i^{max}, V_i^{min} Upper and lower limits on voltage magnitude at Node i.

V_{ref} Reference voltage, used when generator voltages are not fixed. It does not necessarily have to be the voltage at the reference node.

V_i^{set} A fixed voltage set–point at Node i.

$V\theta$ Swing bus.

X

$\tilde{x}$ Vector of OPF state (i.e. unknown) variables.

x_1 Arbitrary primal variable.

X_k Series reactance of Branch k, at Node i.

X_{ki} Shunt reactive of Branch k, at Node i.

X/R Reactance to resistance ratio of each power system branch.

Y

Y The admittance matrix.

Z

z Arbitrary term used to represent all variables relevant to the context in which z appears.

A.4 MISCELLANEOUS MATHEMATICAL SYMBOLS

Δ Finite change.

$\forall$ For all.

$\in$ In.

j $\sqrt{-1}$.

\# Number of.

$*$ Observed value of the associated variable, around which the OPF formulation is linearised, in Chapter 5 (superscript).

$*$ Complex conjugate. Although identical to the 'observed value', they do not appear in the same context.

$\frac{\partial}{\partial}$, ∇ Partial derivative with respect to (or gradient of).

$\sim$ Tilde, denoting a vector.

$|z|$ Absolute value of z.

$\langle z \rangle$ z is only non–zero if the constraint corresponding to term z is binding.

Appendix B

LAGRANGE MULTIPLIERS

The optimal power flow algorithm used in this thesis makes use of LaGrange multipliers to aid the search for an optimal dispatch. In this appendix a brief definition of LaGrange multipliers is provided. This definition is used to show that LaGrange multipliers are the marginal prices used in an electricity market.

B.1 DEFINITION

An OPF minimises the value of an objective function, say $f(x, y)$, subject to a set of constraint equations such as $g(x, y)$. Assuming only one constraint for simplicity, an OPF attempts to minimise the value of $f(x, y)$ as (x, y) varies over the constraint curve C, where C is the graph of $g(x, y) = 0$ and is shown in Figure B.1 [Anton 1988].

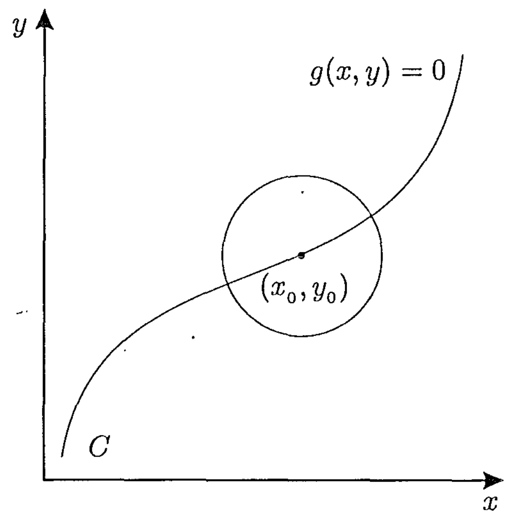


Figure B.1 Curve C is the graph of $g(x, y) = 0$, and has a 'constrained relative minimum' at (x_0, y_0).

Anton stated that $f(x, y)$ is said to be at a 'constrained relative minimum' at (x_0, y_0) if there is a circle centred at (x_0, y_0) such that

$$f(x_0, y_0) \leq f(x, y)$$

for all points (x, y) on C within the circle.

If $g(x, y) = 0$ represents the energy conservation equation then, all values of (x, y) on C represent valid power system dispatches, and (x_0, y_0) represents the optimal dispatch where the objective function (or cost function) has been minimised. Often this constrained relative minimum is also the absolute minimum, especially when $f(x, y)$ is a function of generation costs.

Given an optimal dispatch therefore, , Theorem B.1 holds.

Theorem B.1 *Let f and g be functions of two variables with continuous first partial derivatives on some open set containing the constraint curve $g(x, y) = 0$, and assume that $\nabla g \neq 0$ at any point on this curve. If f has a constrained relative extremum, then this extremum occurs at a point (x_0, y_0) on the constraint curve where the gradient vectors $\nabla f(x_0, y_0)$ and $\nabla g(x_0, y_0)$ are parallel, that is,*

$$\nabla f(x_0, y_0) = -\beta \nabla g(x_0, y_0)$$

for some number β.

The number β is called a LaGrange multiplier [Anton 1988, Section 16.10]. For each constraint there is an associated LaGrange multiplier (or shadow price, in economic terms).

B.2 USING LAGRANGE MULTIPLIERS AS MARGINAL PRICES

The following discussion focuses on only one power system variable and one constraint. Including all variables and constraints from Equations D.2 to D.17 provides no advantage in demonstrating that LaGrange multipliers are the marginal prices of a power system.

Let the objective function equal the cost of reactive power generation at a generator Node i:

$$f(Q_{Gi}) = c^{+}_{Qi} Q_{Gi}$$

c^{+}_{Qi} is the cost of the next unit of reactive power generation. Equation D.16 says that the load at Node i must equal some value set by the customer:

$$Q_{Di} = Q^{set}_{Di} (= Q^{*}_{Di})$$

Hence, the constraint $g(x, y)$ becomes:

$$g(Q_{Di}) = Q_{Di} - Q_{Di}^{set} = 0 \tag{B.1}$$

At a generator node, demand is equal to negative generation:

$$Q_{Gi} = -Q_{Di}$$

Therefore, the objective function becomes:

$$\begin{aligned} f(Q_{Di}) &= -f(Q_{Gi}) \\ &= -c_{Qi}^{+} Q_{Di} \end{aligned} \tag{B.2}$$

A LaGrange equation can be formed from the objective function and constraint equation [Wood and Wollenberg 1996]. For one variable, this is written as:

$$\mathcal{L}(Q_{Di}) = f(Q_{Di}) + \beta_{Qi} g(Q_{Di})$$

where the first partial derivatives are continuous. β_{Qi} is the LaGrange multiplier or shadow price on Equation D.16. By Theorem B.1 (and in Wood and Wollenberg [1996]) the gradient of this equation is zero when the dispatch is optimised by minimising the objective function:

$$\begin{aligned} \nabla\mathcal{L}(Q_{Di}) &= \nabla f(Q_{Di}) + \beta_{Qi} \nabla g(Q_{Di}) = 0 \\ &= \frac{\partial f(Q_{Di})}{\partial Q_{Di}} + \beta_{Qi} \frac{\partial g(Q_{Di})}{\partial Q_{Di}} \end{aligned} \tag{B.3}$$

This is known as the 'Gradient method' of solution.

If the LaGrange equation (B.3) is rearranged and the partial derivatives Equations B.1 and B.2 are substituted into it, the following is obtained:

$$\beta_{Qi} = \frac{\partial f(Q_{Di})}{\partial Q_{Di}} = c_{Qi}^{+}$$

This demonstrates the LaGrange multiplier is equal to the increment in the total generation cost (f), with respect to an incremental change in reactive power load at a generator node. Economically, the shadow price is equal to the unit generation cost (marginal price) of reactive power at the generator node.

In the full *ex post* OPF formulation of this thesis (Equations D.2 to D.17) there are 15 constraint equations, each with its own shadow price. If a primal equality constraint is not binding, the location of the constrained relative minimum in space is not changed by this primal constraint, and the corresponding dual shadow price is zero (Duality 5.5).

Wood and Wollenberg [1996, Section 13.2] provide a similar discussion using all power system variables and constraints.

The value of every multiplier is dependent on every other multiplier because multi–constraint, multi–variable LaGrange multiplier problems are simultaneous in nature. For example, the shadow price of reactive power demand (β_{Qi}) at a pq node will be dependent on the shadow prices of any binding constraints such as the shadow price of a voltage constraint (μ_{Vn}) at a node n. β_{Qi} is also influenced by all other non–zero shadow prices.

Appendix C

THE ORIGINAL DISPATCH BASED PRICING MODEL

C.1 INTRODUCTION

This appendix contains the original non–linear OPF formulation (Equations C.1 to C.13) and the consequential Dispatch Based Pricing model (Equations C.14 to C.22) proposed by Read and Ring (see Read and Ring [1995d], and Ring [1995]). The derivation process used by Read and Ring to derive their original Dispatch Based Pricing model from their original non–linear OPF formulation, is summarised in Chapters 3 to 5 and Appendix D.

Read and Ring used power–flow terminology to describe the behaviour of the different types of power system nodes. Thus, generator nodes are described as PV nodes and non–generator nodes are described as PQ nodes. Furthermore, the power–flow swing bus (denoted by 'S') is included in their non–linear OPF formulation and in their Dispatch Based Pricing equations.

THE ORIGINAL NON–LINEAR OPF FORMULATION

$$\underset{\mathbf{P}_{\mathrm{D}}^{\mathrm{PXS}},\mathbf{Q}_{\mathrm{D}}^{\mathrm{PXS}},\mathbf{P}_{\mathrm{G}}^{\mathrm{PXS}},\mathbf{Q}_{\mathrm{G}}^{\mathrm{PXS}},\mathbf{V}^{\mathrm{PXS}}}{\textbf{Minimise}} \qquad Costs(\mathbf{P}_{\mathrm{G}}^{\mathrm{PXS}},\mathbf{Q}_{\mathrm{G}}^{\mathrm{PQS}}) \tag{C.1}$$

subject to:

CONSERVATION OF POWER

$$\sum_{i\in \mathrm{PXS}} (P_{Gi} - P_{Di}) - L_P\left(\mathbf{P}_{\mathrm{G}}^{\mathrm{PX}} - \mathbf{P}_{\mathrm{D}}^{\mathrm{PX}}, \mathbf{Q}_{\mathrm{G}}^{\mathrm{PQ}} - \mathbf{Q}_{\mathrm{D}}^{\mathrm{PQ}}, \mathbf{V}^{\mathrm{PVS}}\right) = 0 \tag{C.2}$$

$$\sum_{i\in \mathrm{PXS}} (Q_{Gi} - Q_{Di}) - L_Q\left(\mathbf{P}_{\mathrm{G}}^{\mathrm{PX}} - \mathbf{P}_{\mathrm{D}}^{\mathrm{PX}}, \mathbf{Q}_{\mathrm{G}}^{\mathrm{PQ}} - \mathbf{Q}_{\mathrm{D}}^{\mathrm{PQ}}, \mathbf{V}^{\mathrm{PVS}}\right) = 0 \tag{C.3}$$

DEPENDENT REACTIVE POWER INJECTION AT PV NODES

$$-Q_n\left(\mathbf{P}_{\mathrm{G}}^{\mathrm{PX}} - \mathbf{P}_{\mathrm{D}}^{\mathrm{PX}}, \mathbf{Q}_{\mathrm{G}}^{\mathrm{PQ}} - \mathbf{Q}_{\mathrm{D}}^{\mathrm{PQ}}, \mathbf{V}^{\mathrm{PVS}}\right) + (Q_{Gn} - Q_{Dn}) = 0 \;\forall n \in \mathrm{PV} \tag{C.4}$$

DEPENDENT VOLTAGE AT PQ NODES

$$-V_n\left(\mathbf{P}_{\mathrm{G}}^{\mathrm{PX}} - \mathbf{P}_{\mathrm{D}}^{\mathrm{PX}}, \mathbf{Q}_{\mathrm{G}}^{\mathrm{PQ}} - \mathbf{Q}_{\mathrm{D}}^{\mathrm{PQ}}, \mathbf{V}^{\mathrm{PVS}}\right) + V_n = 0 \qquad \forall n \in \mathrm{PQ} \tag{C.5}$$

TRANSMISSION LINE FLOWS

$$-\bar{P}_k\left(\mathbf{P}_{\mathrm{G}}^{\mathrm{PX}} - \mathbf{P}_{\mathrm{D}}^{\mathrm{PX}}, \mathbf{Q}_{\mathrm{G}}^{\mathrm{PQ}} - \mathbf{Q}_{\mathrm{D}}^{\mathrm{PQ}}, \mathbf{V}^{\mathrm{PVS}}\right) + \bar{P}_k = 0 \qquad \forall k \in \mathrm{K} \tag{C.6}$$

$$-\bar{Q}_k\left(\mathbf{P}_{\mathrm{G}}^{\mathrm{PX}} - \mathbf{P}_{\mathrm{D}}^{\mathrm{PX}}, \mathbf{Q}_{\mathrm{G}}^{\mathrm{PQ}} - \mathbf{Q}_{\mathrm{D}}^{\mathrm{PQ}}, \mathbf{V}^{\mathrm{PVS}}\right) + \bar{Q}_k = 0 \qquad \forall k \in \mathrm{K} \tag{C.7}$$

REAL AND REACTIVE GENERATION AND VOLTAGE SETTINGS

$$P_{Gi}^{min} \le P_{Gi} \le P_{Gi}^{max} \qquad \forall i \in \mathrm{PXS} \tag{C.8}$$

$$Q_{Gi}^{min} \le Q_{Gi} \le Q_{Gi}^{max} \qquad \forall i \in \mathrm{PXS} \tag{C.9}$$

$$V_i^{min} \le V_i \le V_i^{max}. \qquad \forall i \in \mathrm{PXS} \tag{C.10}$$

REAL AND REACTIVE POWER LOAD SETTINGS

$$P_{Di} = P_{Di}^{set} \qquad \forall i \in \mathrm{PXS} \tag{C.11}$$

$$Q_{Di} = Q_{Di}^{set} \qquad \forall i \in \mathrm{PXS} \tag{C.12}$$

TRANSMISSION LINE THERMAL LIMITS

$$\bar{P}_k^2 + \bar{Q}_k^2 \le T_k^{max} \qquad \forall k \in \mathrm{K} \tag{C.13}$$

THE ORIGINAL DISPATCH BASED PRICING MODEL

A DUAL OBJECTIVE FUNCTION

(See Read and Ring [1995d])

$$\underset{\substack{\chi^{K},\, \upsilon_P^{+PXS},\, \upsilon_P^{-PXS},\, \upsilon_Q^{+PXS},\, \upsilon_Q^{-PXS},\, \upsilon_V^{+PXS},\, \upsilon_V^{-PXS} \geq 0 \\ \lambda_P,\, \lambda_Q,\, \mu_Q^{PV},\, \mu_V^{PQ},\, \beta_P^{PXS},\, \beta_Q^{PXS}}}{\text{MAXIMISE}} \tag{C.14}$$

subject to:

<u>*Marginal Price Equations*</u>

$$\beta_{Pi} = \lambda_P\left(1 - \frac{\partial L_P}{\partial P_i}\right) - \lambda_Q \frac{\partial L_Q}{\partial P_i} - \sum_{n \in PV} \langle \mu_{Qn} \rangle \left\langle \frac{\partial Q_n}{\partial P_i} \right\rangle - \sum_{n \in PQ} \langle \mu_{Vn} \rangle \left\langle \frac{\partial V_n}{\partial P_i} \right\rangle - 2\sum_{k \in K} \langle \chi_k \rangle \left(\left\langle \bar{P}_k^* \frac{\partial \bar{P}_k}{\partial P_i} \right\rangle + \left\langle \bar{Q}_k^* \frac{\partial \bar{Q}_k}{\partial P_i} \right\rangle \right) \quad \forall i \in PXS \tag{C.15}$$

$$\beta_{Qi} = -\lambda_P \frac{\partial L_P}{\partial Q_i} + \lambda_Q\left(1 - \frac{\partial L_Q}{\partial Q_i}\right) - \sum_{n \in PV} \langle \mu_{Qn} \rangle \left\langle \frac{\partial Q_n}{\partial Q_i} \right\rangle - \sum_{n \in PQ} \langle \mu_{Vn} \rangle \left\langle \frac{\partial V_n}{\partial Q_i} \right\rangle - 2\sum_{k \in K} \langle \chi_k \rangle \left(\left\langle \bar{P}_k^* \frac{\partial \bar{P}_k}{\partial Q_i} \right\rangle + \left\langle \bar{Q}_k^* \frac{\partial \bar{Q}_k}{\partial Q_i} \right\rangle \right) \quad \forall i \in PQ \tag{C.16}$$

$$\beta_{Vi} = -\lambda_P \frac{\partial L_P}{\partial V_i} - \lambda_Q \frac{\partial L_Q}{\partial V_i} - \sum_{n \in PV} \langle \mu_{Qn} \rangle \left\langle \frac{\partial Q_n}{\partial V_i} \right\rangle - \sum_{n \in PQ} \langle \mu_{Vn} \rangle \left\langle \frac{\partial V_n}{\partial V_i} \right\rangle - 2\sum_{k \in K} \langle \chi_k \rangle \left(\left\langle \bar{P}_k^* \frac{\partial \bar{P}_k}{\partial V_i} \right\rangle + \left\langle \bar{Q}_k^* \frac{\partial \bar{Q}_k}{\partial V_i} \right\rangle \right) \quad \forall i \in PVS \tag{C.17}$$

$$\mu_{Qn} = \beta_{Qn} - \lambda_Q \quad \forall n \in PVS \tag{C.18}$$

$$\mu_{Vn} = \beta_{Vn} \quad \forall n \in PQ \tag{C.19}$$

<u>*Price Bound Equations*</u>

$$c_{Pi}^{-} - \left\langle \upsilon_{Pi}^{-} \right\rangle \leq \beta_{Pi} \leq c_{Pi}^{+} + \left\langle \upsilon_{Pi}^{+} \right\rangle \quad \forall i \in PXS \tag{C.20}$$

$$c_{Qi}^{-} - \left\langle \upsilon_{Qi}^{-} \right\rangle \leq \beta_{Qi} \leq c_{Qi}^{+} + \left\langle \upsilon_{Qi}^{+} \right\rangle \quad \forall i \in PXS \tag{C.21}$$

$$\beta_{Vi} = \left\langle \upsilon_{Vi}^{-} \right\rangle - \left\langle \upsilon_{Vi}^{+} \right\rangle \quad \forall i \in PXS \tag{C.22}$$

Appendix D

THE PRIMAL AND DUAL LINEAR PROGRAMS

The aim of the Dispatch Based Pricing model is to calculate *ex post* marginal prices for an observed dispatch. Ring [1995] derived this model by linearising their original non–linear OPF equations (D.2–D.17), around an operating point to obtain a primal linear programming problem. The operating point is the observed (or existing) dispatch. The primal problem was used to form a dual linear program, from which the *ex post* marginal prices can be solved directly.

This appendix summarises the linearisation process detailed in Read and Ring [1995d], but with respect to the non–linear pq/PvG OPF formulation (Equations 4.1 to 4.13). After this, the primal problem and dual problem are presented.

D.1 LINEARISING THE PRIMAL OPF PROBLEM

The linearisation processes for the different pq/PvG OPF equations are summarised below.

D.1.1 The Objective Function

Theorem B.1 in Appendix B states that the first partial derivatives of the objective function must be continuous. In Dispatch Based Pricing analysis however, it is possible for the first derivatives of Equation 4.1 to be discontinuous. As an example, there is a discontinuity when the marginal generator is generating a full capacity, but where the demand for power is such that the next least expensive generator has not been dispatched. On one side of this discontinuity, the unit cost of generation is equal to the unit cost of the current marginal generator. On the other side, the unit generation cost is equal to the unit cost of the next marginal generator. Therefore, two linearised variables (c^+ and c^-) are required to describe the cost of the next and last units of generation. Hence, the objective function becomes:

$$\begin{aligned}
\mathrm{Cost}\left(\mathrm{P_G}, \mathrm{Q_G}\right) \Rightarrow\ & \mathrm{Cost}\left(\mathrm{P^*_G}, \mathrm{Q^*_G}\right) \\
& + \sum_{i \in \mathrm{PX}} \left(c^+_{Pi} P^+_{Gi} - c^-_{Pi} P^-_{Gi}\right) \\
& + \sum_{i \in \mathrm{PX}} \left(c^+_{Qi} Q^+_{Gi} - c^-_{Qi} Q^-_{Gi}\right)
\end{aligned}$$

In this objective function, the real and reactive power generation variables have also been linearised. For example, reactive power generation at Node i becomes:

$$Q_{Gi} = Q^{*}_{Gi} + Q^{+}_{Gi} - Q^{-}_{Gi} \tag{D.1}$$

where:

- Q^{*}_{Gi} indicates the observed reactive power generation, and
- Q^{+}_{Gi} *and* Q^{-}_{Gi} describe the next and last units of reactive power generation.

The '*' symbol indicates the values of the observed dispatch around which the pq/PvG OPF equations are linearised. This point of linearisation corresponds to point (x_0, y_o) in Appendix B.

D.1.2 The Constraints

The constraint equations are linearised using a first order Taylor expansion. For example, a general constraint equation $g(x)$ is linearised thus:

$$g(x) = g(x^{*}) + \sum_{i \in I} \frac{\partial g}{\partial x_i}\left(x_i - x^{*}_i\right)$$

Applying this to the reactive power energy conservation constraint (Equation 4.3) produces:

$$\begin{aligned}\sum_{i \in \mathrm{PX}} \left(Q^{*}_{Gi} + Q^{+}_{Gi} - Q^{-}_{Gi} - Q_{Di}\right) - L^{*}_{Q} \\ - \sum_{i \in \mathrm{PX}} \frac{\partial L_Q}{\partial P_i}\left(\left(P^{*}_{Gi} + P^{+}_{Gi} - P^{-}_{Gi} - P_{Di}\right) - \left(P^{*}_{Gi} - P^{*}_{Di}\right)\right) \\ - \sum_{i \in \mathrm{pq}} \frac{\partial L_Q}{\partial Q_i}\left(\left(Q^{*}_{Gi} + Q^{+}_{Gi} - Q^{-}_{Gi} - Q_{Di}\right) - \left(Q^{*}_{Gi} - Q^{*}_{Di}\right)\right) \\ - \sum_{i \in \mathrm{PvG}} \frac{\partial L_Q}{\partial V_i}\left(V_i - V^{*}_i\right) = 0\end{aligned}$$

Note that the variables P_i and Q_i have been linearised according to the method demonstrated by Equation D.1.

Inequality constraints are linearised in the same way. For example, the constraint describing the reactive power generation at Node i:

$$Q^{min}_{Gi} \leq Q_{Gi} \leq Q^{max}_{Gi}$$

linearises to:

$$-Q^{+}_{Gi} \geq -Q^{max}_{Gi} + Q^{*}_{Gi} \quad \text{and} \quad -Q^{-}_{Gi} \geq Q^{min}_{Gi} - Q^{*}_{Gi}.$$

Demand functions for power (e.g. Q_D) are already linear functions.

D.2 THE PRIMAL, LINEAR PROGRAMMING FORMULATION

Equations D.2 to D.17 are the linearised optimal power flow equations. They form the primal linear programming problem. The primal problem has been simplified. Common multiples have been cancelled out of the equations, and all constant terms have been transferred to the right hand side of the equations. All inequality constraints are arranged as 'greater than or equal to' inequalities. This format is called the 'canonical' form and simplifies the process of forming the dual. The example in Section 4.4.2 is also formulated in the canonical form.

The variable to the right of each primal constraint is the shadow price (i.e. LaGrange multiplier or dual variable) for that constraint.

D.3 THE DUAL, LINEAR PROGRAMMING FORMULATION

The process used to derive the dual, linear programming pricing equations from the primal pricing equations is the same as that used to obtain the example dual linear program in Section 5.2. Most linear programming texts, such as Bazaraa and Jarvis [1977], provide more information on linear programming duality theory. However, the following steps summarise the process of obtaining the dual equations. Each item below has a corresponding superscripted number. These numbers also appear in the primal and dual equations. They identify the part of the equation being described by the corresponding item.

- the right hand side of each primal constraint becomes the coefficient of the corresponding shadow price in the dual objective function^1;
- each primal objective function coefficient becomes the right hand side of the dual constraint that corresponds to the original primal variable^2;
- for each primal variable, the coefficients are multiplied by the shadow prices of the primal constraints in which they are found. These terms are then summed to produce a dual constraint. The shadow price (i.e. Lagrange multiplier) of this new dual constraint is the original primal variable^3.

The dual linear programming problem obtained by this process is described by Equations D.18 to D.28. Chapter 5 reports a simplified set of these equations (5.1–5.9), which is the final pq/PvG Dispatch Based Pricing model.

THE pq/PvG PRIMAL LINEAR PROGRAM

$$\underset{\substack{P_G^{+\text{PX}},\, P_G^{-\text{PX}},\, Q_G^{+\text{PX}},\, Q_G^{-\text{PX}} \ge 0 \\ P_D^{\text{PX}},\, Q_D^{\text{PX}},\, V^{\text{PX}}}}{\text{MINIMISE}} \quad \sum_{i\in PX}\left(c^+_{Pi}P^+_{Gi} - \boxed{c^-_{Pi}}^{2} P^-_{Gi}\right) + \sum_{i\in PX}\left(c^+_{Qi}Q^+_{Gi} - c^-_{Qi}Q^-_{Gi}\right) \tag{D.2}$$

subject to:

CONSERVATION OF POWER

$$\sum_{i\in\text{PX}}\left(P^+_{Gi} - P^-_{Gi} - P_{Di}\right) - \sum_{i\in\text{PX}}\frac{\partial L_P}{\partial P_i}\left(P^+_{Gi} - P^-_{Gi} - P_{Di}\right) - \sum_{i\in\text{pq}}\frac{\partial L_P}{\partial Q_i}\left(Q^+_{Gi} - Q^-_{Gi} - \boxed{Q_{Di}}^{3}\right) - \sum_{i\in\text{PvG}}\frac{\partial L_P}{\partial V_i}V_i$$

$$= \boxed{L^*_P + \sum_{i\in\text{PX}}\frac{\partial L_P}{\partial P_i}P^*_{Di} + \sum_{i\in\text{pq}}\frac{\partial L_P}{\partial Q_i}Q^*_{Di} - \sum_{i\in\text{PvG}}\frac{\partial L_P}{\partial V_i}V^*_i - \sum_{i\in\text{PX}}P^*_{Gi}}^{1} \quad : \lambda_P \tag{D.3}$$

$$\sum_{i\in\text{PX}}\left(Q^+_{Gi} - Q^-_{Gi} - \boxed{Q_{Di}}^{3}\right) - \sum_{i\in\text{PX}}\frac{\partial L_Q}{\partial P_i}\left(P^+_{Gi} - P^-_{Gi} - P_{Di}\right) - \sum_{i\in\text{pq}}\frac{\partial L_Q}{\partial Q_i}\left(Q^+_{Gi} - Q^-_{Gi} - \boxed{Q_{Di}}^{3}\right) - \sum_{i\in\text{PvG}}\frac{\partial L_Q}{\partial V_i}V_i$$

$$= L^*_Q + \sum_{i\in\text{PX}}\frac{\partial L_Q}{\partial P_i}P^*_{Di} + \sum_{i\in\text{pq}}\frac{\partial L_Q}{\partial Q_i}Q^*_{Di} - \sum_{i\in\text{PvG}}\frac{\partial L_Q}{\partial V_i}V^*_i - \sum_{i\in\text{PX}}Q^*_{Gi} \quad : \lambda_Q \tag{D.4}$$

DEPENDENT REACTIVE POWER INJECTION AT PVG NODES

$$-\sum_{i\in\text{PX}}\frac{\partial Q_n}{\partial P_i}\left(P^+_{Gi} - P^-_{Gi} - P_{Di}\right) - \sum_{i\in\text{pq}}\frac{\partial Q_n}{\partial Q_i}\left(Q^+_{Gi} - Q^-_{Gi} - \boxed{Q_{Di}}^{3}\right) - \sum_{i\in\text{PvG}}\frac{\partial Q_n}{\partial V_i}V_i + \left(Q^+_{Gn} - Q^-_{Gn} - Q_{Dn}\right)$$

$$= -Q^*_{Dn} + \sum_{i\in\text{PX}}\frac{\partial Q_n}{\partial P_i}P^*_{Di} + \sum_{i\in\text{pq}}\frac{\partial Q_n}{\partial Q_i}Q^*_{Di} - \sum_{i\in\text{PvG}}\frac{\partial Q_n}{\partial V_i}V^*_i \quad : \mu_{Qn} \quad \forall n \in \text{PvG} \tag{D.5}$$

Continued overleaf

THE pq/PvG PRIMAL LINEAR PROGRAM (CONTINUED)

DEPENDENT VOLTAGE AT PQ NODES

$$-\sum_{i\in \mathrm{PX}} \frac{\partial V_n}{\partial P_i}\left(P_{Gi}^{+} - P_{Gi}^{-} - P_{Di}\right) - \sum_{i\in \mathrm{pq}} \frac{\partial V_n}{\partial Q_i}\left(Q_{Gi}^{+} - Q_{Gi}^{-} - \boxed{Q_{Di}}^{3}\right) - \sum_{i\in \mathrm{PvG}} \frac{\partial V_n}{\partial V_i} V_i + V_n$$

$$= V_n^{*} + \sum_{i\in \mathrm{PX}} \frac{\partial V_n}{\partial P_i} P_{Di}^{*} + \sum_{i\in \mathrm{pq}} \frac{\partial V_n}{\partial Q_i} Q_{Di}^{*} - \sum_{i\in \mathrm{PvG}} \frac{\partial V_n}{\partial V_i} V_i^{*} \qquad : \mu_{Vn} \qquad \forall n \in \mathrm{pq} \qquad \text{(D.6)}$$

TRANSMISSION LINE FLOWS

$$-\sum_{i\in \mathrm{PX}} \frac{\partial \bar{P}_k}{\partial P_i}\left(P_{Gi}^{+} - P_{Gi}^{-} - P_{Di}\right) - \sum_{i\in \mathrm{pq}} \frac{\partial \bar{P}_k}{\partial Q_i}\left(Q_{Gi}^{+} - Q_{Gi}^{-} - \boxed{Q_{Di}}^{3}\right) - \sum_{i\in \mathrm{PvG}} \frac{\partial \bar{P}_k}{\partial V_i} V_i + \bar{P}_k$$

$$= \bar{P}_k^{*} + \sum_{i\in \mathrm{PX}} \frac{\partial \bar{P}_k}{\partial P_i} P_{Di}^{*} + \sum_{i\in \mathrm{pq}} \frac{\partial \bar{P}_k}{\partial Q_i} Q_{Di}^{*} - \sum_{i\in \mathrm{PvG}} \frac{\partial \bar{P}_k}{\partial V_i} V_i^{*} \qquad : \eta_{Pk} \qquad \forall k \in \mathrm{K} \qquad \text{(D.7)}$$

$$-\sum_{i\in \mathrm{PX}} \frac{\partial \bar{Q}_k}{\partial P_i}\left(P_{Gi}^{+} - P_{Gi}^{-} - P_{Di}\right) - \sum_{i\in \mathrm{pq}} \frac{\partial \bar{Q}_k}{\partial Q_i}\left(Q_{Gi}^{+} - Q_{Gi}^{-} - \boxed{Q_{Di}}^{3}\right) - \sum_{i\in \mathrm{PvG}} \frac{\partial \bar{Q}_k}{\partial V_i} V_i + \bar{Q}_k$$

$$= \bar{Q}_k^{*} + \sum_{i\in \mathrm{PX}} \frac{\partial \bar{Q}_k}{\partial P_i} P_{Di}^{*} + \sum_{i\in \mathrm{pq}} \frac{\partial \bar{Q}_k}{\partial Q_i} Q_{Di}^{*} - \sum_{i\in \mathrm{PvG}} \frac{\partial \bar{Q}_k}{\partial V_i} V_i^{*} \qquad : \eta_{Qk} \qquad \forall k \in \mathrm{K} \qquad \text{(D.8)}$$

Continued overleaf

THE pq/PvG PRIMAL LINEAR PROGRAM (CONTINUED)

REAL AND REACTIVE GENERATION AND VOLTAGE SETTINGS

$$-P^{+}_{Gi} \geq -P^{max}_{Gi} + P^{*}_{Gi} \quad : \upsilon^{+}_{Pi} \quad \forall i \in \mathrm{PX} \tag{D.9}$$

$$-P^{-}_{Gi} \geq P^{min}_{Gi} - P^{*}_{Gi} \quad : \upsilon^{-}_{Pi} \quad \forall i \in \mathrm{PX} \tag{D.10}$$

$$-Q^{+}_{Gi} \geq -Q^{max}_{Gi} + Q^{*}_{Gi} \quad : \upsilon^{+}_{Qi} \quad \forall i \in \mathrm{PX} \tag{D.11}$$

$$-Q^{-}_{Gi} \geq Q^{min}_{Gi} - Q^{*}_{Gi} \quad : \upsilon^{-}_{Qi} \quad \forall i \in \mathrm{PX} \tag{D.12}$$

$$-V_{i} \geq -V^{max}_{i} \quad : \upsilon^{+}_{Vi} \quad \forall i \in \mathrm{PX} \tag{D.13}$$

$$V_{i} \geq V^{min}_{i} \quad : \upsilon^{-}_{Vi} \quad \forall i \in \mathrm{PX} \tag{D.14}$$

REAL AND REACTIVE POWER LOAD SETTINGS

$$P_{Di} = P^{*}_{Di} \quad : \beta_{Pi} \quad \forall i \in \mathrm{PX} \tag{D.15}$$

$$\boxed{Q_{Di}}^{3} = Q^{*}_{Di} \quad : \beta_{Qi} \quad \forall i \in \mathrm{PX} \tag{D.16}$$

TRANSMISSION LINE THERMAL LIMITS

$$-2\bar{P}^{*}_{k}\bar{P}_{k} - 2\bar{Q}^{*}_{k}\bar{Q}_{k} \geq -T^{max}_{k} - \bar{P}^{*2}_{k} - \bar{Q}^{*2}_{k} \quad : \chi_{k} \quad \forall k \in \mathrm{K} \tag{D.17}$$

THE OBJECTIVE FUNCTION OF THE pq/PvG DUAL LINEAR PROGRAM

$$
\begin{aligned}
&\underset{\substack{\chi^{\mathrm{K}},\, \upsilon_P^{+\mathrm{PX}},\, \upsilon_P^{-\mathrm{PX}},\, \upsilon_Q^{+\mathrm{PX}},\, \upsilon_Q^{-\mathrm{PX}},\, \upsilon_V^{+\mathrm{PX}},\, \upsilon_V^{-\mathrm{PX}} \geq 0 \\ \lambda_P,\, \lambda_Q,\, \beta_P^{\mathrm{PX}},\, \beta_Q^{\mathrm{PX}},\, \mu_Q^{\mathrm{PvG}},\, \mu_V^{\mathrm{pq}},\, \eta_P^{\mathrm{K}},\, \eta_Q^{\mathrm{K}}}}{\text{MAXIMISE}} \quad \lambda_P \left(\boxed{L_P^* + \sum_{i\in \mathrm{PX}} \frac{\partial L_P}{\partial P_i} P_{Di}^* + \sum_{i\in \mathrm{pq}} \frac{\partial L_P}{\partial Q_i} Q_{Di}^* - \sum_{i\in \mathrm{PvG}} \frac{\partial L_P}{\partial V_i} V_i^* - \sum_{i\in \mathrm{PX}} P_{Gi}^*}^{\,1} \right) \\
&+\lambda_Q \left(L_Q^* + \sum_{i\in \mathrm{PX}} \frac{\partial L_Q}{\partial P_i} P_{Di}^* + \sum_{i\in \mathrm{pq}} \frac{\partial L_Q}{\partial Q_i} Q_{Di}^* - \sum_{i\in \mathrm{PvG}} \frac{\partial L_Q}{\partial V_i} V_i^* - \sum_{i\in \mathrm{PX}} Q_{Gi}^* \right) \\
&+ \sum_{n\in \mathrm{PvG}} \mu_{Qn} \left(-Q_{Dn}^* + \sum_{i\in \mathrm{PX}} \frac{\partial Q_n}{\partial P_i} P_{Di}^* + \sum_{i\in \mathrm{pq}} \frac{\partial Q_n}{\partial Q_i} Q_{Di}^* - \sum_{i\in \mathrm{PvG}} \frac{\partial Q_n}{\partial V_i} V_i^* \right) \\
&+ \sum_{n\in \mathrm{pq}} \mu_{Vn} \left(V_n^* + \sum_{i\in \mathrm{PX}} \frac{\partial V_n}{\partial P_i} P_{Di}^* + \sum_{i\in \mathrm{pq}} \frac{\partial V_n}{\partial Q_i} Q_{Di}^* - \sum_{i\in \mathrm{PvG}} \frac{\partial V_n}{\partial V_i} V_i^* \right) \\
&+ \sum_{k\in \mathrm{K}} \eta_{Pk} \left(\bar{P}_k^* + \sum_{i\in \mathrm{PX}} \frac{\partial \bar{P}_k}{\partial P_i} P_{Di}^* + \sum_{i\in \mathrm{pq}} \frac{\partial \bar{P}_k}{\partial Q_i} Q_{Di}^* - \sum_{i\in \mathrm{PvG}} \frac{\partial \bar{P}_k}{\partial V_i} V_i^* \right) \\
&+ \sum_{k\in \mathrm{K}} \eta_{Qk} \left(\bar{Q}_k^* + \sum_{i\in \mathrm{PX}} \frac{\partial \bar{Q}_k}{\partial P_i} P_{Di}^* + \sum_{i\in \mathrm{pq}} \frac{\partial \bar{Q}_k}{\partial Q_i} Q_{Di}^* - \sum_{i\in \mathrm{PvG}} \frac{\partial \bar{Q}_k}{\partial V_i} V_i^* \right) \\
&+ \sum_{i\in \mathrm{PX}} \left(\overset{+}{\upsilon}_{Pi} \left(-P_{Gi}^{max} + P_{Gi}^* \right) + \overset{-}{\upsilon}_{Pi} \left(P_{Gi}^{min} - P_{Gi}^* \right) \right) \\
&+ \sum_{i\in \mathrm{PX}} \left(\overset{+}{\upsilon}_{Qi} \left(-Q_{Gi}^{max} + Q_{Gi}^* \right) + \overset{-}{\upsilon}_{Qi} \left(Q_{Gi}^{min} - Q_{Gi}^* \right) \right) + \sum_{i\in \mathrm{PX}} \left(-\overset{+}{\upsilon}_{Vi} V_i^{max} + \overset{-}{\upsilon}_{Vi} V_i^{min} \right) \\
&+ \sum_{i\in \mathrm{PX}} \beta_{Pi} P_{Di}^* + \sum_{i\in \mathrm{PX}} \beta_{Qi} Q_{Di}^* + \sum_{k\in \mathrm{K}} \chi_k \left(-T_k^{max} - \bar{P}_k^{*2} - \bar{Q}_k^{*2} \right)
\end{aligned}
\tag{D.18}
$$

Continued overleaf

The Constraints of the pq/PvG Dual Linear Program

subject to:

Marginal Prices Defined by OPF Demand Terms

$$-\lambda_P\left(1-\frac{\partial L_P}{\partial P_i}\right)+\lambda_Q\frac{\partial L_Q}{\partial P_i}+\sum_{n\in \mathrm{PvG}}\left(\mu_{Qn}\frac{\partial Q_n}{\partial P_i}\right)+\sum_{n\in \mathrm{pq}}\left(\mu_{Vn}\frac{\partial V_n}{\partial P_i}\right)+\sum_{k\in \mathrm{K}}\left(\eta_{Pk}\frac{\partial \bar{P}_k}{\partial P_i}+\eta_{Qk}\frac{\partial \bar{Q}_k}{\partial P_i}\right)+\beta_{Pi}=0 \qquad : P_{Di} \quad \forall i\in \mathrm{PX} \qquad (D.19)$$

$$\lambda_P\frac{\partial L_P}{\partial Q_i}-\lambda_Q\left(1-\frac{\partial L_Q}{\partial Q_i}\right)+\sum_{n\in \mathrm{PvG}}\left(\mu_{Qn}\frac{\partial Q_n}{\partial Q_i}\right)+\sum_{n\in \mathrm{pq}}\left(\mu_{Vn}\frac{\partial V_n}{\partial Q_i}\right)+\sum_{k\in \mathrm{K}}\left(\eta_{Pk}\frac{\partial \bar{P}_k}{\partial Q_i}+\eta_{Qk}\frac{\partial \bar{Q}_k}{\partial Q_i}\right)+\beta_{Qi}=0 \qquad : \boxed{Q_{Di}}^{3} \quad \forall i\in \mathrm{pq} \qquad (D.20)$$

$$-\lambda_Q-\mu_{Qn}+\beta_{Qn}=0 \qquad : Q_{Di} \quad \forall i\in \mathrm{PvG} \qquad (D.21)$$

Marginal Price Constraints Set by Unit Generation Costs

$$\pm\lambda_P\left(1-\frac{\partial L_P}{\partial P_i}\right)\mp\lambda_Q\frac{\partial L_Q}{\partial P_i}\mp\sum_{n\in \mathrm{PvG}}\left(\mu_{Qn}\frac{\partial Q_n}{\partial P_i}\right)\mp\sum_{n\in \mathrm{pq}}\left(\mu_{Vn}\frac{\partial V_n}{\partial P_i}\right)\mp\sum_{k\in \mathrm{K}}\left(\eta_{Pk}\frac{\partial \bar{P}_k}{\partial P_i}+\eta_{Qk}\frac{\partial \bar{Q}_k}{\partial P_i}\right)-v^{\pm}_{Pi}\leq \boxed{\pm c^{\pm}_{Pi}}^{2} \qquad : P^{\pm}_{Gi} \quad \forall i\in \mathrm{PX} \qquad (D.22)$$

$$\mp\lambda_P\frac{\partial L_P}{\partial Q_i}\pm\lambda_Q\left(1-\frac{\partial L_Q}{\partial Q_i}\right)\mp\sum_{n\in \mathrm{PvG}}\left(\mu_{Qn}\frac{\partial Q_n}{\partial Q_i}\right)\mp\sum_{n\in \mathrm{pq}}\left(\mu_{Vn}\frac{\partial V_n}{\partial Q_i}\right)\mp\sum_{k\in \mathrm{K}}\left(\eta_{Pk}\frac{\partial \bar{P}_k}{\partial Q_i}+\eta_{Qk}\frac{\partial \bar{Q}_k}{\partial Q_i}\right)-v^{\pm}_{Qi}\leq \pm c^{\pm}_{Qi} \qquad : Q^{\pm}_{Gi} \quad \forall i\in \mathrm{pq} \qquad (D.23)$$

$$\pm\lambda_Q\pm\mu_{Qn}-v^{\pm}_{Qn}\leq \pm c^{\pm}_{Qi} \qquad : Q^{\pm}_{Gn} \quad \forall n\in \mathrm{PvG} \qquad (D.24)$$

Continued overleaf

THE CONSTRAINTS OF THE pq/PvG DUAL LINEAR PROGRAM (CONTINUED)

VOLTAGE CONSTRAINT COST RELATIONSHIPS

$$-\lambda_P \frac{\partial L_P}{\partial V_i} - \lambda_Q \frac{\partial L_Q}{\partial V_i} - \sum_{n\in \mathrm{PvG}}\left(\mu_{Qn}\frac{\partial Q_n}{\partial V_i}\right) - \sum_{n\in \mathrm{pq}}\left(\mu_{Vn}\frac{\partial V_n}{\partial V_i}\right) - \sum_{k\in \mathrm{K}}\left(\eta_{Pk}\frac{\partial \bar{P}_k}{\partial V_i} + \eta_{Qk}\frac{\partial \bar{Q}_k}{\partial V_i}\right) - \upsilon^{+}_{Vi} + \upsilon^{-}_{Vi} = 0 \quad : V_i \quad \forall i \in \mathrm{PvG} \tag{D.25}$$

$$\mu_{Vn} - \upsilon^{+}_{Vn} + \upsilon^{-}_{Vn} = 0 \quad : V_n \quad \forall n \in \mathrm{pq} \tag{D.26}$$

PRICING RELATIONSHIPS
FOR THE OPF TRANSMISSION LINE CONSTRAINTS

$$\eta_{Pk} - 2\bar{P}^{*}_{k}\chi_k = 0 \quad : \bar{P}_k \quad \forall k \in \mathrm{K} \tag{D.27}$$

$$\eta_{Qk} - 2\bar{Q}^{*}_{k}\chi_k = 0 \quad : \bar{Q}_k \quad \forall k \in \mathrm{K} \tag{D.28}$$

Appendix E

TEST POWER SYSTEMS

E.1 DESCRIPTION

This appendix contains the schematics and raw data describing the three test power systems used for all experiments within this thesis. The test power systems are:

- the 9–bus power system from Cornell University. This accompanied the Matpower software (Figure E.1);
- the IEEE 14–bus power system representing the American electric power system (Figure E.2);
- the IEEE 30–bus power system representing the American electric power system (Figure E.3).

For the 14–bus and 30–bus power system, the raw data has been converted from the IEEE Common Data Format to the modified PSS/E data format required by Matpower and QOPF [CDF.1973]. The actual numerical values of the data describe a power–flow dispatch rather than an optimal power flow dispatch. Additional information such as reactive power generation limits and cost function data was added during conversion.

E.2 NOMENCLATURE

The Dispatch Based Pricing nomenclature has been used for all discussions within the thesis. Therefore, the Dispatch Based Pricing nomenclature is used to indicate any changes that have been made to the power system data, so as to avoid confusion during discussions. However, this nomenclature is easily translated. For example:

- P_{D4} is the 'Pd' (real power load) field for Bus 4;
- Q_{G2}^{max} is the 'Qmax' (maximum reactive power generation) field for Generator 2;
- V_1^{min} is the 'Vmin' (minimum voltage limit) field for Bus 1.

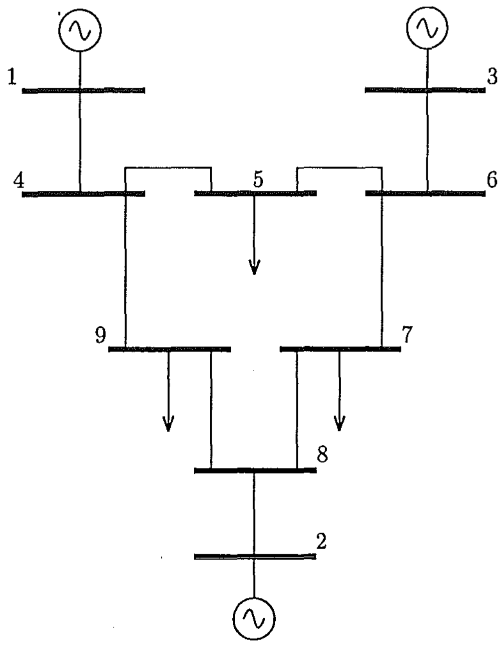


Figure E.1 Schematic of the Cornell University 9–bus Power System

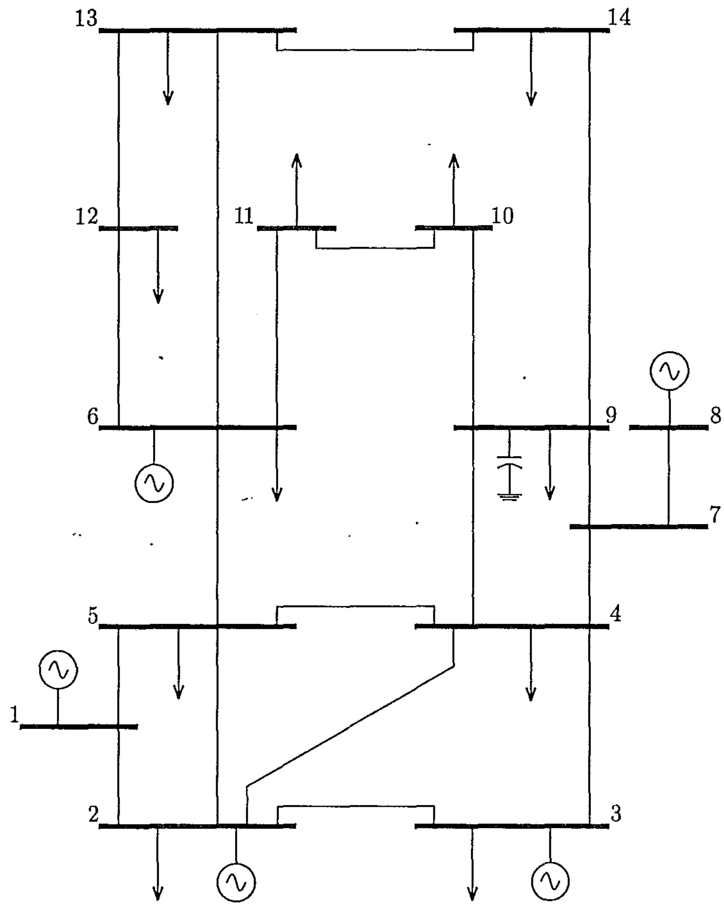


Figure E.2 Schematic of the IEEE 14–bus Power System

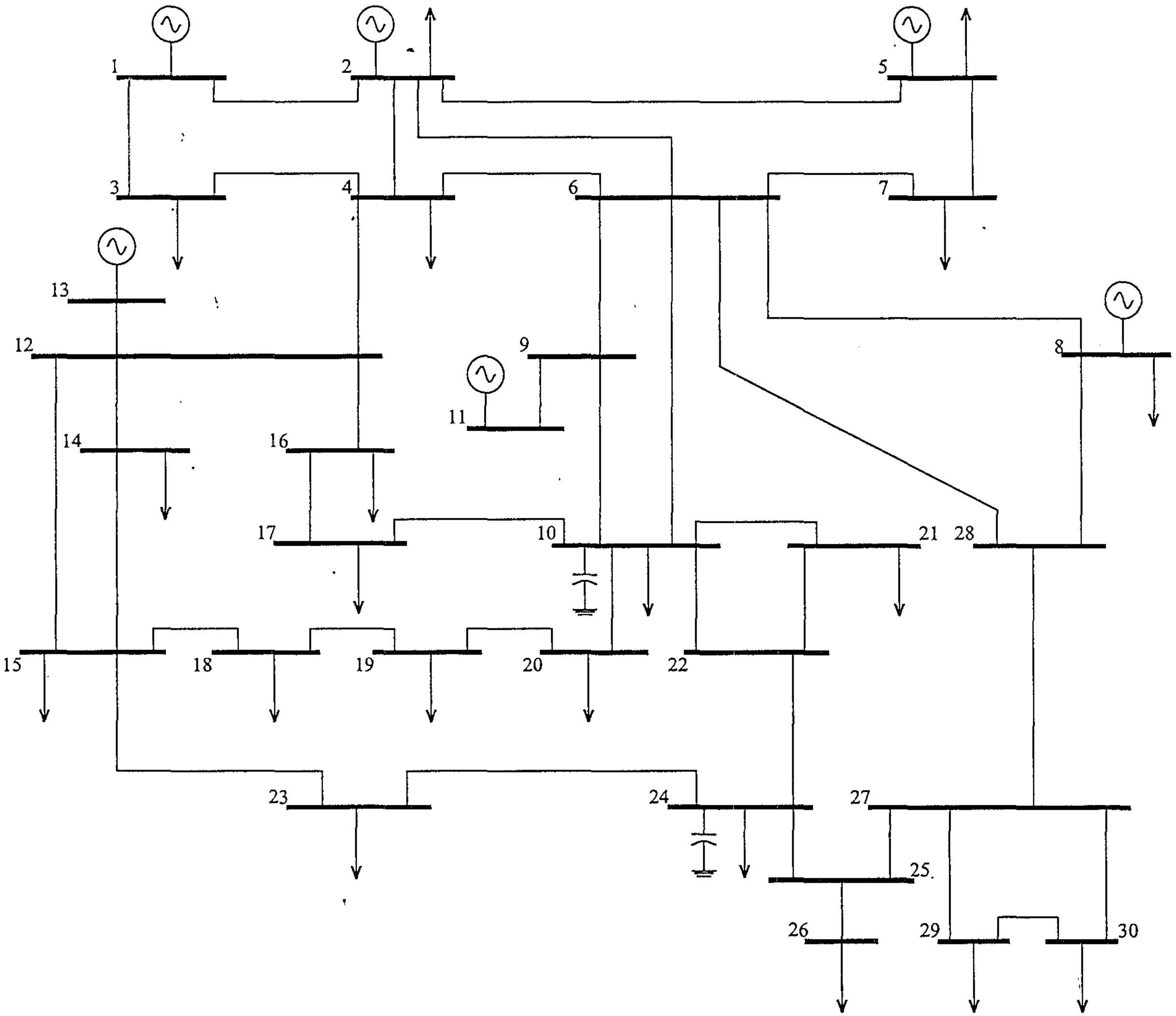


Figure E.3 Schematic of the IEEE 30–bus Power System

```
function [baseMVA, bus, gen, branch, area, gencost] = case
%CASE    Defines the power flow data in a format similar to PTI.
%   [baseMVA, bus, gen, branch, area, gencost] = case
%   The format for the data is similar to PTI format except where noted.
%   An item marked with (+) indicates that it is included in this data
%   but is not part of the PTI format. An item marked with (-) is one that
%   is in the PTI format but is not included here.
%
%   Bus Data Format
%       1   bus number (1 to 29997)
%       2   bus type
%               PQ bus          = 1
%               PV bus          = 2
%               reference bus   = 3
%               isolated bus    = 4
%       3   Pd, real power demand (MW)
%       4   Qd, reactive power demand (MVAR)
%       5   Gs, shunt conductance (MW (demanded?) at V = 1.0 p.u.)
%       6   Bs, shunt susceptance (MVAR (injected?) at V = 1.0 p.u.)
%       7   area number, 1-100
%       8   Vm, voltage magnitude (p.u.)
%       9   Va, voltage angle (degrees)
%   (-)     (bus name)
```

```
%       10  baseKV, base voltage (kV)
%       11  zone, loss zone (1-999)
%   (+) 12  maxVm, maximum voltage magnitude (p.u.)
%   (+) 13  minVm, minimum voltage magnitude (p.u.)
%
%   Generator Data Format
%       1   bus number
%   (-)     (machine identifier, 0-9, A-Z)
%       2   Pg, real power output (MW)
%       3   Qg, reactive power output (MVAR)
%       4   Qmax, maximum reactive power output (MVAR)
%       5   Qmin, minimum reactive power output (MVAR)
%       6   Vg, voltage magnitude setpoint (p.u.)
%   (-)     (remote controlled bus index)
%       7   mBase, total MVA base of this machine, defaults to baseMVA
%   (-)     (machine impedance, p.u. on mBase)
%   (-)     (step up transformer impedance, p.u. on mBase)
%   (-)     (step up transformer off nominal turns ratio)
%       8   status, 1 - machine in service, 0 - machine out of service
%   (-)     (% of total VARS to come from this gen in order to hold V at
%               remote bus controlled by several generators)
%       9   Pmax, maximum real power output (MW)
%       10  Pmin, minimum real power output (MW)
%
%   Branch Data Format
```

```
%       1   f, from bus number
%       2   t, to bus number
%   (-)     (circuit identifier)
%       3   r, resistance (p.u.)
%       4   x, reactance (p.u.)
%       5   b, total line charging susceptance (p.u.)
%       6   rateA, MVA rating A (long term rating)
%       7   rateB, MVA rating B (short term rating)
%       8   rateC, MVA rating C (emergency rating)
%       9   ratio, transformer off nominal turns ratio
%       10  angle, transformer phase shift angle (degrees)
%   (-)     (Gf, shunt conductance at from bus p.u.)
%   (-)     (Bf, shunt susceptance at from bus p.u.)
%   (-)     (Gt, shunt conductance at to bus p.u.)
%   (-)     (Bt, shunt susceptance at to bus p.u.)
%       11  initial branch status, 1 - in service, 0 - out of service
%
% (+) Area Data Format
%       1   i, area number
%       2   price_ref_bus, reference bus for that area
%
% (+) Generator Cost Data Format
%       NOTE: If gen has n rows, then the first n rows of gencost contain
%       the cost for active power produced by the corresponding generators.
%       If gencost has 2*n rows then rows n+1 to 2*n contain the reactive
```

```
%       power costs in the same format.
%       1   model, 1 - piecewise linear, 2 - polynomial
%       2   startup, startup cost in US dollars
%       3   shutdown, shutdown cost in US dollars
%       4   n, number of cost parameters to follow
%       5 and following, cost data, piecewise linear data as:
%                   x0, y0, x1, y1, x2, y2, ...
%           and polynomial data as, e.g.:
%                   c2, c1, c0
%           where the polynomial is c0 + c1*P + c2*P^2
%
% << this file created [97-Aug-26 12:29:15] by PS::System version 1.3 >>

%%-----  Power Flow Data  -----%%
%% system MVA base
baseMVA = 100.0000;
%% bus data
%bus type  Pd      Qd       Gs      Bs     area  Vm      Va      baseKV zone   Vmax   Vmin
bus = [
     1     3     0.0     0.0     0.0     0.0     1     1.0000     0.0000     345.0000     1     1.0000     1.0000;
     2     2     0.0     0.0     0.0     0.0     1     1.0000     0.0000     345.0000     1     1.1000     0.9000;
     3     2     0.0     0.0     0.0     0.0     1     1.0000     0.0000     345.0000     1     1.1000     0.9000;
     4     1     0.0     0.0     0.0     0.0     1     1.0000     0.0000     345.0000     1     1.1000     0.9000;
     5     1     90.00   30.00   0.0     0.0     1     1.0000     0.0000     345.0000     1     1.1000     0.9000;
     6     1     0.0     0.0     0.0     0.0     1     1.0000     0.0000     345.0000     1     1.1000     0.9000;
```

```
    7     1     100.00 35.00    0.0      0.0      1      1.0000      0.0000      345.0000      1      1.1000      0.9000;
    8     1     0.0      0.0    0.0      0.0      1      1.0000      0.0000      345.0000      1      1.1000      0.9000;
    9     1     125.00 50.00    0.0      0.0      1      1.0000      0.0000      345.0000      1      1.1000      0.9000;
];
%% generator data
%bus  Pg        Qg       Qmax      Qmin    Vsp       base   status    Pmax     Pmin
gen = [
    1      0.0000      0.0000      300.0000      -300.0000      1.0000      100.0000      1      250.0000      10.0000;
    2    163.0000      0.0000      300.0000      -300.0000      1.0000      100.0000      1      300.0000      10.0000;
    3     85.0000      0.0000      300.0000      -300.0000      1.0000      100.0000      1      270.0000      10.0000;
];
%% branch data
%fbus tbus      r      x      b        ratea    rateb    ratec ratio    angle   status
branch = [
    1     4     0.0000     0.0576     0.0000     250.0000     250.0000     250.0000     1.0000     0.0000     1;
    4     5     0.0170     0.0920     0.1580     250.0000     250.0000     250.0000     1.0000     0.0000     1;
    5     6     0.0390     0.1700     0.3580     150.0000     150.0000     150.0000     1.0000     0.0000     1;
    3     6     0.0000     0.0586     0.0000     300.0000     300.0000     300.0000     1.0000     0.0000     1;
    6     7     0.0119     0.1008     0.2090     150.0000     150.0000     150.0000     1.0000     0.0000     1;
    7     8     0.0085     0.0720     0.1490     250.0000     250.0000     250.0000     1.0000     0.0000     1;
    8     2     0.0000     0.0625     0.0000     250.0000     250.0000     250.0000     1.0000     0.0000     1;
    8     9     0.0320     0.1610     0.3060     250.0000     250.0000     250.0000     1.0000     0.0000     1;
    9     4     0.0100     0.0850     0.1760     250.0000     250.0000     250.0000     1.0000     0.0000     1;
];
```

```
%%-----  OPF Data  -----%%
%% area data
area = [
     1     2;
];
%% generator cost data
gencost = [
% Real
   2     0.00        0.00         3     0.0  10.7000     0.0;
   2     0.00        0.00         3     0.0  10.9000     0.0;
   2     0.00        0.00         3     0.0  11.0000     0.0;
%Reactive
   2     0.00        0.00         3     0.0  1.07000     0.0;
   2     0.00        0.00         3     0.0  1.09000     0.0;
   2     0.00        0.00         3     0.0  1.10000     0.0;
];
return;
```

E.4 RAW DATA FOR THE IEEE 14–BUS POWER SYSTEM

```
% Reactive power generation limits adjusted to produce an unconstrained
% optimal solution using runopf.  Andrew Ward 8/12/97
function [baseMVA, bus, gen, branch, area, gencost] = ieee14
% 09/25/93 UW ARCHIVE            100.0  1962 W IEEE 14 Bus Test Case
% The system data was originally in IEEE CDF,
% converted to matpower format by cdf2matp.
baseMVA = 100.0;
%bus type  Pd       Qd        Gs       Bs     area  Vm        Va        baseKV zone   Vmax    Vmin
bus = [
   1  3     0.00     0.00    0.000    0.000   1   1.06000    0.000     0.00  1     2.06000  0.94000;
   2  2    21.70    12.70    0.000    0.000   1   1.04500   -4.980     0.00  1     2.06000  0.94000;
   3  2    94.20    19.00    0.000    0.000   1   1.01000  -12.720     0.00  1     2.06000  0.94000;
   4  1    47.80    -3.90    0.000    0.000   1   1.01900  -10.330     0.00  1     2.06000  0.94000;
   5  1     7.60     1.60    0.000    0.000   1   1.02000   -8.780     0.00  1     2.06000  0.94000;
   6  2    11.20     7.50    0.000    0.000   1   1.07000  -14.220     0.00  1     2.06000  0.94000;
   7  1     0.00     0.00    0.000    0.000   1   1.06200  -13.370     0.00  1     2.06000  0.94000;
   8  2     0.00     0.00    0.000    0.000   1   1.09000  -13.360     0.00  1     2.06000  0.94000;
   9  1    29.50    16.60    0.000   19.000   1   1.05600  -14.940     0.00  1     2.06000  0.94000;
  10  1     9.00     5.80    0.000    0.000   1   1.05100  -15.100     0.00  1     2.06000  0.94000;
  11  1     3.50     1.80    0.000    0.000   1   1.05700  -14.790     0.00  1     2.06000  0.94000;
  12  1     6.10     1.60    0.000    0.000   1   1.05500  -15.070     0.00  1     2.06000  0.94000;
  13  1    13.50     5.80    0.000    0.000   1   1.05000  -15.160     0.00  1     2.06000  0.94000;
  14  1    14.90     5.00    0.000    0.000   1   1.03600  -16.040     0.00  1     2.06000  0.94000;
```

```
];
%bus  Pg       Qg       Qmax     Qmin    Vsp      base   status   Pmax     Pmin
gen = [
   1  232.40  -16.90   20.00   -20.00  1.06000   100.00   1      332.40     0.00;
   2   40.00   42.40   50.00   -40.00  1.04500   100.00   1      140.00     0.00;
   3    0.00   23.40   40.00   -10.00  1.01000   100.00   1      100.00     0.00;
   6    0.00   12.20   24.00    -6.00  1.07000   100.00   1      100.00     0.00;
   8    0.00   17.40   24.00    -6.00  1.09000   100.00   1      100.00     0.00;
];
%fbus tbus      r       x       b        ratea    rateb    ratec ratio    angle   status
branch = [
   1    2   0.01938  0.05917  0.05280  9900.00     0.00     0.00  0.00000   0.000    1;
   1    5   0.05403  0.22304  0.04920  9900.00     0.00     0.00  0.00000   0.000    1;
   2    3   0.04699  0.19797  0.04380  9900.00     0.00     0.00  0.00000   0.000    1;
   2    4   0.05811  0.17632  0.03740  9900.00     0.00     0.00  0.00000   0.000    1;
   2    5   0.05695  0.17388  0.03400  9900.00     0.00     0.00  0.00000   0.000    1;
   3    4   0.06701  0.17103  0.03460  9900.00     0.00     0.00  0.00000   0.000    1;
   4    5   0.01335  0.04211  0.01280  9900.00     0.00     0.00  0.00000   0.000    1;
   4    7   0.00000  0.20912  0.00000  9900.00     0.00     0.00  0.97800   0.000    1;
   4    9   0.00000  0.55618  0.00000  9900.00     0.00     0.00  0.96900   0.000    1;
   5    6   0.00000  0.25202  0.00000  9900.00     0.00     0.00  0.93200   0.000    1;
   6   11   0.09498  0.19890  0.00000  9900.00     0.00     0.00  0.00000   0.000    1;
   6   12   0.12291  0.25581  0.00000  9900.00     0.00     0.00  0.00000   0.000    1;
   6   13   0.06615  0.13027  0.00000  9900.00     0.00     0.00  0.00000   0.000    1;
   7    8   0.00000  0.17615  0.00000  9900.00     0.00     0.00  0.00000   0.000    1;
```

```
   7    9   0.00000  0.11001  0.00000  9900.00     0.00       0.00  0.00000     0.000      1;
   9   10   0.03181  0.08450  0.00000  9900.00     0.00       0.00  0.00000     0.000      1;
   9   14   0.12711  0.27038  0.00000  9900.00     0.00       0.00  0.00000     0.000      1;
  10   11   0.08205  0.19207  0.00000  9900.00     0.00       0.00  0.00000     0.000      1;
  12   13   0.22092  0.19988  0.00000  9900.00     0.00       0.00  0.00000     0.000      1;
  13   14   0.17093  0.34802  0.00000  9900.00     0.00       0.00  0.00000     0.000      1;
];
area = [
   1      1;
];
gencost = [
%Real
   2     0.00     0.00     3     0.0  10.7000     0.0;
   2     0.00     0.00     3     0.0  10.7000     0.0;
   2     0.00     0.00     3     0.0  11.0000     0.0;
   2     0.00     0.00     3     0.0  10.9000     0.0;
   2     0.00     0.00     3     0.0  11.1000     0.0;
%Reactive
   2     0.00     0.00     3     0.0  1.07000     0.0;
   2     0.00     0.00     3     0.0  1.07000     0.0;
   2     0.00     0.00     3     0.0  1.10000     0.0;
   2     0.00     0.00     3     0.0  1.09000     0.0;
   2     0.00     0.00     3     0.0  1.11000     0.0;
];
return;
```

E.5 RAW DATA FOR THE IEEE 30–BUS POWER SYSTEM

```
function [baseMVA, bus, gen, branch, area, gencost] = ieee30
% 09/25/93 UW ARCHIVE           100.0  1961 W IEEE 30 Bus Test Case
% The system data was originally in IEEE CDF,
% converted to matpower format by cdf2matp.

baseMVA = 100.0;
%bus type  Pd      Qd       Gs       Bs     area  Vm       Va       baseKV zone   Vmax    Vmin
bus = [
   1  3     0.00     0.00    0.000    0.000   1   1.06000    0.000   132.00  1    1.06000  0.94000;
   2  2    21.70    12.70    0.000    0.000   1   1.04300   -5.480   132.00  1    1.06000  0.94000;
   3  1     2.40     1.20    0.000    0.000   1   1.02100   -7.960   132.00  1    1.06000  0.94000;
   4  1     7.60     1.60    0.000    0.000   1   1.01200   -9.620   132.00  1    1.06000  0.94000;
   5  2    94.20    19.00    0.000    0.000   1   1.01000  -14.370   132.00  1    1.06000  0.94000;
   6  1     0.00     0.00    0.000    0.000   1   1.01000  -11.340   132.00  1    1.06000  0.94000;
   7  1    22.80    10.90    0.000    0.000   1   1.00200  -13.120   132.00  1    1.06000  0.94000;
   8  2    30.00    30.00    0.000    0.000   1   1.01000  -12.100   132.00  1    1.06000  0.94000;
   9  1     0.00     0.00    0.000    0.000   1   1.05100  -14.380     1.00  1    1.06000  0.94000;
  10  1     5.80     2.00    0.000   19.000   1   1.04500  -15.970    33.00  1    1.06000  0.94000;
  11  2     0.00     0.00    0.000    0.000   1   1.08200  -14.390    11.00  1    1.06000  0.94000;
  12  1    11.20     7.50    0.000    0.000   1   1.05700  -15.240    33.00  1    1.06000  0.94000;
  13  2     0.00     0.00    0.000    0.000   1   1.07100  -15.240    11.00  1    1.06000  0.94000;
  14  1     6.20     1.60    0.000    0.000   1   1.04200  -16.130    33.00  1    1.06000  0.94000;
  15  1     8.20     2.50    0.000    0.000   1   1.03800  -16.220    33.00  1    1.06000  0.94000;
```

```
 16  1     3.50    1.80   0.000   0.000  1  1.04500  -15.830   33.00  1   1.06000  0.94000;
 17  1     9.00    5.80   0.000   0.000  1  1.04000  -16.140   33.00  1   1.06000  0.94000;
 18  1     3.20    0.90   0.000   0.000  1  1.02800  -16.820   33.00  1   1.06000  0.94000;
 19  1     9.50    3.40   0.000   0.000  1  1.02600  -17.000   33.00  1   1.06000  0.94000;
 20  1     2.20    0.70   0.000   0.000  1  1.03000  -16.800   33.00  1   1.06000  0.94000;
 21  1    17.50   11.20   0.000   0.000  1  1.03300  -16.420   33.00  1   1.06000  0.94000;
 22  1     0.00    0.00   0.000   0.000  1  1.03300  -16.410   33.00  1   1.06000  0.94000;
 23  1     3.20    1.60   0.000   0.000  1  1.02700  -16.610   33.00  1   1.06000  0.94000;
 24  1     8.70    6.70   0.000   4.300  1  1.02100  -16.780   33.00  1   1.06000  0.94000;
 25  1     0.00    0.00   0.000   0.000  1  1.01700  -16.350   33.00  1   1.06000  0.94000;
 26  1     3.50    2.30   0.000   0.000  1  1.00000  -16.770   33.00  1   1.06000  0.94000;
 27  1     0.00    0.00   0.000   0.000  1  1.02300  -15.820   33.00  1   1.06000  0.94000;
 28  1     0.00    0.00   0.000   0.000  1  1.00700  -11.970  132.00  1   1.06000  0.94000;
 29  1     2.40    0.90   0.000   0.000  1  1.00300  -17.060   33.00  1   1.06000  0.94000;
 30  1    10.60    1.90   0.000   0.000  1  0.99200  -17.940   33.00  1   1.06000  0.94000;
];
%bus  Pg       Qg       Qmax     Qmin    Vsp       base   status   Pmax     Pmin
gen = [
   1  260.20   -16.10   10.00   -10.00  1.06000  100.00     1     360.20     0.00;
   2   40.00    50.00   50.00   -40.00  1.04500  100.00     1     140.00     0.00;
   5    0.00    37.00   40.00   -40.00  1.01000  100.00     1     100.00     0.00;
   8    0.00    37.30   40.00   -10.00  1.01000  100.00     1     100.00     0.00;
  11    0.00    16.20   24.00    -6.00  1.08200  100.00     1     100.00     0.00;
  13    0.00    10.60   24.00    -6.00  1.07100  100.00     1     100.00     0.00;
];
```

```
%fbus tbus     r      x      b         ratea   rateb   ratec ratio   angle  statu
branch = [
   1    2  0.01920  0.05750  0.05280  9900.00     0.00      0.00  0.00000   0.000     1;
   1    3  0.04520  0.18520  0.04080  9900.00     0.00      0.00  0.00000   0.000     1;
   2    4  0.05700  0.17370  0.03680  9900.00     0.00      0.00  0.00000   0.000     1;
   3    4  0.01320  0.03790  0.00840  9900.00     0.00      0.00  0.00000   0.000     1;
   2    5  0.04720  0.19830  0.04180  9900.00     0.00      0.00  0.00000   0.000     1;
   2    6  0.05810  0.17630  0.03740  9900.00     0.00      0.00  0.00000   0.000     1;
   4    6  0.01190  0.04140  0.00900  9900.00     0.00      0.00  0.00000   0.000     1;
   5    7  0.04600  0.11600  0.02040  9900.00     0.00      0.00  0.00000   0.000     1;
   6    7  0.02670  0.08200  0.01700  9900.00     0.00      0.00  0.00000   0.000     1;
   6    8  0.01200  0.04200  0.00900  9900.00     0.00      0.00  0.00000   0.000     1;
   6    9  0.00000  0.20800  0.00000  9900.00     0.00      0.00  0.97800   0.000     1;
   6   10  0.00000  0.55600  0.00000  9900.00     0.00      0.00  0.96900   0.000     1;
   9   11  0.00000  0.20800  0.00000  9900.00     0.00      0.00  0.00000   0.000     1;
   9   10  0.00000  0.11000  0.00000  9900.00     0.00      0.00  0.00000   0.000     1;
   4   12  0.00000  0.25600  0.00000  9900.00     0.00      0.00  0.93200   0.000     1;
  12   13  0.00000  0.14000  0.00000  9900.00     0.00      0.00  0.00000   0.000     1;
  12   14  0.12310  0.25590  0.00000  9900.00     0.00      0.00  0.00000   0.000     1;
  12   15  0.06620  0.13040  0.00000  9900.00     0.00      0.00  0.00000   0.000     1;
  12   16  0.09450  0.19870  0.00000  9900.00     0.00      0.00  0.00000   0.000     1;
  14   15  0.22100  0.19970  0.00000  9900.00     0.00      0.00  0.00000   0.000     1;
  16   17  0.08240  0.19230  0.00000  9900.00     0.00      0.00  0.00000   0.000     1;
  15   18  0.10730  0.21850  0.00000  9900.00     0.00      0.00  0.00000   0.000     1;
  18   19  0.06390  0.12920  0.00000  9900.00     0.00      0.00  0.00000   0.000     1;
```

```
   19   20   0.03400  0.06800  0.00000  9900.00    0.00     0.00  0.00000    0.000     1;
   10   20   0.09360  0.20900  0.00000  9900.00    0.00     0.00  0.00000    0.000     1;
   10   17   0.03240  0.08450  0.00000  9900.00    0.00     0.00  0.00000    0.000     1;
   10   21   0.03480  0.07490  0.00000  9900.00    0.00     0.00  0.00000    0.000     1;
   10   22   0.07270  0.14990  0.00000  9900.00    0.00     0.00  0.00000    0.000     1;
   21   22   0.01160  0.02360  0.00000  9900.00    0.00     0.00  0.00000    0.000     1;
   15   23   0.10000  0.20200  0.00000  9900.00    0.00     0.00  0.00000    0.000     1;
   22   24   0.11500  0.17900  0.00000  9900.00    0.00     0.00  0.00000    0.000     1;
   23   24   0.13200  0.27000  0.00000  9900.00    0.00     0.00  0.00000    0.000     1;
   24   25   0.18850  0.32920  0.00000  9900.00    0.00     0.00  0.00000    0.000     1;
   25   26   0.25440  0.38000  0.00000  9900.00    0.00     0.00  0.00000    0.000     1;
   25   27   0.10930  0.20870  0.00000  9900.00    0.00     0.00  0.00000    0.000     1;
   28   27   0.00000  0.39600  0.00000  9900.00    0.00     0.00  0.96800    0.000     1;
   27   29   0.21980  0.41530  0.00000  9900.00    0.00     0.00  0.00000    0.000     1;
   27   30   0.32020  0.60270  0.00000  9900.00    0.00     0.00  0.00000    0.000     1;
   29   30   0.23990  0.45330  0.00000  9900.00    0.00     0.00  0.00000    0.000     1;
    8   28   0.06360  0.20000  0.04280  9900.00    0.00     0.00  0.00000    0.000     1;
    6   28   0.01690  0.05990  0.01300  9900.00    0.00     0.00  0.00000    0.000     1;
];
area = [
    1      1;
];
gencost = [
%Real
   2     0.00         0.00         3      0.0  10.8000     0.0;
```

```
    2      0.00        0.00        3     0.0  10.6000    0.0;
    2      0.00        0.00        3     0.0  11.0000    0.0;
    2      0.00        0.00        3     0.0  10.9000    0.0;
    2      0.00        0.00        3     0.0  11.1000    0.0;
    2      0.00        0.00        3     0.0  10.7000    0.0;
%Reactive
    2      0.00        0.00        3     0.0  1.08000    0.0;
    2      0.00        0.00        3     0.0  1.06000    0.0;
    2      0.00        0.00        3     0.0  1.10000    0.0;
    2      0.00        0.00        3     0.0  1.09000    0.0;
    2      0.00        0.00        3     0.0  1.11000    0.0;
    2      0.00        0.00        3     0.0  1.07000    0.0;
]; return;
```

Appendix F

EXAMPLE OF MARGINAL PRICES

F.1 INTODUCTION

This appendix presents the outputs from QOPF and NODAL2, for the optimal dispatch of the IEEE 14 bus power system. This dispatch is typical of:

- the dispatches used to ensure that the NODAL2 source code is reliable,
- the dispatches used to develop the pq and PvG pricing models, and
- the dispatches used to investigate the behaviour of reactive power marginal prices.

These dispatches were only possible once QOPF and Matpower^{pq} had been developed.

The procedures used to obtain outputs from QOPF and NODAL2 are described. Then, outputs are presented for a specific dispatch.

F.2 THE QOPF OUTPUT

The dispatch of the 14–bus power system was optimised with QOPF. The power system data used by QOPF to obtain this dispatch are is presented, verbatim, in Appendix E.

In this dispatch, all generator voltage magnitudes were fixed at the levels specified in Appendix E. The generator voltages were fixed using the QOPF 'Standard, Polynomial Fixed Generator Voltage' option. This option produces a slightly different dispatch to that obtained by setting $V_i^{min} = V_i^{max}$ for all generator nodes. When this option is used QOPF does not display in the 'Voltage Constraints' section, the non–zero shadow prices associated with the fixed generator voltages.

The objective function minimised by QOPF is:

$$\begin{aligned}\text{Total Generation Cost} = {} & 10.7P_{G1} + 10.7P_{G2} + 11.0P_{G3} + 10.9P_{G6} + 11.1P_{G8} \\ & + 1.07Q_{G1} + 1.07Q_{G2} + 1.1Q_{G3} + 1.09Q_{G6} + 1.11Q_{G8}\end{aligned}$$

In the QOPF output (Section F.4) the marginal price at each generator node is equal to the unit generation cost of that generator. This indicates that all generators are marginal for real power and reactive power. This is also evident in the 'Generation Constraints' part of the output.

F.3 THE NODAL2 OUTPUT

NODAL2 was used to calculate *ex post* marginal price sets for the (observed) dispatch calculated by QOPF. This was achieved by running the dispatch through NODAL2, as illustrated in Figure 6.2.

Price sets are presented in Sections F.5 and F.6. They are generated by the pq and PvG pricing models respectively. Both models are implemented by the NODAL2 software. NODAL2 implements the pq pricing model when the real and reactive power generation data, from the QOPF output, are converted to negative demands when specified in BUSDATA.DAT. That is:

$$P_{Di} \Rightarrow -P_{Gi} \qquad \text{and} \qquad Q_{Di} \Rightarrow -Q_{Gi}$$

Otherwise, NODAL2 implements the PvG pricing model.

The marginal data and constraint commands used to obtain a price set from the pq pricing model output are presented in Table F.1. No voltage constraint has been specified for Node 1 because NODAL2 automatically applies a voltage constraint to the reference node. All generators have been formulated as marginal for reactive power, via the 'X' and 'Y' commands. Only Generator 8 has been formulated as marginal for real power, via the 'P' command.

Table F.1 MARGINAL.DAT commands required by Nodal2 to calculate marginal prices for the dispatch of the IEEE 14–bus system, using the pq pricing model.

S	Node 1	0.000	X	Node 6	1.090
P	Node 8	11.100	Y	Node 6	1.090
X	Node 1	1.070	X	Node 8	1.110
Y	Node 1	1.070	Y	Node 8	1.110
X	Node 2	1.070	V	Node 2	
Y	Node 2	1.070	V	Node 3	
X	Node 3	1.100	V	Node 6	
Y	Node 3	1.100	V	Node 8	

The marginal data and constraint commands used to obtain a price set from the PvG pricing model output are presented in Table F.2. Again, all generators have been formulated as marginal for reactive power and only Generator 8 has been formulated as marginal for real power. There are no voltage constraints because NODAL2 automatically fixes V at all PvG nodes.

Table F.2 MARGINAL.DAT commands required by Nodal2 to calculate marginal prices for the dispatch of the IEEE 14–bus system, using the PvG pricing model.

S	Node 1	0.000	X	Node 3	1.100
P	Node 8	11.100	Y	Node 3	1.100
X	Node 1	1.070	X	Node 6	1.090
Y	Node 1	1.070	Y	Node 6	1.090
X	Node 2	1.070	X	Node 8	1.110
Y	Node 2	1.070	Y	Node 8	1.110

F.4 AN OUTPUT FROM QOPF

Filename of the OPF case study: example.m

OPF run on: 5-Oct-98 at 13: 5:19

Dispatch Description: This is a voltage constrained example because all generator voltages are fixed. All generators marginal for real and reactive power.

Converged in 24.50 seconds

=== Objective Function Value ===

f = 2873.6690 $/hr

=============================== Bus Data ===================================

Bus	Voltage		Generation		Load		Lamda ($/MVA-hr)	
#	Mag(pu)	Ang(deg)	P (MW)	Q (MVAR)	P (MW)	Q (MVAR)	P	Q
1	1.0600	0.0000	28.5952	27.5917	-	-	10.7000	1.0700
2	1.0450	-0.0474	84.9285	-3.3396	21.7000	12.7000	10.7000	1.0700
3	1.0100	-2.1406	73.0356	-8.3524	94.2000	19.0000	11.0000	1.1000
4	1.0281	-2.6668	-	-	47.8000	-3.9000	11.1131	1.0727
5	1.0308	-1.9821	-	-	7.6000	1.6000	10.9919	1.1001
6	1.0700	-2.4043	49.1221	1.5796	11.2000	7.5000	10.9000	1.0900
7	1.0648	-3.0249	-	-	-	-	11.1868	1.1241
8	1.0900	-0.8010	25.5686	16.0797	-	-	11.1000	1.1100
9	1.0572	-4.6442	-	-	29.5000	16.6000	11.2826	1.1550

10	1.0516	-4.5374	-	-	9.0000	5.8000	11.2994	1.1926
11	1.0568	-3.6143	-	-	3.5000	1.8000	11.1495	1.1663
12	1.0555	-3.3637	-	-	6.1000	1.6000	11.1205	1.1655
13	1.0502	-3.5553	-	-	13.5000	5.8000	11.2109	1.2055
14	1.0360	-5.1721	-	-	14.9000	5.0000	11.5343	1.2763

============================ Branch Data ==================================

From	To	Avg. Flow		Loss		"From" End		"To" End	
Bus	Bus	P(MW)	Q(MVAR)	P(MW)	Q(MVAR)	P(MW)	Q(MVAR)	P(MW)	Q(MVAR)
1	2	9.29	23.60	0.113	-5.50	9.3470	20.8446	-9.2342	-26.3493
1	5	19.14	8.98	0.222	-4.46	19.2482	6.7471	-19.0265	-11.2100
2	3	22.52	12.74	0.299	-3.37	22.6656	11.0593	-22.3667	-14.4256
2	4	28.06	0.63	0.426	-2.72	28.2781	-0.7368	-27.8517	-1.9881
2	5	21.40	1.45	0.243	-2.92	21.5189	-0.0129	-21.2756	-2.9072
3	4	1.16	-11.24	0.083	-3.38	1.2023	-12.9268	-1.1195	9.5446
4	5	-29.19	2.79	0.108	-1.01	-29.1340	2.2798	29.2423	-3.2947
4	7	3.35	-6.85	0.000	0.11	3.3462	-6.7969	-3.3462	6.9055
4	9	6.96	0.74	0.000	0.24	6.9590	0.8606	-6.9590	-0.6177
5	6	3.46	15.54	0.000	0.54	3.4598	15.8119	-3.4598	-15.2721
6	11	12.51	1.06	0.132	0.28	12.5774	1.1961	-12.4450	-0.9188
6	12	8.36	2.01	0.080	0.17	8.3998	2.0972	-8.3194	-1.9298
6	13	20.27	5.80	0.262	0.52	20.4046	6.0584	-20.1428	-5.5429
7	8	-25.57	-15.40	0.000	1.35	-25.5686	-14.7271	25.5686	16.0797
7	9	28.91	7.39	0.000	0.87	28.9148	7.8217	-28.9148	-6.9511

9	10	0.12	6.84	0.013	0.04	0.1277	6.8557	-0.1143	-6.8202
9	14	6.21	5.27	0.077	0.16	6.2461	5.3472	-6.1692	-5.1836
10	11	-8.92	0.95	0.059	0.14	-8.8857	1.0202	8.9450	-0.8812
12	13	2.21	0.33	0.010	0.01	2.2194	0.3298	-2.2094	-0.3207
13	14	8.79	-0.06	0.121	0.25	8.8522	0.0636	-8.7308	0.1836

Total Losses = 2.249915 + j-18.706873

========== Voltage Constraints =========

Bus#	Vmin mu	Vmin	|V|	Vmax	Vmax mu
1	-	1.060	1.060	1.060	-
2	-	0.940	1.045	2.060	-
3	-	0.940	1.010	2.060	-
4	-	0.940	1.028	2.060	-
5	-	0.940	1.031	2.060	-
6	-	0.940	1.070	2.060	-
7	-	0.940	1.065	2.060	-
8	-	0.940	1.090	2.060	-
9	-	0.940	1.057	2.060	-
10	-	0.940	1.052	2.060	-
11	-	0.940	1.057	2.060	-
12	-	0.940	1.055	2.060	-
13	-	0.940	1.050	2.060	-
14	-	0.940	1.036	2.060	-

============ Generation Constraints ==========

Bus #	Pmin mu	Active Power Limits Pmin	P	Pmax	Pmax mu
1	-	0.00	28.60	332.40	-
2	-	0.00	84.93	140.00	-
3	-	0.00	73.04	100.00	-
6	-	0.00	49.12	100.00	-
8	-	0.00	25.57	100.00	-

Bus #	Qmin mu	Reactive Power Limits Qmin	Q	Qmax	Qmax mu
1	-	-20.00	27.59	40.00	-
2	-	-40.00	-3.34	50.00	-
3	-	-10.00	-8.35	40.00	-
6	-	-6.00	1.58	24.00	-
8	-	-6.00	16.08	24.00	-

================= Line Flow Constraints ==================

From Bus	"From" End |Sf| mu	|Sf|	Limit |Smax|	"To" End |St|	|St| mu	To Bus
1	-	22.84	9900.00	27.92	-	2
1	-	20.40	9900.00	22.08	-	5
2	-	25.22	9900.00	26.62	-	3
2	-	28.29	9900.00	27.92	-	4
2	-	21.52	9900.00	21.47	-	5
3	-	12.98	9900.00	9.61	-	4
4	-	29.22	9900.00	29.43	-	5
4	-	7.58	9900.00	7.67	-	7
4	-	7.01	9900.00	6.99	-	9
5	-	16.19	9900.00	15.66	-	6
6	-	12.63	9900.00	12.48	-	11
6	-	8.66	9900.00	8.54	-	12
6	-	21.29	9900.00	20.89	-	13
7	-	29.51	9900.00	30.20	-	8
7	-	29.95	9900.00	29.74	-	9
9	-	6.86	9900.00	6.82	-	10
9	-	8.22	9900.00	8.06	-	14
10	-	8.94	9900.00	8.99	-	11
12	-	2.24	9900.00	2.23	-	13
13	-	8.85	9900.00	8.73	-	14

F.5 AN OUTPUT FROM THE pq PRICING MODEL (NODAL2)

```
Trans Power Transmission System Pricing Model (full model), Compiled on Jun 26 1997, @10:53:37

5/10/1998 13:17:45: Price and Power Injection Data for, pq pricing model example

Transmission Rentals, Objective = MINIMISE, P at PV, P at PQ, P at S, Q at PV, Q at PQ, Q at S
Active Power Rentals =,  32.5   $
Active Power Payments =, -1679.7 $
Active Power Earnings =,  1712.1 $
Reactive Power Rentals =,  76.9   $
Reactive Power Payments =,  5.8    $
Reactive Power Earnings =,  71.1    $
Voltage Rentals =,  2.6    $
Voltage Payments =,  2.6    $
Voltage Earnings =,  0.0    $
Other Rentals =,  0.0    $
Other Payments =,  0.0    $
Other Earnings =,  0.0    $

Total Generation =,  0.000000 , MW,    0.000000 , MVAr
Losses =  2.249915  MW,   -39.940858 MVAr

Swing/Marginal bus =, G_1
LambdaP =,  10.7000  $/MW
```

```
LambdaQ =,  1.0700   $/MVar
Production Cost (Min & Max) for Active Power Known at:
     G_1,  0.000 ,  1000000000000000000.000
     G_8,  11.100,  11.100
Production Cost (Min & Max) for Reactive Power Known at:
     G_1,  1.070 ,  1.070
     G_2,  1.070 ,  1.070
     G_3,  1.100 ,  1.100
     G_6,  1.090 ,  1.090
     G_8,  1.110 ,  1.110
Voltage Congestion at Nodes:
     G_2, Cost =,  1.597 , has reached an UPPER constraint
     G_3, Cost =,  1.940 , has reached an UPPER constraint
     G_6, Cost =,  0.725 , has reached an UPPER constraint
     G_8, Cost =, -0.222 , has reached a LOWER constraint

Price Data for P,Q Buses Follows:

Bus,                 MW Inj,  MW Price,          Q, Volt Price, Reac Price,    Voltage
D_10            ,     -9.00,   11.2994,    -5.8000,     0.0000,     1.1926,     1.0516
D_11            ,     -3.50,   11.1495,    -1.8000,     0.0000,     1.1663,     1.0568
D_12            ,     -6.10,   11.1205,    -1,6000,     0.0000,     1.1655,     1.0555
D_13            ,    -13.50,   11.2109,    -5.8000,     0.0000,     1.2055,     1.0502
D_14            ,    -14.90,   11.5343,    -5.0000,     0.0000,     1.2763,     1.0360
D_4             ,    -47.80,   11.1131,     3.9000,     0.0000,     1.0727,     1.0281
```

```
D_5                ,      -7.60,    10.9919,    -1.6000,     0.0000,     1.1001,     1.0308
D_7                ,       0.00,    11.1868,     0.0000,     0.0000,     1.1241,     1.0648
D_9                ,     -29.50,    11.2826,   -16.6000,     0.0000,     1.1550,     1.0572
G_2                ,      63.23,    10.7000,   -16.0396,     1.5975,     1.0700,     1.0450
G_3                ,     -21.16,    11.0000,   -27.3524,     1.9402,     1.1000,     1.0100
G_6                ,      37.92,    10.9000,    -5.9204,     0.7247,     1.0900,     1.0700
G_8                ,      25.57,    11.1000,    16.0797,    -0.2217,     1.1100,     1.0900

Voltage, Price Data for Voltage Controlled Buses Follows:

Bus,                    MW Inj,   MW Price,          Q, Volt Price, Reac Price,    Voltage
G_1                ,      28.60,    10.7000,    27.5917,    -2.4212,     1.0700,     1.0600

The End
```

F.6 AN OUTPUT FROM THE PvG PRICING MODEL (NODAL2)

```
Trans Power Transmission System Pricing Model (full model), Compiled on Jun 26 1997, @10:53:37

5/10/1998 13:19:45: Price and Power Injection Data for, PvG pricing model example

Transmission Rentals, Objective = MINIMISE, P at PV, P at PQ, P at S, Q at PV, Q at PQ, Q at S
Active Power Rentals =,  30.9   $
Active Power Payments =, -1665.6 $
Active Power Earnings =,  1696.5 $
Reactive Power Rentals =,  40.9.  $
Reactive Power Payments =,  0.0    $
Reactive Power Earnings =,  40.9   $
Voltage Rentals =, -2.3    $
Voltage Payments =,  0.0    $
Voltage Earnings =, -2.2    $
Other Rentals =,  0.0    $
Other Payments =,  0.0    $
Other Earnings =,  0.0    $

Total Generation =,  134.149915, MW,   -5.640934 , MVAr
Losses =  2.249915  MW,   -39.940858 MVAr

Swing/Marginal bus =, G_1
LambdaP =,  10.6271  $/MW
```

```
LambdaQ =,  1.0700   $/MVar
Production Cost (Min & Max) for Active Power Known at:
     G_1,  0.000 ,  1000000000000000000000.000
     G_8,  11.000,  11.000
Production Cost (Min & Max) for Reactive Power Known at:
     G_1,  1.070 ,  1.070
     G_2,  1.070 ,  1.070
     G_3,  1.100 ,  1.100
     G_6,  1.090 ,  1.090
     G_8,  1.110 ,  1.110

Price Data for P,Q Buses Follows:

Bus,                  MW Inj,  MW Price,         Q, Volt Price, Reac Price,    Voltage
D_10           ,       -9.00,   11.1922,   -5.8000,     0.0000,     1.1903,     1.0516
D_11           ,       -3.50,   11.0380,   -1.8000,     0.0000,     1.1648,     1.0568
D_12           ,       -6.10,   11.0044,   -1.6000,     0.0000,     1.1647,     1.0555
D_13           ,      -13.50,   11.0950,   -5.8000,     0.0000,     1.2041,     1.0502
D_14           ,      -14.90,   11.4224,   -5.0000,     0.0000,     1.2736,     1.0360
D_4            ,      -47.80,   11.0190,    3.9000,     0.0000,     1.0735,     1.0281
D_5            ,       -7.60,   10.8998,   -1.6000,     0.0000,     1.1008,     1.0308
D_7            ,        0.00,   11.0852,    0.0000,     0.0000,     1.1230,     1.0648
D_9            ,      -29.50,   11.1780,  -16.6000,     0.0000,     1.1528,     1.0572
```

```
Voltage, Price Data for Voltage Controlled Buses Follows:

Bus,                 MW Inj,  MW Price,          Q, Volt Price, Reac Price,   Voltage
G_1               ,   28.60,   10.6271,    27.5917,    -2.4706,     1.0700,    1.0600
G_2               ,   63.23,   10.6186,   -16.0396,     1.4419,     1.0700,    1.0450
G_3               ,  -21.16,   10.9016,   -27.3524,     2.2143,     1.1000,    1.0100
G_6               ,   37.92,   10.7857,    -5.9204,     1.0815,     1.0900,    1.0700
G_8               ,   25.57,   11.0000,    16.0797,    -0.0157,     1.1100,    1.0900

The End
```

Appendix G

MERIT ORDER DISPATCH AND MULTIPLE MARGINAL GENERATORS

G.1 INTRODUCTION

The configuration of a power system, unit generation costs, real and reactive power generation capacity, and binding constraints all have an effect of the dispatch of reactive power. Economic theory dictates that reactive power must be dispatched according to a merit order in a reactive power market, in the absence of binding power system constraints (see Definition DBP 5.4 in Chapter 5). In this thesis however, the cases where QOPF has been used have generally resulted in 'out–of–merit–order dispatches'.

Out–of–merit–order dispatches cause multiple marginal generators for real and reactive power, even though the system appears to be unconstrained. Hence, the purpose of this appendix is to demonstrate that reactive power is dispatched according to a merit order. However, the reasons for the occurrence of multiple marginal generators for reactive power are also described.

Case studies are used to show that multiple marginal generators for reactive power occur when dispatching reactive power, if several constraints are binding (in particular, fixed generator voltages). Also, reactive power losses are shown to contribute to the occurrence of multiple marginal generators for real and reactive power. Before presenting these reactive power cases however, cases are presented to identify the primary cause of multiple marginal generators for real power.

All cases are based on the dispatch of the IEEE 14 bus power system. For each case, any changes made to the original data for this power system are specified.

G.2 MULTIPLE MARGINAL GENERATORS FOR REAL POWER

Read and Ring [1995b] clearly demonstrated that real power is dispatched according to a merit order, in the absence of binding constraints. Ring [1995] however, commented that losses can cause an out–of–merit–order dispatch, resulting in multiple marginal generators for real power. The purpose of Cases 1 and 2 is to verify this comment.

For these cases, the IEEE 14–bus power system is used. When dispatching this power system, reactive power has a zero unit generation cost. This ensures that the real power dispatch is not affected by reactive power costs. For both cases, the generator voltage magnitudes were fixed when using QOPF to determine optimal dispatches. Figure G.1 shows the real and reactive power generation from each generator, for both cases.

Case 1

Case 1 is used as a reference case for Case 2. Case 1 is an example of an unconstrained dispatch where multiple marginal generators of real power have occurred. The following lower and upper reactive power generation limits in the original IEEE 14–bus power system were relaxed sufficiently, to prevent the generators from becoming non–marginal for reactive power:

- $Q^{min}_{G1,2,3,6,8} = -400\,\text{MVAr}$
- $Q^{max}_{G1} = 200\,\text{MVAr}$

The real power generation limits did not need to be changed.

The unit generation costs of real and reactive power generation for each generator are:

$$
\begin{aligned}
&c_{P1} = \$10.7/\text{MW},\ c_{P2} = \$10.8/\text{MW},\ c_{P3} = \$11.0/\text{MW}\\
&c_{P6} = \$10.9/\text{MW},\ c_{P8} = \$11.1/\text{MW}\\
&c_{Q1,2,3,6,8} = \$0/\text{MVAr}
\end{aligned}
$$

These unit generation costs form the following objective function, used by QOPF:

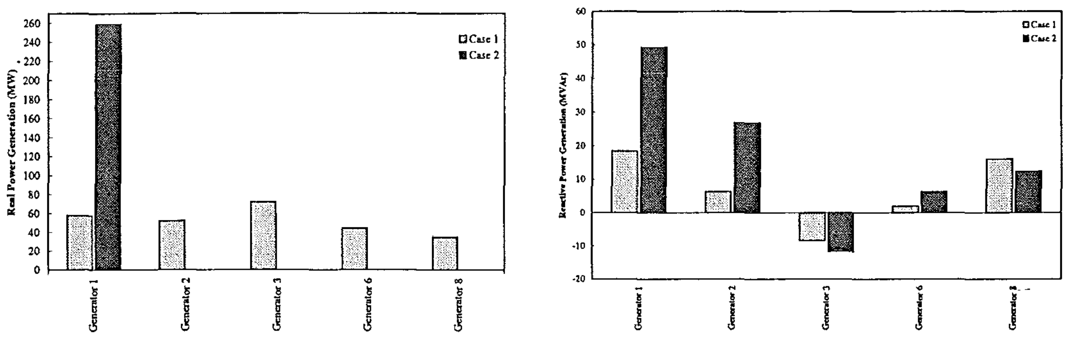
$$
f\,(cost)_{P,Q} = c_{P1}\,P_{G1} + c_{P2}\,P_{G2} + c_{P3}\,P_{G3} + c_{P6}\,P_{G6} + c_{P8}\,P_{G8} + c_{Q1,2,3,6,8}\,Q_{G1,2,3,6,8} \tag{G.1}
$$

$$
= 10.7\,P_{G1} + 10.8\,P_{G2} + 11.0\,P_{G3} + 10.9\,P_{G6} + 11.1\,P_{G8} + 0\,Q_{G1,2,3,6,8} \tag{G.2}
$$

This case resulted in an unconstrained dispatch of the power system (aside from the fixed generator voltages). However, all generators have been made marginal in order to supply the real power loads of the power system.

Case 2

This case demonstrates that resistance, and hence real power losses, are a major factor in the occurrence of multiple marginal generators for real power. The real power



(a) MW Generation (b) MVAr Generation

Figure G.1 Optimal dispatch of real power, with and without branch series resistances. Reactive power has a zero unit generation cost.

losses are removed by setting to zero the series resistances of all the power system branches. Hence, the changes made to the 14–bus power system are:

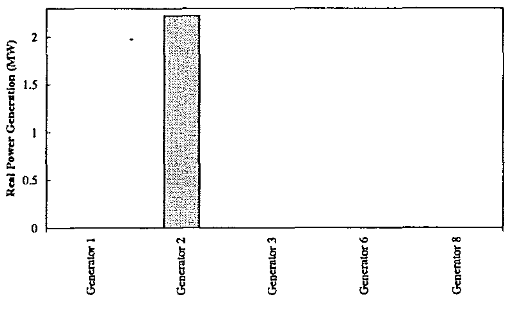
- $Q^{min}_{G1,2,3,6,8} = -400$ MVAr
- $Q^{max}_{G1} = 200$ MVAr
- $R_k = 0$ pu $\quad \forall k \in K$

R_k is the series resistance of Branch k, and K is the set of all branches in the power system. The marginal unit costs for this case are the same as that of Case 1 (i.e. Equation G.1 still applies). Case 2 has resulted in an unconstrained dispatch, apart from the fixed generator voltages. Figure G.1(a) shows that Generator 1 is the only marginal generator for Case 2.

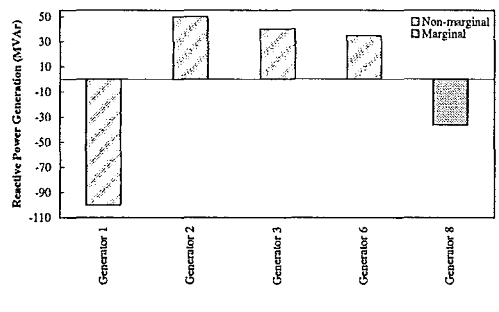
Discussion

With respect to definition DBP 5.6, the five fixed generator voltage constraints are explained by all five generators being marginal for reactive power and Generator 1 being marginal for real power; this applies to both cases. In Case 1 however, Generators 2, 3, 6 and 8 are also marginal for real power, but with no extra constraints to explain why they are marginal.

Figure G.1(a) reveals that a merit order dispatch for real power is the outcome (i.e. Generators 2, 3, 6 and 8 become non–marginal) when the real power losses are removed in Case 2. Therefore, when determining an optimal dispatch of real power, real power losses can act as implicit constraints through the unit generation costs in the



(a) MW Generation



(b) MVAr Generation

Figure G.2 A merit order dispatch of reactive power. Generator voltages are variable. Generators 1, 2, 3 and 6 are non–marginal for reactive power.

objective function. These implicit loss constraints economically justify the occurrence of the real power marginal generators: 2, 3, 6 and 8.

The fact that the generator voltages are fixed for both cases indicates that generator voltage constraints have no influence in causing multiple marginal generators for real power. This was confirmed by rerunning Case 1 with variable generator voltages (V_1 was fixed to 1.06 pu as a reference voltage to prevent voltage drift). In this new unconstrained dispatch, Generators 2, 3, 6 and 8 were still marginal for real power.

The multiple marginal generators for reactive power can be explained by the conclusions of Cases 3 to 5. These multiple marginal generators for reactive power are the result of the fixed generator voltages. When the generator voltages are allowed to vary, the multiple marginal generators for reactive power disappear.

G.3 MERIT ORDER DISPATCH OF REACTIVE POWER

In the pq pricing model, Equation 7.14 and Equation 7.15 have equivalent formats. That is, reactive power marginal prices and real power marginal prices have the same structure in a pq–type market. This infers a pq–type OPF will dispatch reactive power in the same way as real power. That is, according to a merit order dispatch. In this section, Case 3 is used to validate this inference. The real and reactive power generator outputs of the Case 3 dispatch are depicted in Figure G.2.

<u>Case 3</u>

The following changes have been made to the IEEE 14–bus power system:

- $V_1^{min} = V_1^{max} = 1.06$ pu (used as a reference voltage for the power system)

- Set $P_D = 0$ and $Q_D = 0$ for all nodes, except for setting a new reactive power load at Node 13: $Q_{D13} = 35.8\,\text{MVAr}$
- $Q^{min}_{G1,2,3,6,8} = -100\,\text{MVAr}$
- $Q^{max}_{G1} = 50\,\text{MVAr}$
- $Q^{max}_{G6} = 35\,\text{MVAr}$

The values chosen for the generation limits are arbitrary, They have been made the same as the changes made in Cases 4 to 8. This enables the dispatch of this case and the dispatches of subsequent cases to be compared. The negative lower generation limits (i.e. -100 MVAr) have been used so that generators forced against their lower limits are easily identified. This however, produces as unrealistic dispatch. Figure G.2(b) shows that the companies owning Generators 1 and 8 are paying the system to absorb reactive power. Generally, the generating companies would increase their Q^{min}_G to 0 MVAr, in order to avoid paying this cost.

All real and reactive power loads have been replaced by one reactive power load at Node 13. Having only one load enables the behaviour of the optimal reactive power dispatch to be observed more clearly.

This modified 14–bus power system has been dispatched with variable generator voltages. The dispatch has been optimised with respect to the following unit generation costs for real and reactive power (ref. Equation G.1):

$$c_{P1,2,3,6,8} = \$10.7/\text{MW},\ \$10.7/\text{MW},\ \$11.0/\text{MW},\ \$10.9/\text{MW},\ \$11.1/\text{MW}$$
$$c_{Q1,2,3,6,8} = \$5.10/\text{MVAr},\ \$1.07/\text{MVAr},\ \$2.10/\text{MVAr},\ \$1.09/\text{MVAr},\ \$4.10/\text{MVAr}$$

A merit order dispatch favours cheap generators over expensive generators. Hence, Generators 2, 3 and 6 have been made cheaper than Generators 1 and 8, with respect to reactive power. All generators have different reactive power unit generation costs, so as to clearly demonstrate a merit order dispatch.

Figure G.2(a) depicts Generator 3 as being marginal for real power; the other generators have not been dispatched. Generator 8 is marginal for reactive power. The hatched bars indicate generators that are non–marginal for reactive power. Generator 1 has been forced against its lower generation limit (i.e. $Q^{min}_{G1} = -100\,\text{MVAr}$). Generators 2, 3 and 6 have been forced against their upper generation limits (i.e. $Q^{max}_{G2} = 50\,\text{MVAr}$, $Q^{max}_{G3} = 40\,\text{MVAr}$, $Q^{max}_{G6} = 35\,\text{MVAr}$).

Discussion

In this case, the three cheapest generators of reactive power have been dispatched before the two expensive generators. QOPF has used all the reactive power from these three cheap generators, causing them to be fully loaded. Generator 8 is the next least expensive generator. It has been used to supply the balance of the power system's reactive power requirements, which cannot be supplied by the first three generators. This generator is not fully loaded. This means it is marginal for reactive power.

Generator 1 is more expensive than Generator 8. It has therefore been forced against its lower generation limit because Generator 8 is still able to supply any marginal increase in reactive power demand.

To conclude, a merit order dispatch of reactive power has occurred, as defined by definition DBP 5.4. The generators have been dispatched in the merit order defined by their reactive power unit generation costs. That is, the cheapest generators have been generated first (Generators 2, 3 and 6) and the most expensive last (Generator 1).

There is a point of clarification. A merit order dispatch allows only one marginal generator at any point in time. But, there are two marginal generators in this dispatch. These are, Generator 3 for real power and Generator 8 for reactive power. The 'Marginal Price Criterion' (i.e. DBP 5.6) can be used to justify this observation. There is one binding constraint, which is the fixed generator voltage at Node 1. Accordingly, DBP 5.6 requires that two generators be marginal at any point in time. These are Generators 3 and 8.

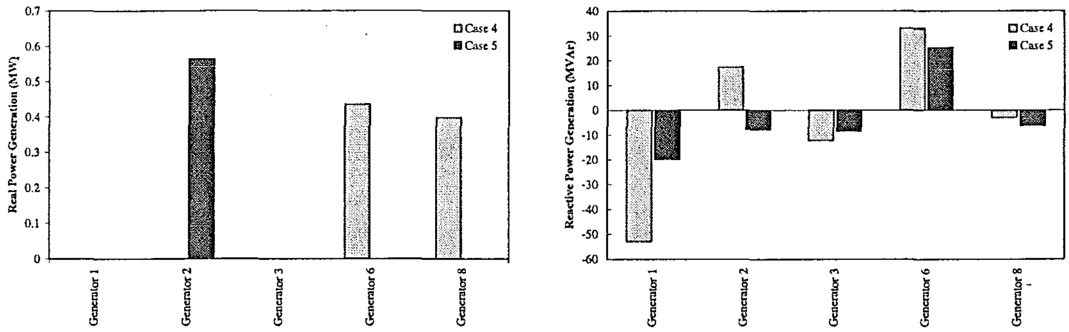
G.4 MULTIPLE MARGINAL GENERATORS FOR REACTIVE POWER - MARGINAL LOSSES

The extreme reactive power unit generation costs of Case 3 were contrived. They were used to demonstrate that reactive power is dispatched according to a merit order. In an actual market, competition can be expected to force the reactive power unit generation costs throughout a power system to be moderate and comparable with each other. In this section, two cases are used to shown that a merit order dispatch does not usually occur when reactive power marginal prices are comparable with each other. They show instead, that multiple marginal generators become marginal for reactive power. That is, an out-of-merit-order dispatch occurs.

Case 4 is contrived, because real power is assigned a zero unit generation cost. It demonstrates that marginal losses are one cause of these multiple marginal generators for reactive power, when optimally dispatching the power system. Case 5 is used to show that the results of Case 4 apply to more realistic dispatches.

The following changes have been made to the IEEE 14-bus power system, for both cases:

- $V_1^{min} = V_1^{max} = 1.06\,\text{pu}$
- Set $P_D = 0$ and $Q_D = 0$ for all nodes, except $Q_{D13} = 35.8\,\text{MVAr}$
- $Q_{G1,2,3,6,8}^{min} = -100\,\text{MVAr}$



(a) MW Generation (b) MVAr Generation

Figure G.3 Marginal reactive power losses cause multiple marginal generators for reactive power. Real power has a zero unit generation cost in Case 4.

- $Q_{G1}^{max} = 50\,\text{MVAr}$
- $Q_{G6}^{max} = 35\,\text{MVAr}$

The changes in the reactive power generation limits are arbitrary. The changes just ensure that all generators are marginal for reactive power.

This modified 14–bus power system is dispatched with variable generator voltages in each case. The only differences between the two dispatches are the unit generation costs. These are specified with each case. The real and reactive power generator outputs of the Case 4 and Case 5 dispatches are depicted in Figure G.3.

Case 4

The unit generation costs of real and reactive power for this case are:

$$c_{P1,2,3,6,8} = \$0/\text{MW}$$
$$c_{Q1,2,3,6,8} = \$1.07/\text{MVAr},\ \$1.07/\text{MVAr},\ \$1.10/\text{MVAr},\ \$1.09/\text{MVAr},\ \$1.11/\text{MVAr}$$

The unit cost of real power is the same for all generators, that is \$0/MW. This has been done to ensure that real power prices do not influence the dispatch of reactive power through the cost of real power losses.

The resultant optimal dispatch only has one binding constraint. This is the fixed reference voltage at Node 1. Comparing the generator limit data with Figure G.3 shows that only Generators 6 and 8 are marginal for real power. All generators are marginal for reactive power.

Case 5

The unit generation costs for this case are the unit generation costs specified in Section E.4:

$$c_{P1,2,3,6,8} = \$10.7/\text{MW},\ \$10.7/\text{MW},\ \$11.0/\text{MW},\ \$10.9/\text{MW},\ \$11.1/\text{MW}$$
$$c_{Q1,2,3,6,8} = \$1.07/\text{MVAr},\ \$1.07/\text{MVAr},\ \$1.10/\text{MVAr},\ \$1.09/\text{MVAr},\ \$1.11/\text{MVAr}$$

They represent unit generation costs typical of a competitive market. Hence, these unit generation costs are moderate and comparable with each other.

Compare the specified generation limit data with the generator outputs (depicted in Figure G.3). It is evident that only Generator 2 is marginal for real power, and that all generators are marginal for reactive power.

Discussion

In both cases, the fixed generator voltage at Node 1 is the only binding constraint. Thus, definition DBP 5.6 is violated in both cases. In Case 4 for example, DBP 5.6 states there should be six binding constraints to explain the seven marginal generators. Or, there should be only two marginal generators because there is only on binding constraint.

QOPF minimises the total cost of real and reactive power generation. In Case 4, Equation G.1 becomes:

$$f\left(cost\right)_{P,Q} = 0P_{Gi} + c_{Qi}Q_{Gi} \qquad \forall i \in \text{PQG}$$

This objective function illustrates that QOPF is minimising just the cost of reactive power generation. QOPF optimises the real and reactive power losses to help minimise the reactive power generation.

Duality theory shows that only marginal losses for reactive power and the voltage constraint have an associated cost component in the marginal price of reactive power demand (ref. Equation 7.15). The real power marginal price component disappears because the unit generation cost (λ_P) is zero:

$$\beta_{Q13} = -\lambda_P \frac{\partial L_P}{\partial Q_{13}} + \lambda_Q \left(1 - \frac{\partial L_Q}{\partial Q_{13}}\right) - \mu_{V1} \frac{\partial V_1}{\partial Q_{13}} \tag{G.3}$$
$$= -0\frac{\partial L_P}{\partial Q_{13}} + \lambda_Q - \lambda_Q \frac{\partial L_Q}{\partial Q_{13}} - \mu_{V1} \frac{\partial V_1}{\partial Q_{13}} \tag{G.4}$$

DBP 5.6 states that the voltage constraint term (μ_{V1}) only accounts for two of the seven marginal generators. Hence, $\lambda_Q \frac{\partial L_Q}{\partial Q_{13}}$ is the only cost component available to economically justify the other five marginal generators. In other words, QOPF optimises the marginal reactive power losses (i.e. $\frac{\partial L_Q}{\partial Q_{13}}$) to ensure that the reactive power unit generation costs of these five generators are consistent with each other (ref. DBP 5.1).

The losses are optimised in such a way that the reactive power losses are minimised. Once optimised, the marginal real and reactive power losses cannot be optimised further to further minimise the reactive power generation costs. Therefore, the marginal reactive power losses are acting as implicit constraints. They explain the five extra marginal generators, which are unaccounted for by DBP 5.1.

The marginal real power losses (with respect to a change in reactive power demand) are only constraints, in that QOPF optimises them to help minimise the cost of reactive power generation.

Case 5 demonstrates a more typical scenario. In this case, multiple generators have still been dispatched for reactive power. However, real power now has a unit generation cost. This is the only significant difference between Cases 4 and 5. This difference is evident in the changes in the generator outputs from Case 4 to Case 5 (see Figure G.3).

The changes in the generator outputs have occurred because the real power marginal losses are now acting as implicit constraints, in the same way that reactive power marginal losses are already acting as implicit constraints. This is because, λ_p is no longer zero in Equation G.3 (i.e. $\lambda_p \neq 0$). Thus, the real power loss constraints and the reactive power loss constraints have combined to explain the occurrence of multiple marginal generators for reactive power.

Both cases only have a single reactive power load at Node 13. Actual power systems however, have numerous real and reactive power loads. These extra loads only increase the likelihood of multiple marginal generators for reactive power. This is because, using moderately expensive local generation to supply changes in a reactive power load can result in a lower total generation cost that using less expensive remote generation to supply those load changes. This can be demonstrated by dispatching with variable generator voltages, the unmodified 14–bus power system presented in Section E.4.

When there are numerous loads, the optimal solution found by QOPF may be to use multiple reactive power generators. This is because the power system may become unstable if a single marginal generator is used to supply all changes in the numerous remote and local loads. In this scenario, consumers will be forced to use local generation, regardless of cost, in order to maintain their desired load profile. This means generating companies can set high reactive power unit generation costs and the consumers will be forced to use this expensive reactive power.

G.5 MULTIPLE MARGINAL GENERATORS FOR REACTIVE POWER - BINDING CONSTRAINTS

Marginal losses are not the only causes of multiple marginal generators (and an out–of–merit–order dispatch) for reactive power. Binding constraints also cause multiple marginal generators for reactive power. For example, the fixed generator voltage in the dispatch of Case 3 caused two generators to be marginal for reactive power. This section demonstrates that binding constraints and marginal loss constraints work together to cause multiple marginal generators for reactive power.

When a power–flow problem is being solved, the voltages of all the generator nodes are always fixed. Following this procedure, fixed generator voltages are used as the binding constraints for the dispatch cases presented in this section.

Case 6 is just Case 3 dispatched with fixed generator voltages. The generator voltages are fixed to the values specified in Section E.4 (refer to the "Generator Voltage Magnitude Setpoint" generator data field). The modifications made to the IEEE 14–bus power system for this case, are specified in Section G.3.

The following changes have been made to the IEEE 14–bus power system for Cases 7 to 9:

- $V_1^{min} = V_1^{max} = 1.06\,\text{pu}$
- None of the loads have been changed or removed
- $Q_{G1,2,3,6,8}^{min} = -100\,\text{MVAr}$
- $Q_{G1}^{max} = 50\,\text{MVAr}$
- $Q_{G6}^{max} = 35\,\text{MVAr}$

The only differences between the cases are the unit generation costs and the fixed or variable generator voltages. Increasing Q_{G1}^{max} to 50 MVAr allows the fixed generator voltages of dispatches of Cases 6, 8 and 9 to converge.

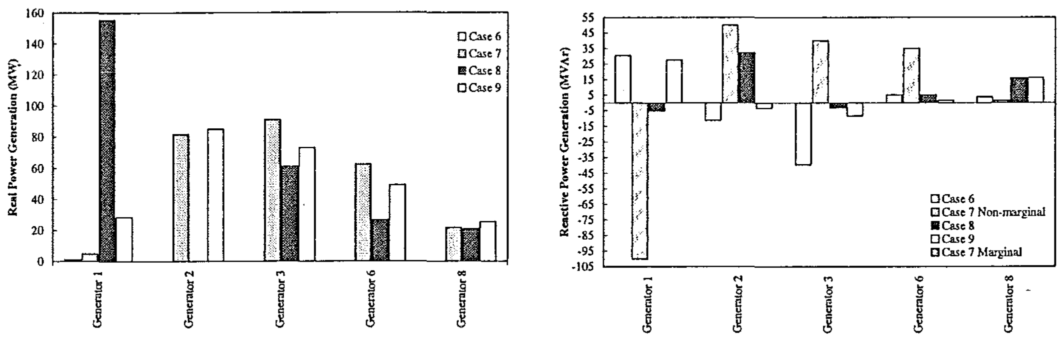
The real and reactive power generator outputs of the Case 6 through Case 9 dispatches are depicted in Figure G.4.

Case 6

Case 6 shows the effect of numerous binding constraints when there is only one reactive power load. This dispatch is the fixed generator voltage equivalent of Case 3 and the unloaded equivalent of Case 8. The unit generation costs for this dispatch are the same as those of the Case 3 dispatch:

$$c_{P1,2,3,6,8} = \$10.7/\text{MW},\ \$10.7/\text{MW},\ \$11.0/\text{MW},\ \$10.9/\text{MW},\ \$11.1/\text{MW}$$

$$c_{Qi,2,3,6,8} = \$5.10/\text{MVAr},\ \$1.07/\text{MVAr},\ \$2.10/\text{MVAr},\ \$1.09/\text{MVAr},\ \$4.10/\text{MVAr}$$



(a) MW Generation

(b) MVAr Generation

Figure G.4 Binding constraints and marginal reactive power losses work together to cause multiple marginal generators for reactive power.

In this fixed generator voltage dispatch, Generator 1 is marginal for real power and all generators are marginal for reactive power (see Figure G.4).

Case 7

This case is a loaded equivalent of Case 3. That is, none of the real and reactive power loads specified in Section E.4 have been changed or removed. The unit generation costs for this case are:

$$c_{P1,2,3,6,8} = \$10.7/\text{MW},\ \$10.7/\text{MW},\ \$11.0/\text{MW},\ \$10.9/\text{MW},\ \$11.1/\text{MW}$$
$$c_{Q1,2,3,6,8} = \$5.10/\text{MVAr},\ \$1.07/\text{MVAr},\ \$2.10/\text{MVAr},\ \$1.09/\text{MVAr},\ \$4.10/\text{MVAr}$$

The generator voltages are allowed to vary in this dispatch, except the reference voltage at Node 1. This case acts as a reference dispatch for Case 8. It enables the effect of fixing voltages to be observed.

Figure G.4 identifies all generators as being marginal for real power and Generator 8 as being marginal for reactive power. The hatched bars indicate non-marginal generators. The three cheapest generators have been forced against their upper reactive power generation limits. The most expensive Generator 1 has been forced against its lower generation limit.

Case 8

This case is used in conjunction with Case 7. They demonstrate that numerous binding constraints cause multiple marginal generators for reactive power. Like Case 7, this case is a loaded equivalent of Case 3. However, the generator voltages are fixed for this dispatch. Hence, this case is also the loaded equivalent of Case 6.

The unit generation costs for this dispatch are:

$$c_{P1,2,3,6,8} = \$10.7/\text{MW},\ \$10.7/\text{MW},\ \$11.0/\text{MW},\ \$10.9/\text{MW},\ \$11.1/\text{MW}$$
$$c_{Q1,2,3,6,8} = \$5.10/\text{MVAr},\ \$1.07/\text{MVAr},\ \$2.10/\text{MVAr},\ \$1.09/\text{MVAr},\ \$4.10/\text{MVAr}$$

In this dispatch, Generators 1, 3, 6 and 8 are marginal for real power. All generators are marginal for reactive power.

Case 9

In this case, the modified 14–bus power system has been dispatched with fixed generator voltages. The objective of this case is to demonstrate the effect on the dispatch of changing the reactive power unit generation costs, when numerous constraints are binding. The unit generation costs for this dispatch are:

$$c_{P1,2,3,6,8} = \$10.7/\text{MW},\ \$10.7/\text{MW},\ \$11.0/\text{MW},\ \$10.9/\text{MW},\ \$11.1/\text{MW}$$
$$c_{Q1,2,3,6,8} = \$1.07/\text{MVAr},\ \$1.07/\text{MVAr},\ \$1.10/\text{MVAr},\ \$1.09/\text{MVAr},\ \$1.11/\text{MVAr}$$

These are the unit generation costs specified in Section E.4.

This case represents a typical power system dispatch. This is because there are numerous real and reactive power loads in the power system, and because the unit generation costs are comparable (see Section G.4).

Figure G.4 identifies all generators as being marginal for real power and all generators are being marginal for reactive power.

Discussion

In the Case 6 dispatch there are five binding constraints, one for each of the fixed generator voltages. By the 'Marginal Price Criterion' (i.e. DBP 5.6) only six marginal generators are allowed to be marginal for real and/or reactive power. Therefore, the dispatch of Case 6 satisfies DBP 5.6 because all five generators are marginal for reactive power and Generator 1 is also marginal real power.

An inspection of Figure G.4(b) reveals that more reactive power is being injected into the power system by the very expensive Generator 1 (at \$5.10/MVAr) than by the cheap Generator 2 (at \$1.07/MVAr). In fact, Generator 2 is drawing reactive power out of the power system. This preference for expensive generation can also be observed in the dispatches of Cases 8 and 9.

The preference demonstrates that QOPF has determined this to be the most optimal way of dispatching reactive power, so as to maintain the fixed generator voltage profile. Alternatively written, QOPF cannot further minimise the costs of real power generation and reactive power generation without changing the generator voltage profile. Therefore, the fixed generator voltages have forced an out-of-merit-order dispatch. That is, QOPF has been forced to make multiple generators, marginal for reactive power.

When preparing the case studies for this thesis it has been observed that fixing the generator voltages generally results in all generators being marginal for reactive power. This can be observed in Case 6. It is possible this observation is the result of the strong relationship between 'voltage magnitude' and 'reactive power' and a high X/R ratio (see Section 3.6). This needs to be proved rigorously. However, Cases 1 and 2 provide some credibility for this observation. Fixing generator voltages does not cause multiple marginal generators for real power in those cases. This emphasises the weak relationship between 'real power' and 'voltage magnitude'. This relationship is only weak for high X/R ratios.

In Case 3, the variable generator voltage dispatch satisfied DBP 5.6. Likewise, the fixed generator voltage dispatch of Case 6 satisfied DBP 5.6. For Case 7, DBP 5.6 states there should only be two marginal generators because only the Node 1 voltage is fixed. However, there are six marginal generators. Similarly for Case 8, there are nine marginal generators instead of the six generators stated by DBP 5.6.

The only difference between Case 3 and Case 7, and between Case 6 and Case 8 is that Cases 7 and 8 have numerous loads. QOPF has found it necessary to violate DBP 5.6 by making four extra generators marginal for Case 7 and three extra for Case 8. These extra marginal generators are required by QOPF to supply the numerous demands, while maintaining the fixed generator voltage profiles. Hence, real and reactive power losses are acting as constraints. They work with the binding voltage constraints to produce more marginal generators than accounted for by DBP 5.6.

In summary, the total number of marginal generators will always be at least one greater than the number of binding constraints (ref. DBP 5.6). However, as the number of binding constraints and/or loads increase, marginal losses also begin to act as constraints. When this occurs, QOPF will make the number of marginal generators greater than, the number of binding constraints plus one. That is, the only way for QOPF to work around all the binding constraints while supplying marginal changes in the load profile is to make more generators marginal for real or reactive power. It must be noted that it is not possible to distinguish between those generators marginal due to binding constraints and those generators marginal due to losses.

Case 9 demonstrates a realistic optimal dispatch because the reactive power unit generation costs are comparable with each other. In this dispatch all of the generators are marginal for both real and reactive power. Therefore, this dispatch demonstrates that realistic dispatches are even more likely to result in multiple marginal generators for real and reactive power than the contrived cases: 6, 7 and 8.

G.6 CONCLUSIONS

Case 3 has been used to demonstrate that reactive power is dispatched according to a merit order. Both the reactive power dispatch and the real power dispatch are determined by the unit generation costs of the generators. However, it has been demonstrated that reactive power is generally dispatched out of merit order in realistic pq–type spot markets (modelled by QOPF). The evidence of an out–of–merit–order dispatch for reactive power is multiple marginal generators for reactive power.

Two main causes of out–of–merit–order dispatches for reactive power have been identified: binding constraints and implicit loss constraints. Binding constraints cause multiple marginal generators for reactive power, but the 'Marginal Price Criterion' (i.e. DBP 5.6) is satisfied. Marginal real and reactive power losses act as implicit constraints to cause multiple marginal generators for reactive power. However, the number of resultant marginal generators will violate DBP 5.6. Implicit marginal loss constraints have been demonstrated to coincide with numerous binding constraints and/or numerous real and reactive power loads.

Cases 1 and 2 demonstrated that multiple marginal generators for real power are primarily the result of implicit real power loss constraints. Fixed generator voltages do not cause multiple marginal generators for real power, subject to a high X/R ratio.

Appendix H

TECHNICAL PAPERS

The following technical review has been published.

A. G. Ward (1996) 'Nodal Pricing In Power Systems', *In Post Graduate Research for New Zealand Industry*, Third New Zealand Conference of Postgraduate Students in Engineering and Technology, *July*, pp. 331–332.

The following technical papers are associated with the research of this thesis. They have been submitted for publication.

A. G. Ward, A. J. Turner, N. R. Watson, B. J. Ring and C. P. Arnold. Inversion of Real Time Spot Prices in the Direction of Real Power Flow. *Accepted to IEEE PES, subject to mandatory changes. These changes are yet to be implemented.*, 1998.

A. G. Ward, C. P. Arnold and N. R. Watson. Reactive Power Generation Costs and their Effect on Real and Reactive Power Spot Prices. *Submitted to IPEC99 conference.*, May 1999, Awaiting acceptance.

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